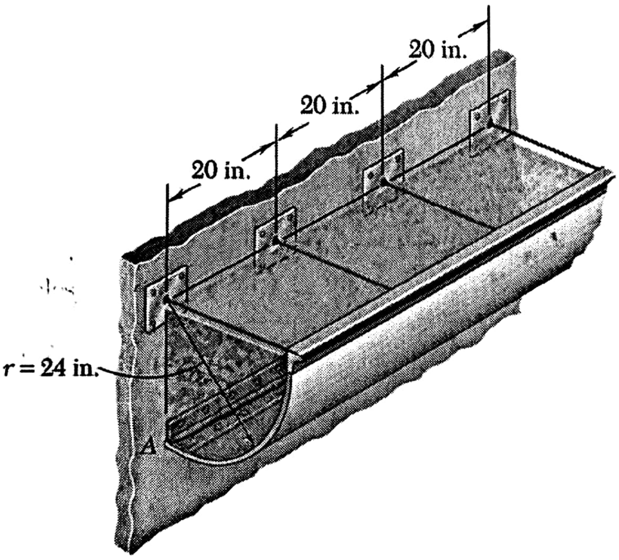
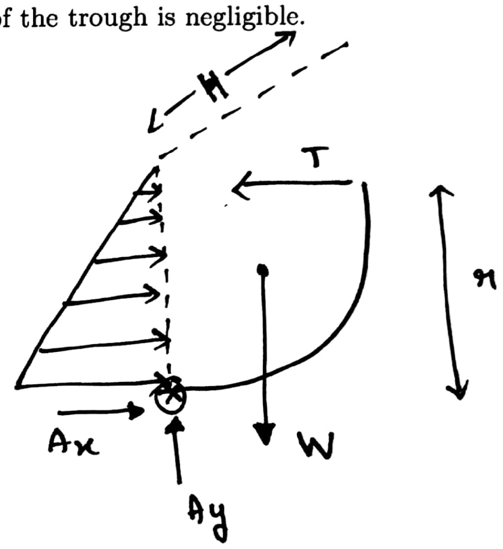
Name: ________________
Group members: ____________________ ____________________ ____________________

TAM 211 - Worksheet 13

1) A long trough is supported by a continuous hinge along its lower edge and by a series of horizontal cables attached to its upper edge. It is completely full of water

a) Draw a 2-D free body diagram that represents the side view of the long trough (orient hinge A along the axis going in and out of the page). Assume the weight of the trough is negligible.





b) Calculate the horizontal component of the equivalent hydrostatic pressure force on the trough, and the corresponding $\bar{y}$ location measured from A.

$$F_H = \frac{\rho g r (H)(r)}{2} = \frac{\rho g r^2 H}{2}$$

acts at $\bar{y} = \frac{r}{3}$ from A.

c) Calculate the vertical component of the equivalent hydrostatic pressure force on the trough, and the corresponding $\bar{x}$ location measured from A. (Hint: Both the $\bar{x}$ and $\bar{y}$ locations of the center of area of a quarter circle measured from the center are $4r/3\pi$.)

$$F_V = mg = \rho V g = \rho g \left[\frac{\pi r^2}{4}\right] H$$

acts at $\bar{x} = \frac{4r}{3\pi}$ from A.

d) Determine the total tension supporting the trough from all of the cables. Use the specific weight of water, $\gamma = 62.4$ lb/ft^3.

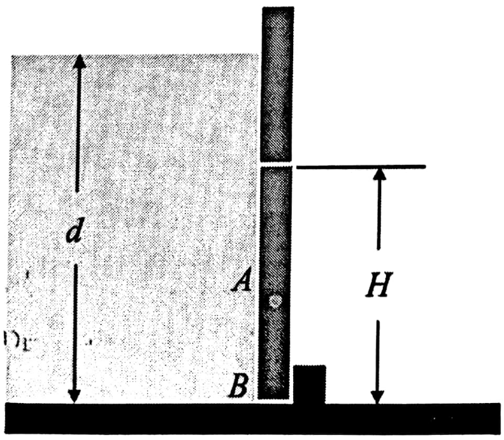
$$\gamma = \rho g = 62.4 \text{ lb/ft}^3 \qquad \Sigma M_A = 0 \Rightarrow -F_H\left(\frac{r}{3}\right) + T(r) - F_V\left(\frac{4r}{3\pi}\right) = 0$$

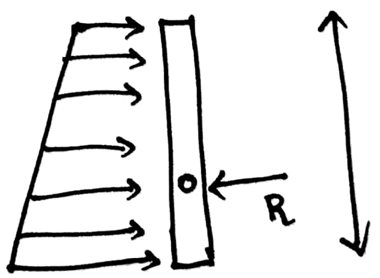
$$\Rightarrow -\frac{\rho g r^2 H}{2}\left[\frac{r}{3}\right] + Tr - \rho g\left[\frac{\pi r^2 H}{4}\right]\left[\frac{4r}{3\pi}\right] = 0$$

$$T = \frac{1}{r}\left[\frac{\gamma r^3 H}{6} + \frac{\gamma \pi r^3 H}{3\pi}\right] \Rightarrow \boxed{T = \frac{\gamma H r^2}{2} = 624 \text{ lb}}$$

2) An automatic valve consists of a rectangular plate (with height $H = 9$ in and width $b = 12$ in) that is pivoted at $0.4H$ through A, measured from the bottom as illustrated below. The specific weight of water is $\gamma = 62.4$ lbf/ft^3.

a) Draw a free body diagram that represents the valve, then use the appropriate equation of equilibrium to show that the valve cannot remain closed (equilibrium) when the equivalent static fluid pressure force is applied above A.





if the equivalent fluid pressure acts above 'A', then there will be a clockwise moment about A which rotates the plate and opens the valve.

b) Express the location of the resultant hydrostatic force acting on the submerged rectangular valve, $\bar{y}$, measured from the bottom surface, as a function of γ, d, and H and b. (Hint: divide the pressure distribution into simple geometries.)

$$\bar{y} = \frac{H[3d-2H]}{3[2d-H]}$$ ↳ centroid of trapezium formula

c) Use the expression from part (b), determine whether the valve will open when $d = 10$ in.

$$\bar{y} = \frac{9[30-18]}{3[20-9]} = \frac{3[12]}{11} = 3.273 \text{ in}$$

$$0.4H = 3.6 \text{ in}$$

as $0.4H > \bar{y}$, the valve will remain closed.

d) Determine the minimum water level, d_{min}, required to open the valve.

$$\bar{y} = 0.4H \Rightarrow 0.4\cancel{H} = \frac{\cancel{H}[3d-2H]}{3[2d-H]}$$

$$\Rightarrow 3d-2H = 1.2[2d-H]$$

$$3d-2H = 2.4d - 1.2H$$

$$0.6d = 0.8H$$

$$d = \frac{8}{6}H = \frac{4}{3}H$$

$$\boxed{d = \frac{4}{3}H}$$