PHYS 598 GTC Homework 1

- 1. Let G be a group, and let $H \subset G$ be a subgroup. Let's get some practice with group theory basics by proving the following two statements:
 - (a) Show that if the index of H in G is two (i.e. |G : H| = 2), then $H \triangleleft G$ is a normal subgroup of G.
 - (b) Let |G| and |H| denote the number of elements in G and H respectively (called the order of the group). Using the coset decomposition of G, show that if |G| is finite then |G| = |G : H||H| (This result is known as Lagrange's theorem).
- 2. Consider a group \overline{G} isomorphic to the point group 6mm.
 - (a) What is the order $|\bar{G}|$ of \bar{G} ?
 - (b) Show that there are three subgroups H_1 , H_2 , and H_3 of \overline{G} , all of which are isomorphic to 2mm.
 - (c) Show that there exists $g \in \overline{G}$ such that $gH_1g^{-1} = H_2$, and $gH_2g^{-1} = H_3$. Are any of $H_1, H_2, or H_3$ normal?
- 3. Let's get some practice with point groups in 2D. For the three decorated squares below, identify the point group that gives the symmetry of the decorated square:



4. Let's get some practice reading space group symbols. Using the Bilbao Crystallographic Server, Bradley and Cracknell, or otherwise, answer the following questions about the space group *P*222₁ (# 17):

- (a) What is the point group \overline{G} of this space group?
- (b) What is the Bravais lattice T of this space group?
- (c) Write a coset decomposition of $P222_1$ relative to T.
- 5. The most common Bravais lattice types (which you may have seen before) are primitive (P), body-centered (I), and face centered (F). Let's look at these for a cubic system:
 - (a) Write down a set of primitive Bravais lattice vectors for a primitive cubic lattice with side length *a*. What is the volume of the primitive unit cell?
 - (b) Do the same for a body-centered cubic lattice (with the sides of the cube still of length *a*). What is the volume of the primitive body-centered unit cell?
 - (c) Finally, do the same for a face-centered Bravais lattice. What is the volume of the primitive face-centered unit cell?