Name: ________________
Group members: ______________________ ______________________ ______________________

TAM 210/211 - Worksheet 3

Objectives:

- Use free body diagrams and equilibrium equations to determine forces in cables and springs.
- Explore an experimental setup to verify theoretical findings.

1) A box of weight W is supported by two springs, as illustrated below.

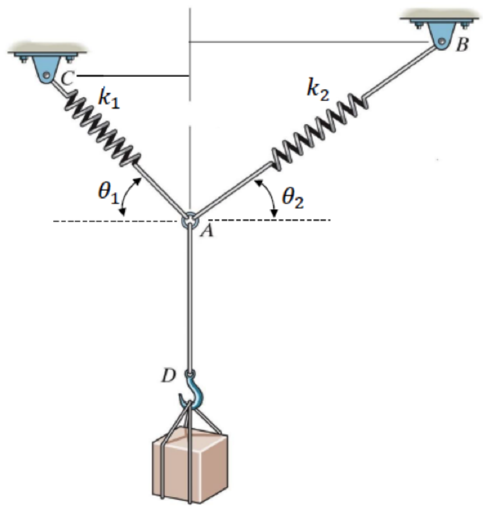
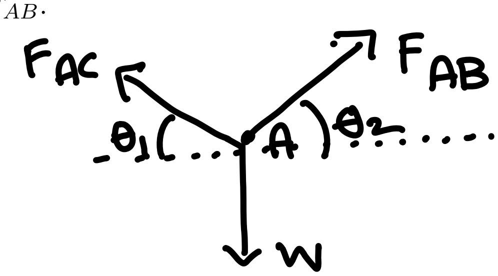


Figure 1: Spring-mass setup.

a) Draw a free body diagram of the forces acting at ring A. Denote the force in spring AC as F_{AC} and the force in spring AB as F_{AB}.



b) Use the equilibrium equations $\sum \mathbf{F} = \mathbf{0}$ to determine F_{AC} and F_{AB}. Your answers should be functions of W, θ_1 and θ_2.

$\sum F_x = 0 \quad F_{AB} \cos\theta_2 - F_{AC} \cos\theta_1 = 0$ — ①

$\sum F_y = 0 \quad F_{AB} \sin\theta_2 + F_{AC} \sin\theta_1 - W = 0$ — ②

from ① $F_{AB} = F_{AC} \dfrac{\cos\theta_1}{\cos\theta_2}$ using this in ② we get

$$F_{AC} = \frac{W\cos\theta_2}{\sin\theta_2\cos\theta_1 + \cos\theta_2\sin\theta_1}$$

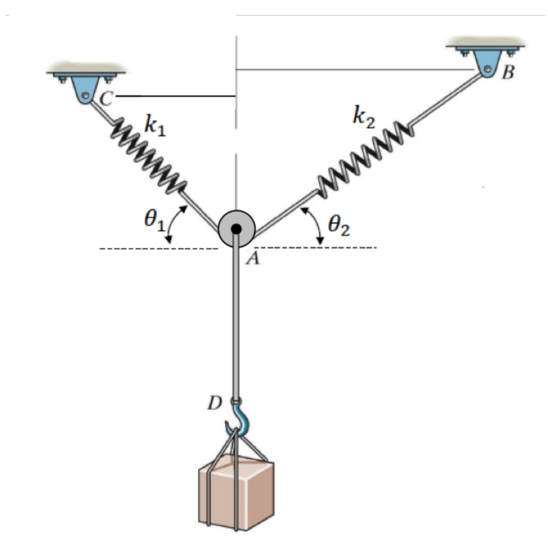
$$F_{AB} = \frac{W\cos\theta_1}{\sin\theta_2\cos\theta_1 + \cos\theta_2\sin\theta_1}$$

2) Use an experimental setup to validate your findings above. Start by calibrating the springs. Make sure to have the reading mark set to "zero" when the spring is hanging unloaded at a vertical position.

a) Find the weight of the object in the toolbox by using a single spring. Record the weight.

b) Use the peg-board, spring, ring, bolts and the weight object to reproduce Question 1 setup. Use the angle measurements and the mass of your object to predict your spring reading using your equations from Question 1(b). Check your solution with actual spring readings.

3) In this next experimental setup, the springs are fixed at uneven positions (different heights) again, but the springs are no longer connected to each other via a ring. Instead, connect the springs using a piece of string, to model the cable that goes through the pulley at A. Use a bobbin to represent the frictionless pulley.



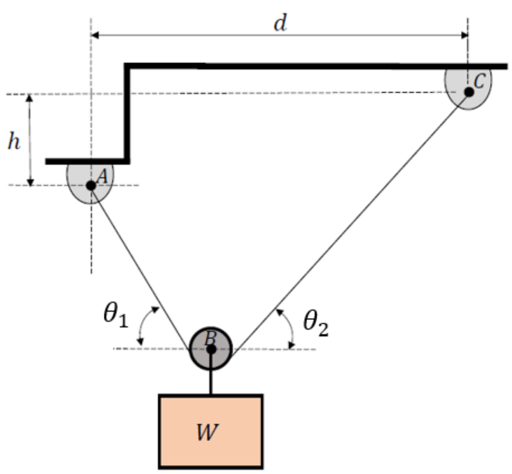
a) After hanging your object, what do you notice about the angles θ_1 and θ_2? What does this tell you about the forces acting on the two ends of the string?

$\theta_1 = \theta_2 \Rightarrow \sum F_n = 0 \quad F_{AB} \cos\theta_2 = F_{AC} \cos\theta_1$

$\Rightarrow F_{AB} = F_{AC}$

b) Use the angle measurements and the mass of your object to predict your spring reading using your equations from Question 1(b). Check your solution with actual spring readings.

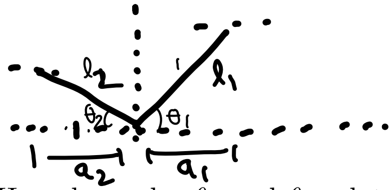
4) An experimental setup is created as illustrated below with string ABC of length L.



a) What happens to angles θ_1 and θ_2 when the weight W is changed by changing the object? Why?

θ_1 & θ_2 donot change with weight. They remain constant and equal.

$\theta_1 = \theta_2$



$l = l_1 + l_2 \qquad d = a_1 + a_2$

b) How do angles θ_1 and θ_2 relate to each other? (Hint: Remember the assumption we make for the tensions at the two ends of the string that goes over a frictionless pulley.) Express the angles **in terms of the given symbolic variables and dimensions** (neglect the size of the frictionless pulley B). How does your theoretical expression validate your conclusions in part (a)?

Tension in a string remains same. $F_{AB} = F_{AC}$

$\Rightarrow \theta_2 = \theta_1 = \theta$. $\quad l_1 = \dfrac{a_1}{\cos\theta_1} \quad l_2 = \dfrac{a_2}{\cos\theta_2} \Rightarrow l_1 + l_2 = \dfrac{a_1 + a_2}{\cos\theta}$

$\Rightarrow l = \dfrac{d}{\cos\theta} \Rightarrow \theta = \cos^{-1}\left(\dfrac{d}{l}\right)$.

c) Express the forces along AB and BC as Cartesian vectors in terms of the given symbolic variables and dimensions.

$F_{AB} = F_{BC} = \dfrac{W}{2\sin\theta}$

$F_{AB} = -F_{AB}\cos\theta_1\,\hat{i} + F_{AB}\sin\theta_1\,\hat{j} = \dfrac{-Wd}{2\sqrt{l^2-d^2}}\,\hat{i} + \dfrac{W}{2}\hat{j}$

$F_{BC} = F_{BC}\cos\theta_2\,\hat{i} + F_{BC}\sin\theta_2\,\hat{j} = \dfrac{Wd}{2\sqrt{l^2-d^2}}\,\hat{i} + \dfrac{W}{2}\hat{j}$

d) If the string ABC were shorter, how would angles θ_1 and θ_2 and the forces along AB and BC change?

$\theta = \cos^{-1}\left(\frac{d}{l}\right)$. If l is reduced $\theta_1 = \theta_2 = \theta$ will reduce

$F_{AB} = F_{BC} = \dfrac{W}{2\sin\theta} \Rightarrow$ as θ is reduced $\sin\theta$ reduces

and as a result F_{AB} & F_{BC} will increase.

e) What implications does part (d) have on the design of systems with different string lengths in terms of the required strengths of the strings?

strings with shorter length should have more strength, as the tension developed in them will be larger.