## PHYS 598 GTC Homework 1

1. Let $G$ be a group, and let $H \subset G$ be a subgroup. Let's get some practice with group theory basics by proving the following two statements:
(a) Show that if the index of $H$ in $G$ is two (i.e. $|G: H|=2$ ), then $H \triangleleft G$ is a normal subgroup of $G$.
(b) Let $|G|$ and $|H|$ denote the number of elements in $G$ and $H$ respectively (called the order of the group). Using the coset decomposition of $G$, show that if $|G|$ is finite then $|G|=|G: H||H|$ (This result is known as Lagrange's theorem).
2. Consider a group $\bar{G}$ isomorphic to the point group 6 mm .
(a) What is the order $|\bar{G}|$ of $\bar{G}$ ?
(b) Show that there are three subgroups $H_{1}, H_{2}$, and $H_{3}$ of $\bar{G}$, all of which are isomorphic to 2 mm .
(c) Show that there exists $g \in \bar{G}$ such that $g H_{1} g^{-1}=H_{2}$, and $g H_{2} g^{-1}=H_{3}$. Are any of $H_{1}, H_{2}$, or $H_{3}$ normal?
3. Let's get some practice with point groups in 2D. For the three decorated squares below, identify the point group that gives the symmetry of the decorated square:

(a)

(b)

(c)
4. Let's get some practice reading space group symbols. Using the Bilbao Crystallographic Server, Bradley and Cracknell, or otherwise, answer the following questions about the space group P2221 (\# 17):
(a) What is the point group $\bar{G}$ of this space group?
(b) What is the Bravais lattice $T$ of this space group?
(c) Write a coset decomposition of $P 222_{1}$ relative to $T$.
5. The most common Bravais lattice types (which you may have seen before) are primitive (P), body-centered (I), and face centered (F). Let's look at these for a cubic system:
(a) Write down a set of primitive Bravais lattice vectors for a primitive cubic lattice with side length $a$. What is the volume of the primitive unit cell?
(b) Do the same for a body-centered cubic lattice (with the sides of the cube still of length a). What is the volume of the primitive body-centered unit cell?
(c) Finally, do the same for a face-centered Bravais lattice. What is the volume of the primitive face-centered unit cell?
