Name: ______________
Group members: ____________________ ____________________ ____________________

TAM 210/211 - Worksheet 12

Composite Bodies

Introduction

The handling and design of many objects in everyday life rely on the center of gravity. For example, holding a heavy stack of books is usually much easier when the books are closer to your body, and thus closer to your center of gravity. This applies things that may appear simple at first, such as restaurant signs that need to hang without breaking or falling as shown in Figure 1.

Figure 1: Example sign for Jimmy John's

The Problem

Sofia would like to follow her dream of opening a pizza restaurant. She wants everything to be perfect on opening day, including the design of her outdoor sign. Before Sofia hangs her sign outside, she would like to know if the supports will be able to hold the sign without breaking.

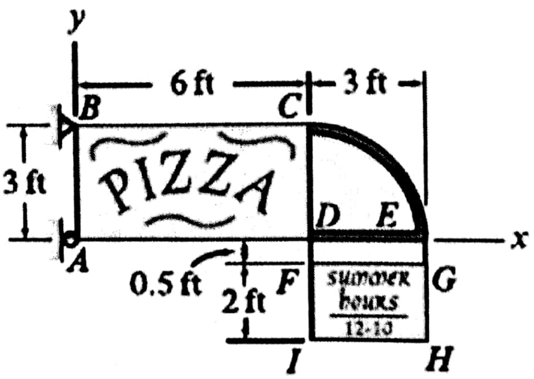


Figure 2: Sofia's almost-perfect sign

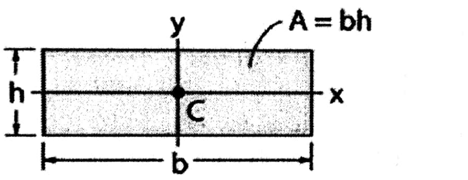
- Sofia sent a 2D drawing and the dimensions of her outdoor sign, shown below in Figure 2. The sign is constructed of a plywood sheets $ABCD$ and $FGHI$, steel bar CED, and steel chains DF and EG.
- The weight of the plywood is 2 lb/ft^2 and the weight of the steel bar is 5 lb/ft. The weight of the chains is negligible.
- Supports A and B will fail at loads of 150 lb and 200 lb, respectively.

Your Task

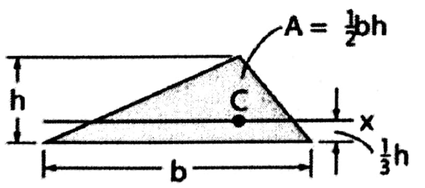
As a team, your job is to discuss issues and perform calculations that will help you evaluate Sofia's outdoor sign. Come up with justified answers to the following questions:

- What is the center of gravity for Sofia's outdoor sign?
- Will the supports be able to hold up the sign?
- What changes can be made to the sign in order to reduce the support load?

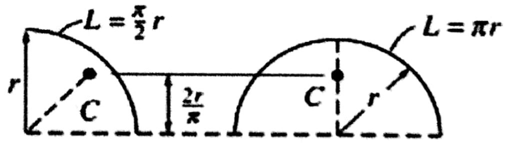
The table below provides the centroid of common geometries:



Rectangular area

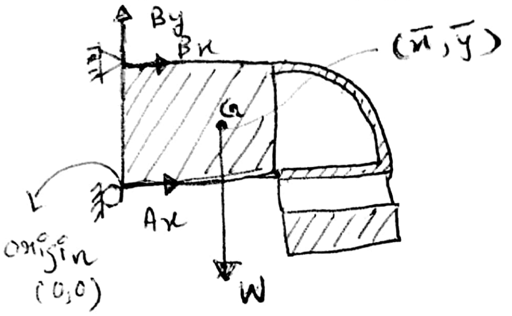


Triangular area



Quarter and semicircle arcs

1. Draw a free-body diagram for the whole outdoor sign, and label the sign's center of gravity, G, at a guessed location $(\bar{x}, \bar{y})$ relative to the coordinate system origin defined at A.



2. Calculate the weight of the two plywood pieces ($ABCD$ and $FGHI$), the curved steel bar (CE) and the straight steel bar (DE) for the outdoor sign.

weights

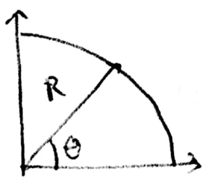
area (ABCD) = $6 \times 3 = 18\ ft^2$ — $= 18 \times 2 = 36$ lb

area (FGHI) = $2 \times 3 = 6\ ft^2$ — $= 6 \times 2 = 12$ lb

length (CE) = 1.5π ft — $= 1.5\pi \times 5 = 7.5\pi$ lb

length (DE) = 3 ft — $= 3 \times 5 = 15$ lb

3. Use the integral method to show that the $\bar{x}$ centroid location for a quarter arc matches with the value in Figure 3. (Hint: The function for quarter circle arc is $y = \sqrt{r^2 - x^2}$, where r is the constant radius)



$$\bar{x} = \frac{\int_0^{\pi/2} \bar{x}\, dL}{\int_0^{\pi/2} dL}$$

$dL = R\, d\theta$

$x = R\cos\theta$

$$\bar{x} = \frac{\int_0^{\pi/2} R\cos\theta\, R\, d\theta}{\int_0^{\pi/2} R\, d\theta} = \frac{R\sin\theta \Big|_0^{\pi/2}}{\pi/2} = \frac{2R}{\pi}$$

4. Find location the center of gravity for Sofia's outdoor sign.

Piece	$\bar{x}_i$	$\bar{y}_i$
ABCD	3	1.5
FGHI	7.5	-1.5
CE	$6 + 6/\pi = 7.91$	$\frac{6}{\pi} = 1.91$
DE	7.5	0

$\bar{x} = \frac{\Sigma x_i w_i}{\Sigma w_i} = 5.739\ ft$ $\quad\bar{y} = \frac{\Sigma y_i w_i}{\Sigma w_i} = 0.9355\ ft$

5. Find magnitude of the support reactions at A and B. Will the supports be able to hold up the sign?

$\Sigma M_B = 0 \Rightarrow A_x(3) = (36 + 12 + 7.5\pi + 15)\ 5.739$

$A_x = 165.589\ lb$

$\Sigma F_x = 0 \Rightarrow A_x + B_x = 0 \Rightarrow B_x = -165.589\ lb$

$\Sigma F_y = 0 \Rightarrow B_y - 86.56 \Rightarrow B_y = 86.56\ lb$

$F_B = \sqrt{165.589^2 + 86.56^2} = 186.84\ lb < 200\ lb$

$F_A = 165.589\ lb > 150\ lb \Rightarrow A$ fails.

6. What changes can be made to the sign in order to reduce the support load?

-> Using lighter materials

-> moving CG closer to the supports