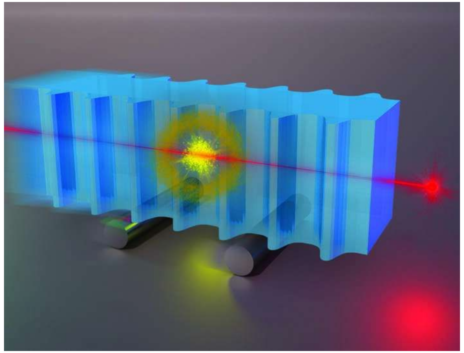
Zero Refractive Index Materials and Topological Photonics

Tree Hirri-O-Tuppa, David Imig, David Jiang, Rohan Joshi, Brittany Karki



Horsley, S.A.R., Woolley, M. Zero-refractive-index materials and topological photonics. *Nat. Phys.* 17, 348–355 (2021).
https://doi.org/10.1038/s41567-020-01082-2

Refractive index and its relation to EM waves

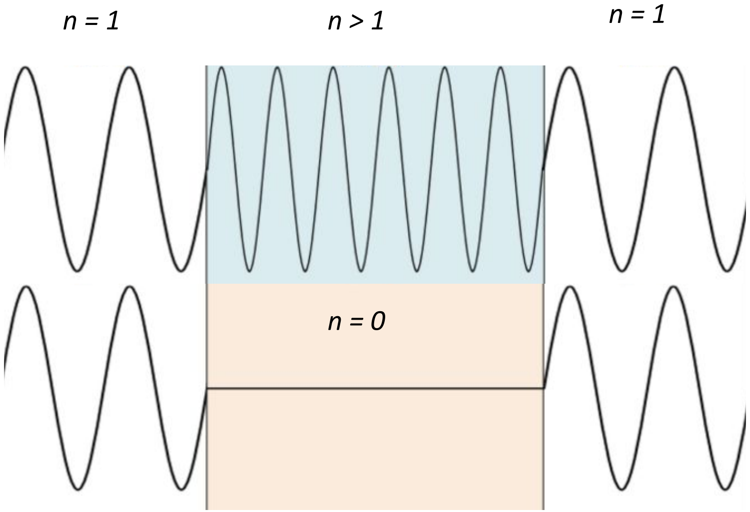
Refractive Index = $\dfrac{\text{speed of light in vacuum}}{\text{speed of light in the material}}$ or $n = c / v$

Can be **complex-valued**!

- Imaginary part: absorption/amplification of wave

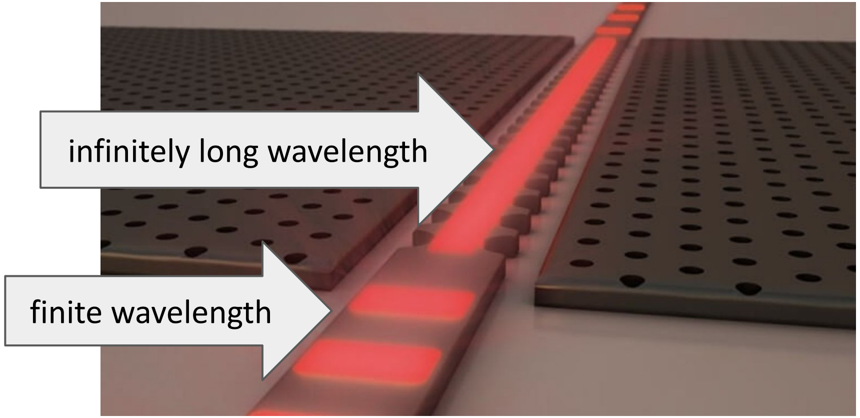
When $n \rightarrow 0$, wave becomes stretched infinitely long

- $v = f\lambda$
- f remains unchanged
- $\lambda \rightarrow \infty$



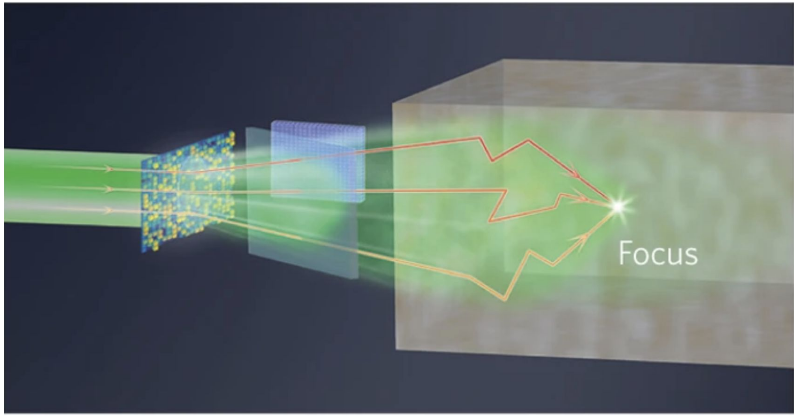
Why Zero Index?

Extraordinary Transmission in Waveguides



https://seas.harvard.edu/drew/news/2017/10/zero-index-waveguide

Wavefront shaping

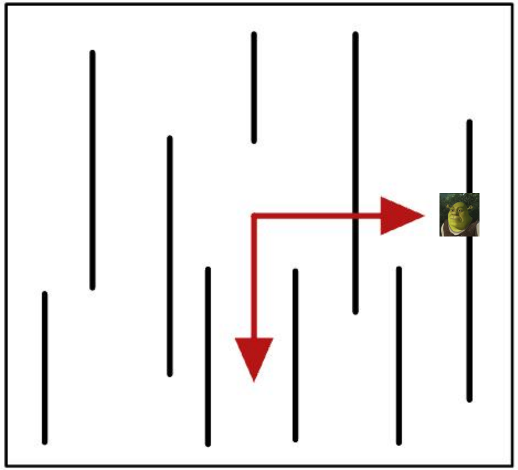


https://www.nature.com/articles/nphoton.2015.140/peacockfigures/4

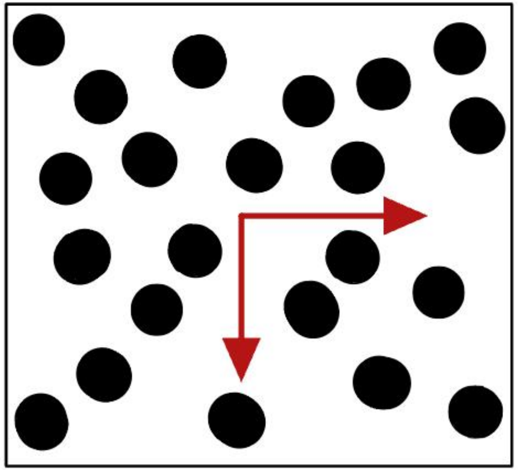
Requirements to study zero index in materials

Materials are generally **anisotropic**.

Need to account for the dependence of waves on their **polarization** and **direction of propagation.**



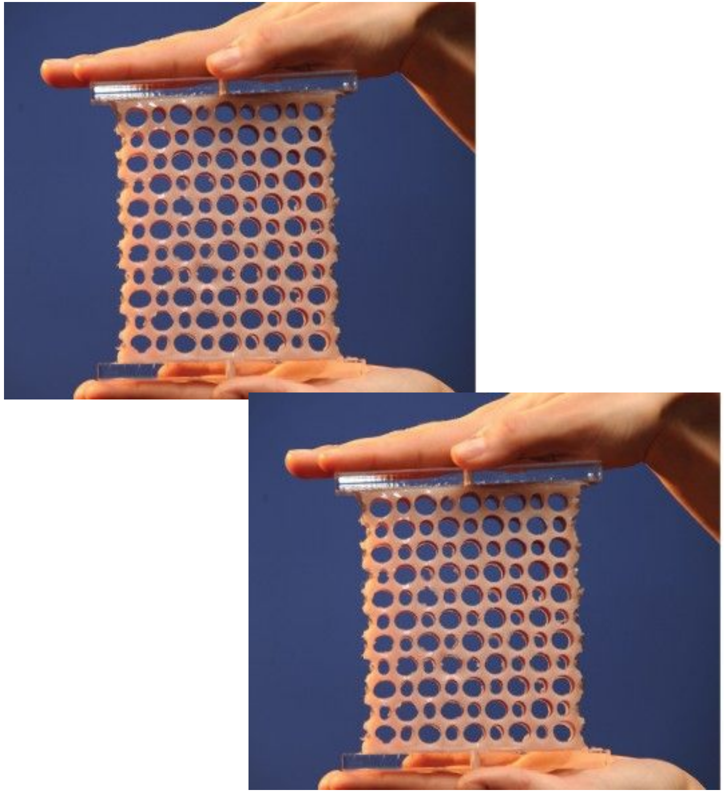
Anisotropic



Isotropic

What are Metamaterials?

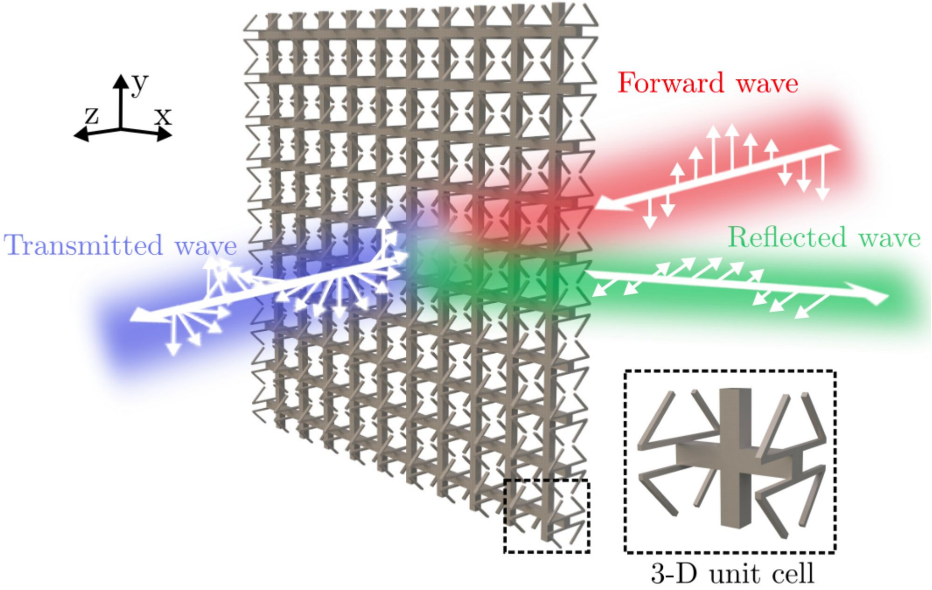
Wonky!!



Credit: Mitch Jacoby/C&EN

What are Metamaterials?

Wonky!!



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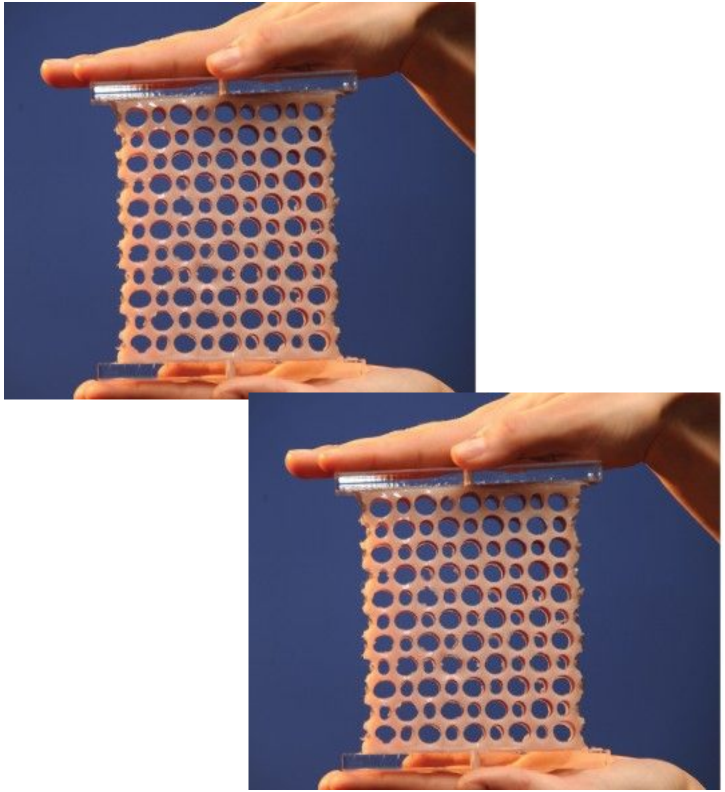
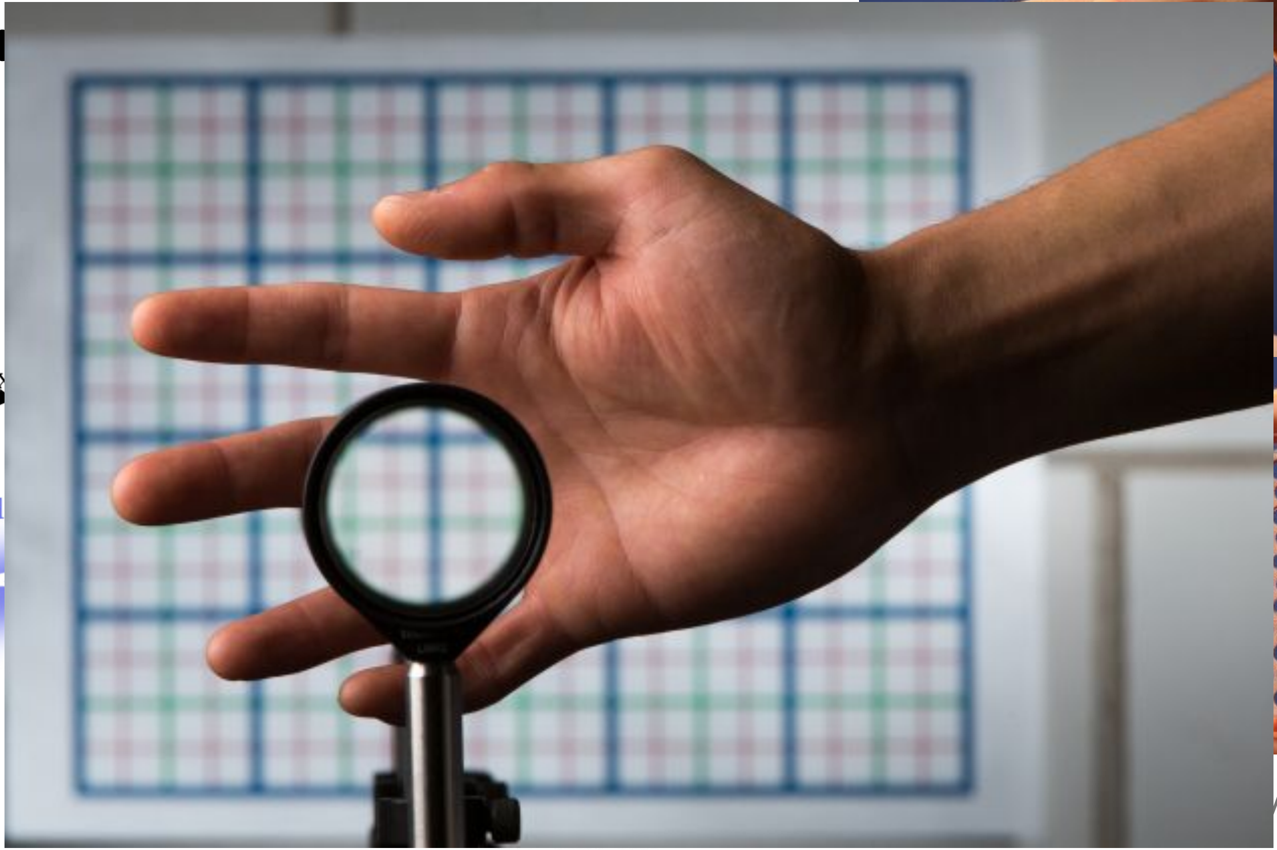
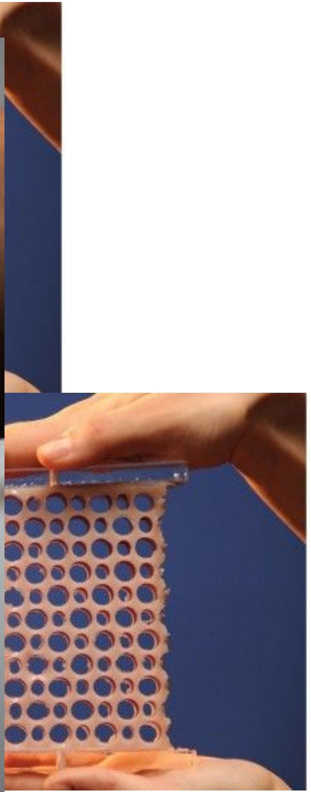


Credit: Mitch Jacoby/C&EN

What ar

Wonky!!





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How to describe wave propagation in 2D material

We use the 2D Dirac equation.

It describes the **relativistic dispersion relation**.

- $E > mc^2$: solutions are propagating, *k* is real
- $E < mc^2$: solutions are exponentially decaying
- $E = mc^2$: transition point
    - corresponds to zero refractive index condition for a material

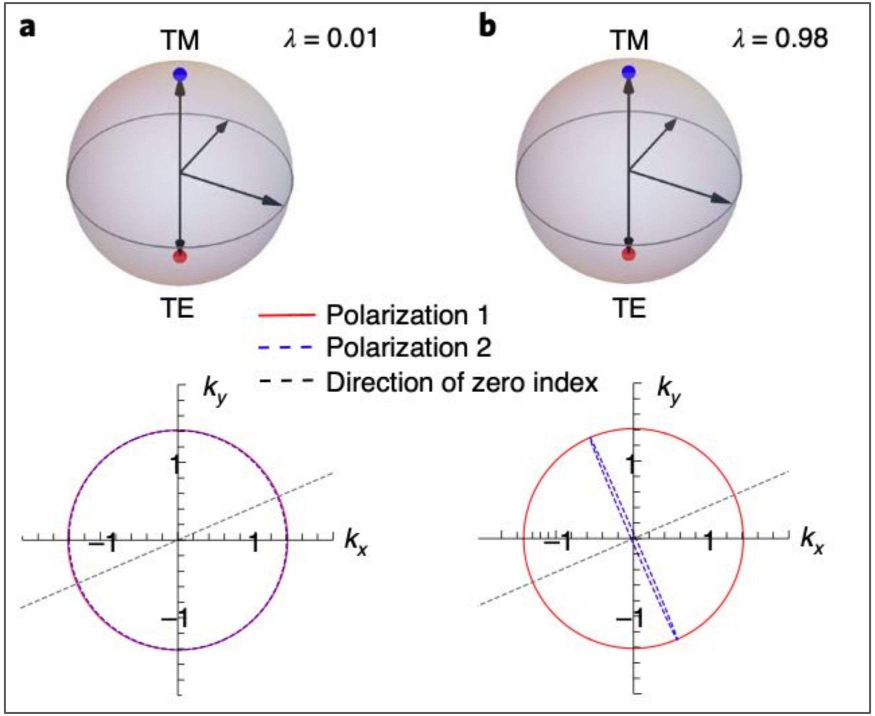
When can materials have zero index?

χ is called the **material tensor.**

- Tells you if materials can have zero index
- Tells you the direction of propagation for zero index, **m**
    - Relates to $\boldsymbol{\lambda}$: as $\boldsymbol{\lambda} \to 1$, the zero index is in the **m** direction

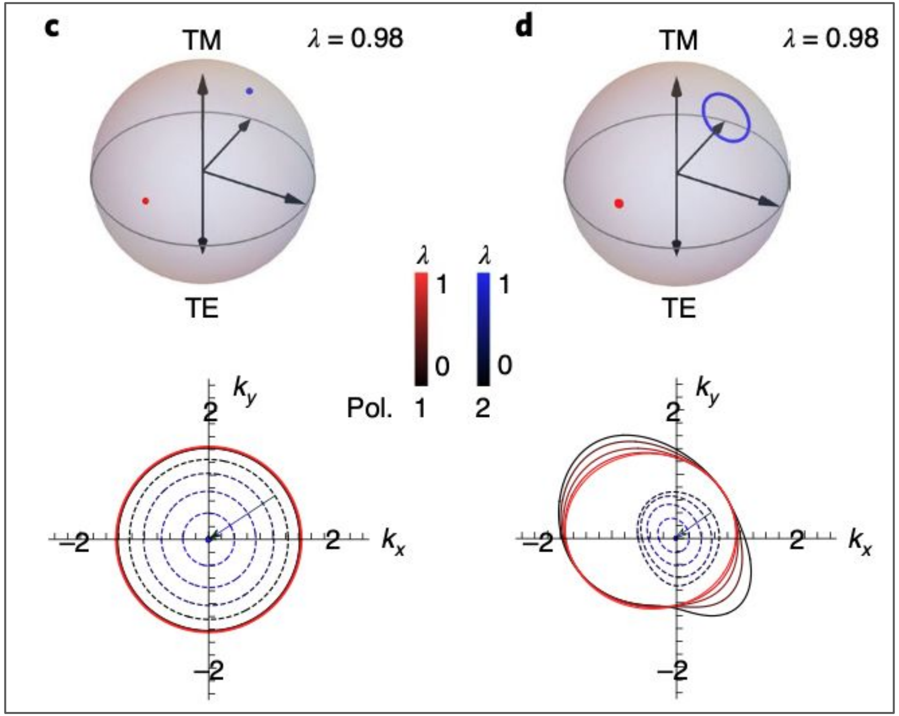
Note: ($\boldsymbol{\lambda}$ is not the wavelength here)

Dispersion relation and polarization in a sample material



the principal axes of the material are designed to make zero index for the blue polarization and the red polarization is less affected

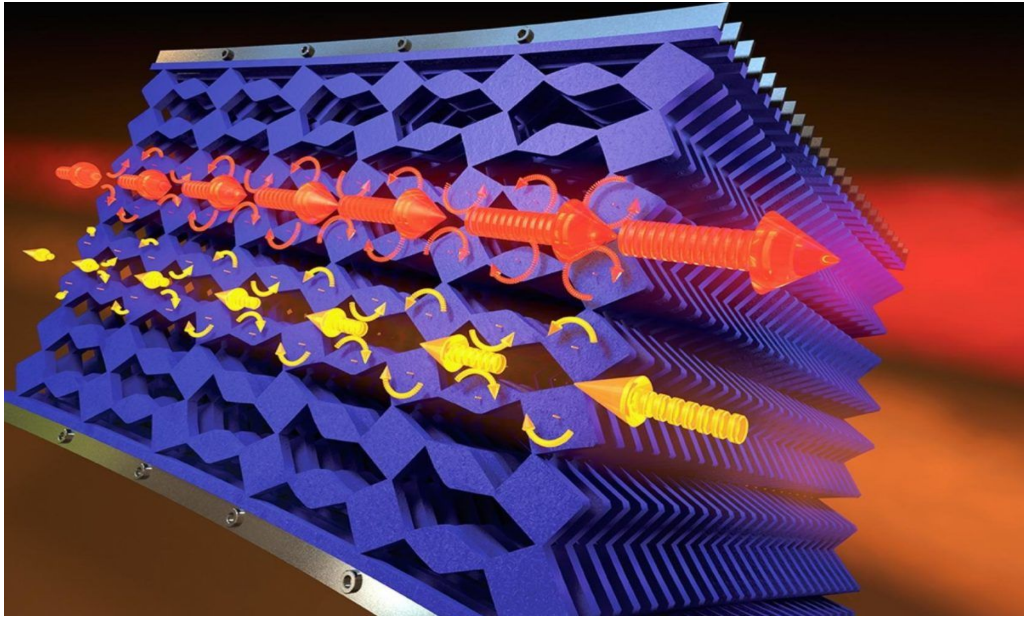
Dispersion relation and polarization in 2 sample materials



the principal axes of the material are designed to make zero index for the blue polarization and the red polarization is less affected

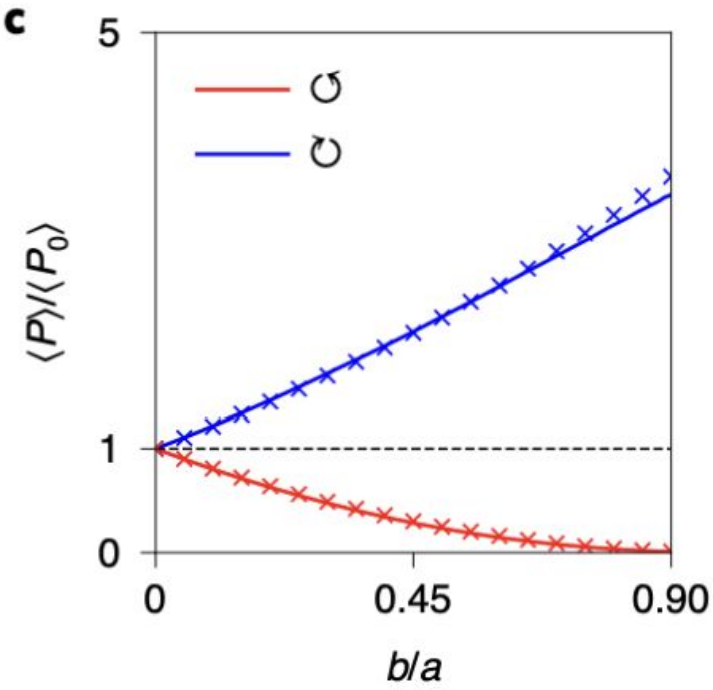
What Happens At Zero Refractive Index?

- Propagation in one direction completely attenuated - Complex Axis Nihility



https://news.utexas.edu/2017/02/13/new-mechanical-metamaterials-can-block-symmetry-of-motion/

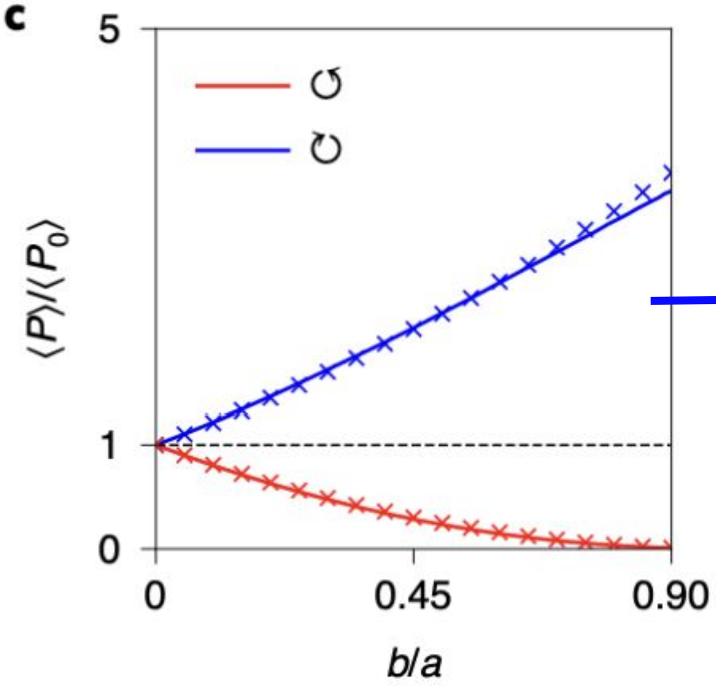
CAN Media Support One-Way Waves!

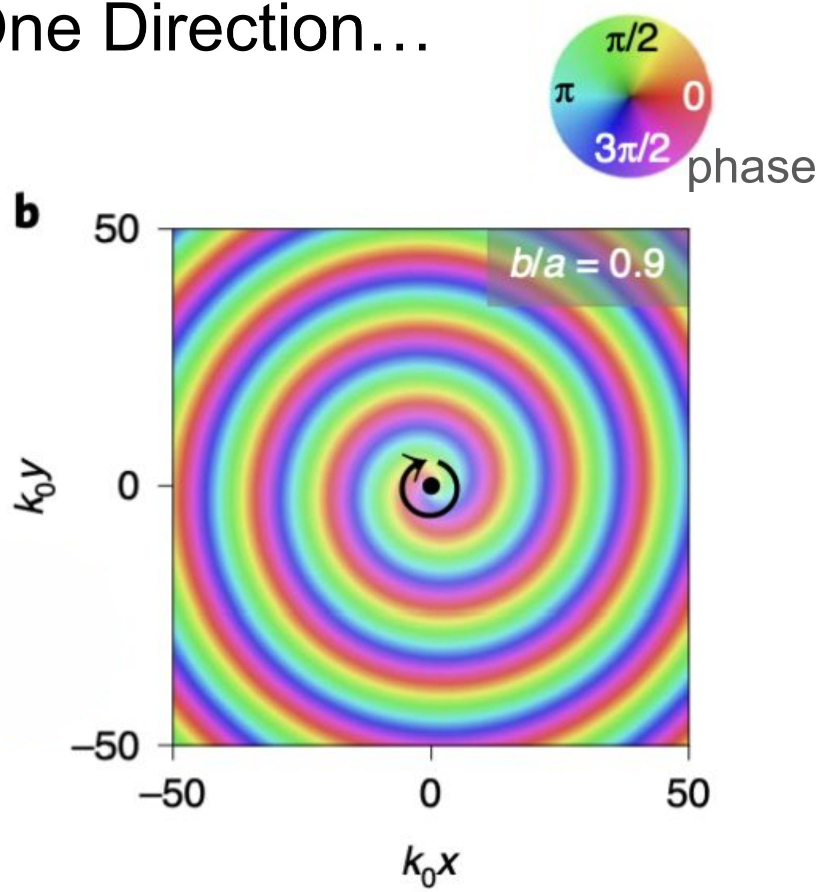


$$\langle P_{\circlearrowleft} \rangle = \frac{\eta_0 \mu_{zz} k_0 |j_0|^2}{16a^2} (a-b)^2$$

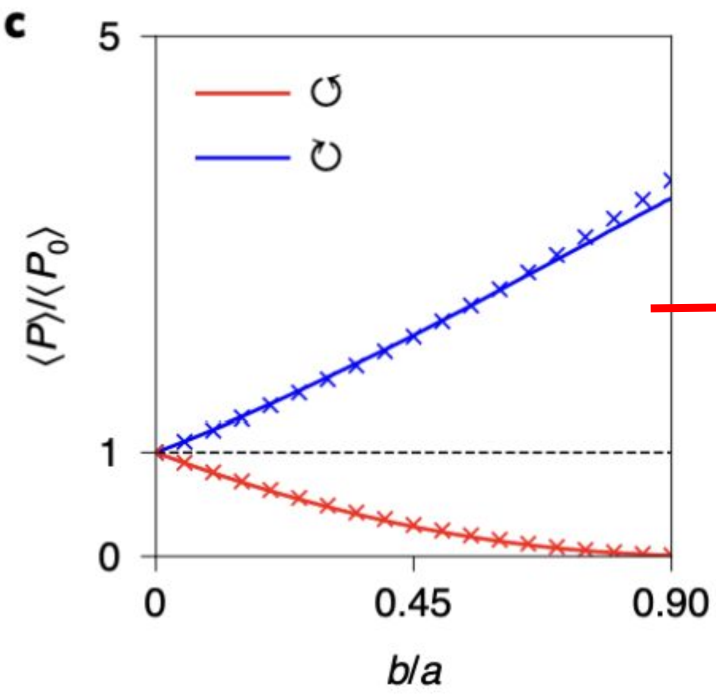
$$\langle P_{\circlearrowright} \rangle = \frac{\eta_0 \mu_{zz} k_0 |j_0|^2}{16a^2} (a+b)^2$$

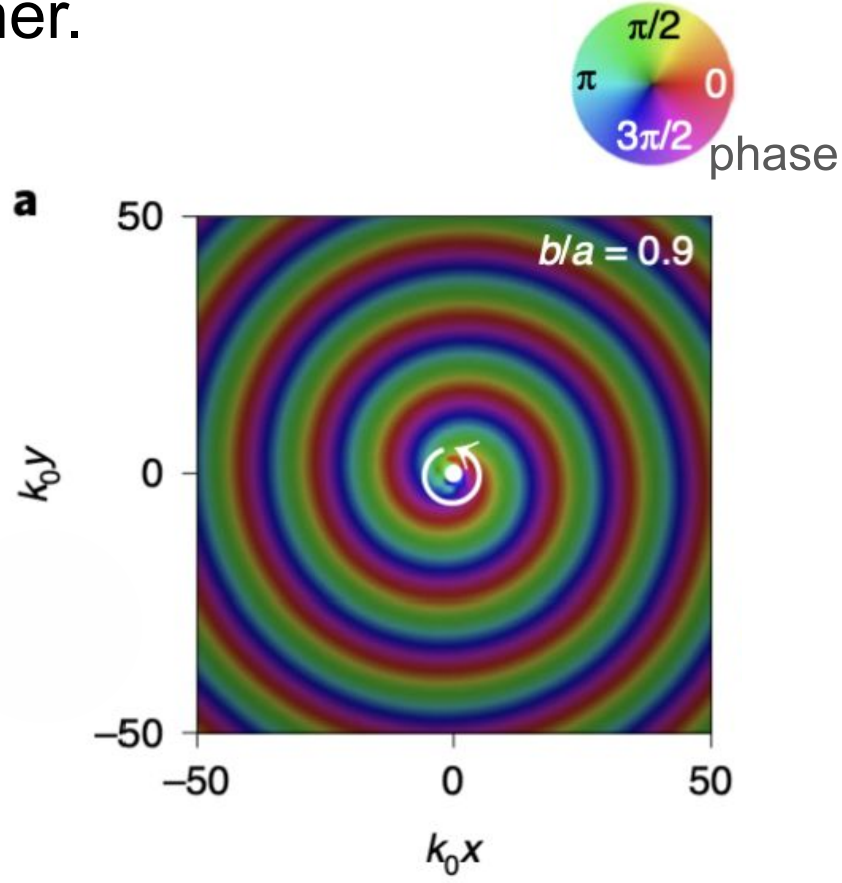
CAN Media Amplify Waves in One Direction…



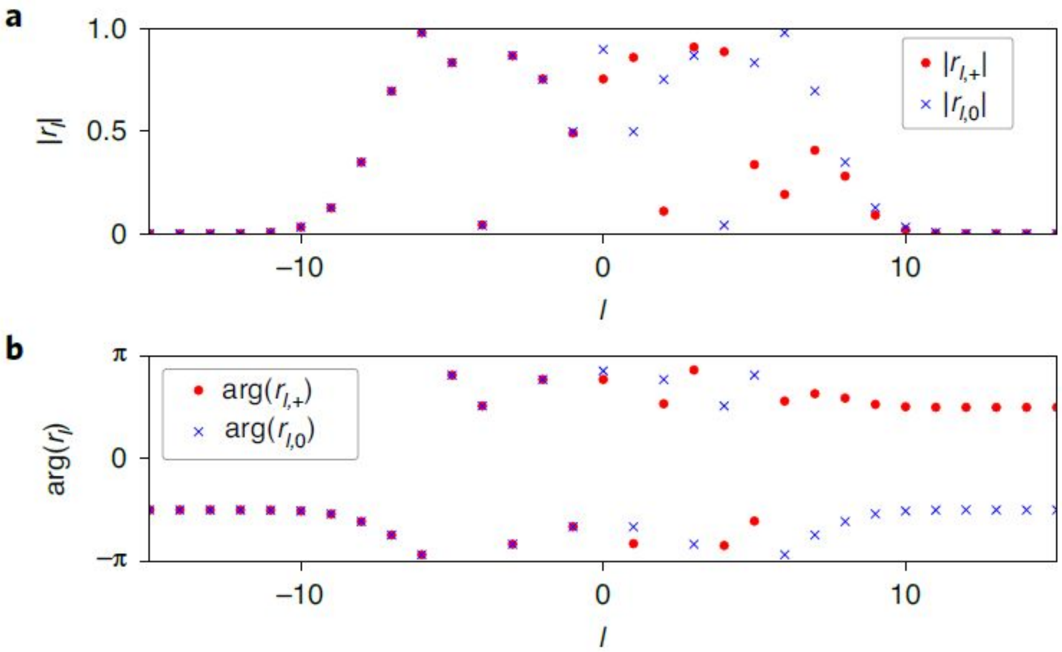


And Attenuate Them in the Other.





Scattering from bianisotropic CAN medium



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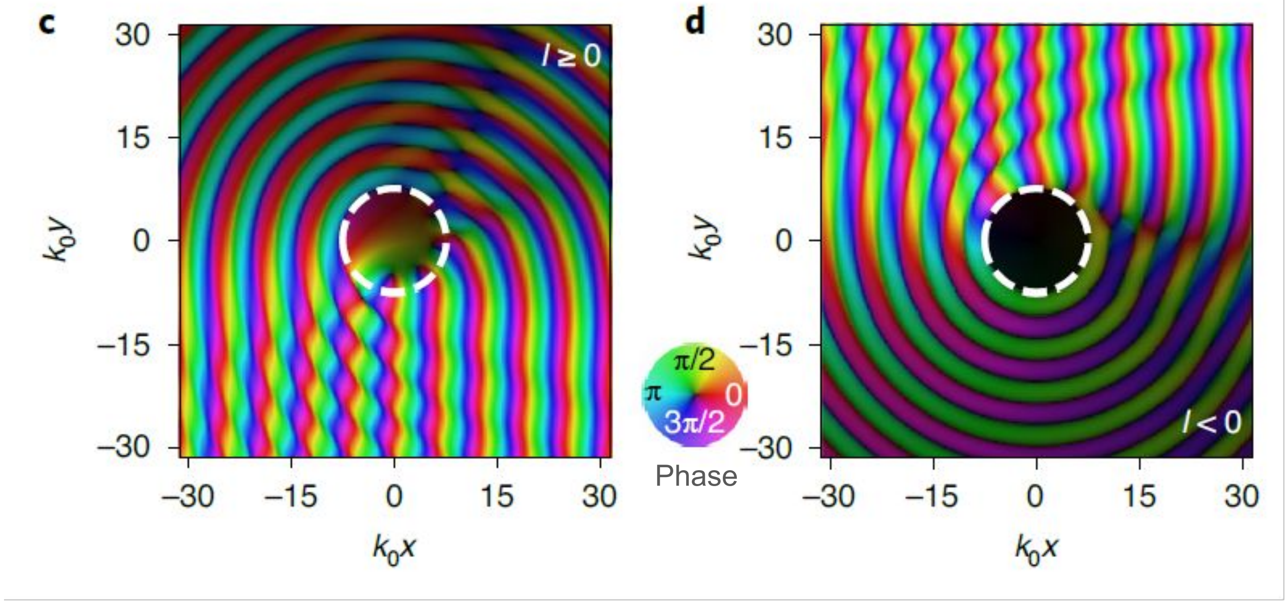
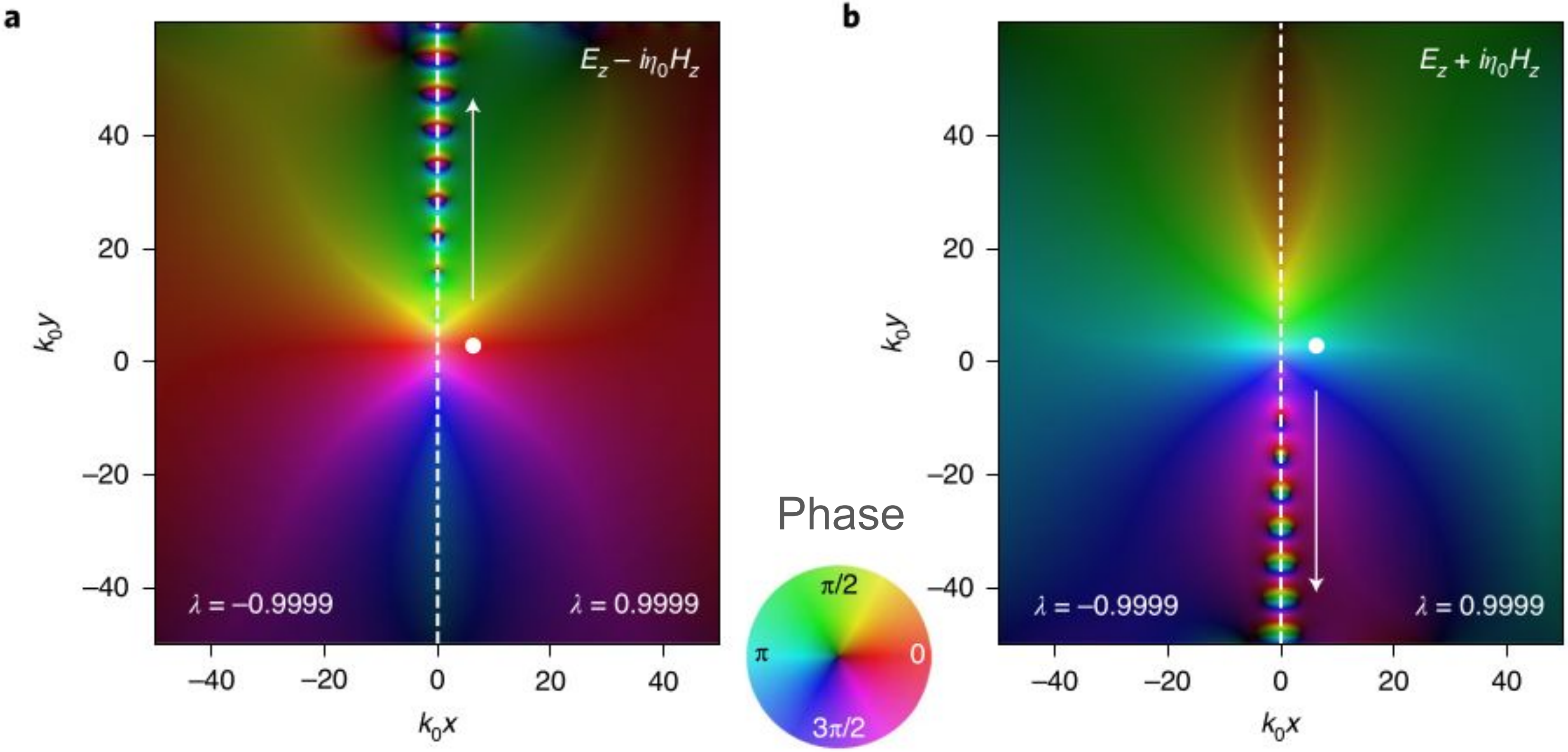
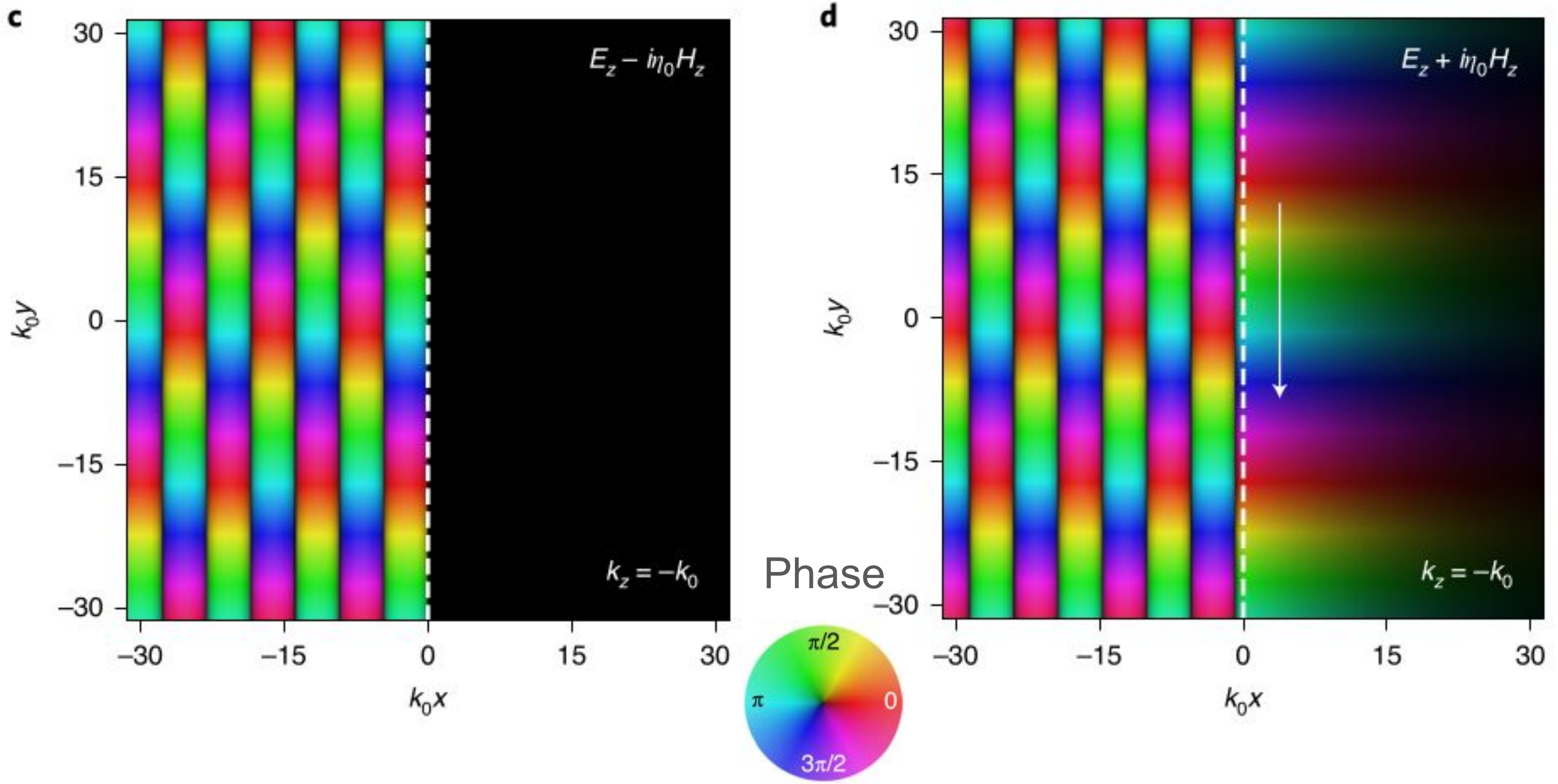


CAN make Unidirectional Edge states at interfaces!

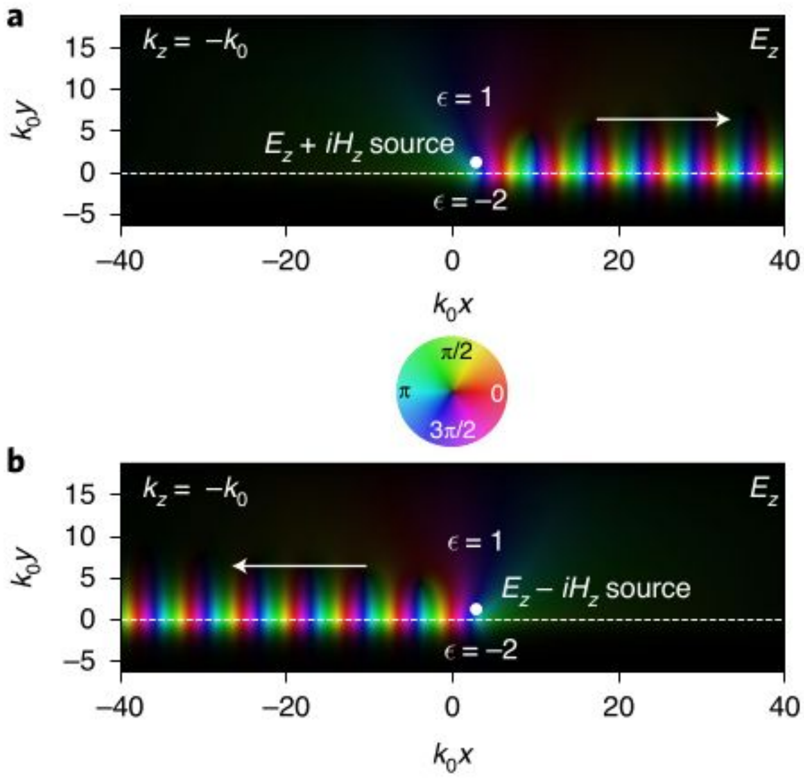


Phase

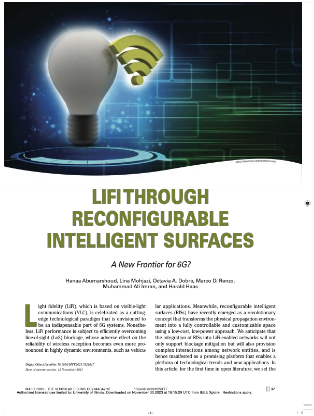
Killing off an Evanescent wave in Total Internal Reflection



More Evanescent Waves: Chiral Antenna



Looking Ahead



LIFI THROUGH RECONFIGURABLE INTELLIGENT SURFACES

A New Frontier for 6G?

Hanaa Abumarshoud, Lina Mohjazi, Octavia A. Dobre, Marco Di Renzo,
Muhammad Ali Imran, and Harald Haas

Light fidelity (LiFi), which is based on visible-light communications (VLC), is celebrated as a cutting-edge technological paradigm that is envisioned to be an indispensable part of 6G systems. Nonetheless, LiFi performance is subject to efficiently overcoming line-of-sight (LoS) blockage, whose adverse effect on the reliability of wireless reception becomes even more pronounced in highly dynamic environments, such as vehicular applications. Meanwhile, reconfigurable intelligent surfaces (RISs) have recently emerged as a revolutionary concept that transforms the physical propagation environment into a fully controllable and customizable space using a low-cost, low-power approach. We anticipate that the integration of RISs into LiFi-enabled networks will not only support blockage mitigation but will also provision complex interactions among network entities, and is hence manifested as a promising platform that enables a plethora of technological trends and new applications. In this article, for the first time in open literature, we set the

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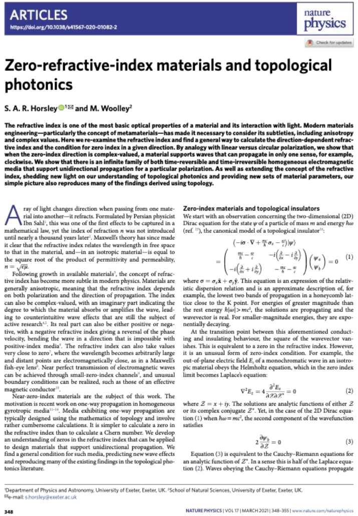

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Zero-refractive-index materials and topological photonics

S. A. R. Horsley[1] and M. Woolley[2]

The refractive index is one of the most basic optical properties of a material and its interaction with light. Modern materials engineering—particularly the concept of metamaterials—has made it necessary to consider its subtleties, including anisotropy and complex values. Here we re-examine the refractive index and find a general way to calculate the direction-dependent refractive index and the condition for zero index in a given direction. By analogy with linear versus circular polarization, we show that when the zero-index direction is complex-valued, a material supports waves that can propagate in only one sense, for example, clockwise. We show that there is an infinite family of both time-reversible and time-irreversible homogeneous electromagnetic media that support unidirectional propagation for a particular polarization. As well as extending the concept of the refractive index, shedding new light on our understanding of topological photonics and providing new sets of material parameters, our simple picture also reproduces many of the findings derived using topology.

A ray of light changes direction when passing from one material into another—it refracts. Formulated by Persian physicist Ibn Sahl[1], this was one of the first effects to be captured in a mathematical law, yet the index of refraction n was not introduced until nearly a thousand years later[2]. Maxwell's theory has since made it clear that the refractive index relates the wavelength in free space to that in the material, and—in an isotropic material—is equal to the square root of the product of permittivity and permeability, $n = \sqrt{\epsilon\mu}$.

Following growth in available materials[3], the concept of refractive index has become more subtle in modern physics. Materials are generally anisotropic, meaning that the refractive index depends on both polarization and the direction of propagation. The index can also be complex-valued, with an imaginary part indicating the degree to which the material absorbs or amplifies the wave, leading to counterintuitive wave effects that are still the subject of active research[4,5]. Its real part can also be either positive or negative, with a negative refractive index giving a reversal of the phase velocity, bending the wave in a direction that is impossible with positive-index media[6]. The refractive index can also take values very close to zero[7], where the wavelength becomes arbitrarily large and distant points are electromagnetically close, as in a Maxwell's fish-eye lens[8]. Near perfect transmission of electromagnetic waves can be achieved through small-zero-index channels[9], and unusual boundary conditions can be realized, such as those of an effective magnetic conductor[10].

Near-zero-index materials are the subject of this work. The motivation is recent work on one-way propagation in homogeneous gyrotropic media[11–14]. Media exhibiting one-way propagation are typically designed using the mathematics of topology and involve rather cumbersome calculations. It is simpler to calculate a zero in the refractive index than to calculate a Chern number. We develop an understanding of zeros in the refractive index that can be applied to design materials that support unidirectional propagation. We find a general condition for such media, predicting new wave effects and reproducing many of the existing findings in the topological photonics literature.

Zero-index materials and topological insulators

We start with an observation concerning the two-dimensional (2D) Dirac equation for the state ψ of a particle of mass m and energy $\hbar\omega$ (ref. [15]), the canonical model of a topological insulator[16]:

$$\left(-i\boldsymbol{\sigma}\cdot\nabla + \frac{mc}{\hbar}\sigma_z - \frac{\omega}{c}\right)|\psi\rangle = \begin{pmatrix} \frac{mc}{\hbar} - \frac{\omega}{c} & -i\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right) \\ -i\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right) & -\frac{mc}{\hbar} - \frac{\omega}{c} \end{pmatrix}\begin{pmatrix}\psi_a \\ \psi_b\end{pmatrix} = 0 \qquad (1)$$

where $\boldsymbol{\sigma} = \sigma_x\hat{\mathbf{x}} + \sigma_y\hat{\mathbf{y}}$. This equation is an expression of the relativistic dispersion relation and is an approximate description of, for example, the lowest two bands of propagation in a honeycomb lattice close to the K point. For energies of greater magnitude than the rest energy $\hbar|\omega| > mc^2$, the solutions are propagating and the wavevector is real. For smaller-magnitude energies, they are exponentially decaying.

At the transition point between this aforementioned conducting and insulating behaviour, the square of the wavevector vanishes. This is equivalent to a zero in the refractive index. However, it is an unusual form of zero-index condition. For example, the out-of-plane electric field E_z of a monochromatic wave in an isotropic material obeys the Helmholtz equation, which in the zero index limit becomes Laplace's equation:

$$\nabla^2 E_z = 4\frac{\partial^2 E_z}{\partial\mathcal{Z}\partial\mathcal{Z}^*} = 0 \qquad (2)$$

where $\mathcal{Z} = x + iy$. The solutions are analytic functions of either $\mathcal{Z}$ or its complex conjugate $\mathcal{Z}^*$. Yet, in the case of the 2D Dirac equation (1) when $\hbar\omega = mc^2$, the second component of the wavefunction satisfies

$$2\frac{\partial\psi_b}{\partial\mathcal{Z}} = 0 \qquad (3)$$

Equation (3) is equivalent to the Cauchy–Riemann equations for an analytic function of $\mathcal{Z}^*$. In a sense this is half of the Laplace equation (2). Waves obeying the Cauchy–Riemann equations propagate

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[1]Department of Physics and Astronomy, University of Exeter, Exeter, UK. [2]School of Natural Sciences, University of Exeter, Exeter, UK.
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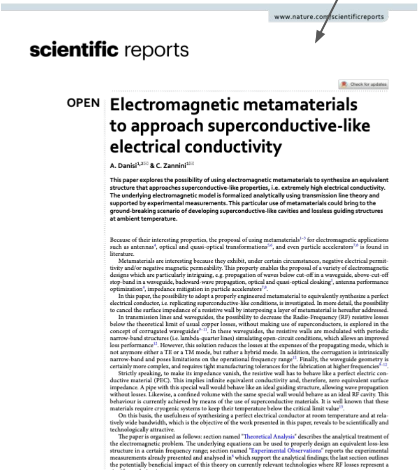

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Electromagnetic metamaterials to approach superconductive-like electrical conductivity

A. Danisi[1,2] & C. Zannini[1]

This paper explores the possibility of using electromagnetic metamaterials to synthesize an equivalent structure that approaches superconductive-like properties, i.e. extremely high electrical conductivity. The underlying electromagnetic model is formalized analytically using transmission line theory and supported by experimental measurements. This particular use of metamaterials could bring to the ground-breaking scenario of developing superconductive-like cavities and lossless guiding structures at ambient temperature.

Because of their interesting properties, the proposal of using metamaterials[1–3] for electromagnetic applications such as antennas[4], optical and quasi-optical transformations[5,6], and even particle accelerators[7,8] is found in literature.

Metamaterials are interesting because they exhibit, under certain circumstances, negative electrical permittivity and/or negative magnetic permeability. This property enables the proposal of a variety of electromagnetic designs which are particularly intriguing, e.g. propagation of waves below cut-off in a waveguide, above-cut-off stop-band in a waveguide, backward-wave propagation, optical and quasi-optical cloaking[3], antenna performance optimization[4], impedance mitigation in particle accelerators[7,8].

In this paper, the possibility to adopt a properly engineered metamaterial to equivalently synthesize a perfect electrical conductor, i.e. replicating superconductive-like conditions, is investigated. In more detail, the possibility to cancel the surface impedance of a resistive wall by interposing a layer of metamaterial is hereafter addressed.

In transmission lines and waveguides, the possibility to decrease the Radio-Frequency (RF) resistive losses below the theoretical limit of usual copper losses, without making use of superconductors, is explored in the concept of corrugated waveguides[9–11]. In these waveguides, the resistive walls are modulated with periodic narrow-band structures (i.e. lambda-quarter lines) simulating open-circuit conditions, which allows an improved loss performance[12]. However, this solution reduces the losses at the expenses of the propagating mode, which is not anymore either a TE or a TM mode, but rather a hybrid mode. In addition, the corrugation is intrinsically narrow-band and poses limitations on the operational frequency range[12]. Finally, the waveguide geometry is certainly more complex, and requires tight manufacturing tolerances for the fabrication at higher frequencies[9–12].

Strictly speaking, to make its impedance vanish, the resistive wall has to behave like a perfect electric conductive material (PEC). This implies infinite equivalent conductivity and, therefore, zero equivalent surface impedance. A pipe with this special wall would behave like an ideal guiding structure, allowing wave propagation without losses. Likewise, a confined volume with the same special wall would behave as an ideal RF cavity. This behaviour is currently achieved by means of the use of superconductive materials. It is well known that these materials require cryogenic systems to keep their temperature below the critical limit value[13].

On this basis, the usefulness of synthesizing a perfect electrical conductor at room temperature and at relatively wide bandwidth, which is the objective of the work presented in this paper, reveals to be scientifically and technologically attractive.

The paper is organised as follows: section named "Theoretical Analysis" describes the analytical treatment of the electromagnetic problem. The underlying equations can be used to properly design an equivalent loss-less structure in a certain frequency range; section named "Experimental Observations" reports the experimental measurements already presented and analysed in[8] which support the analytical findings; the last section outlines the potentially beneficial impact of this theory on currently relevant technologies where RF losses represent a

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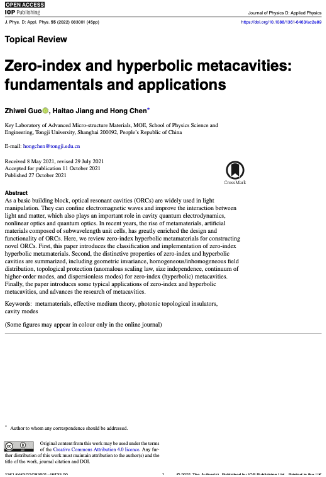

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Topical Review

Zero-index and hyperbolic metacavities: fundamentals and applications

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Abstract

As a basic building block, optical resonant cavities (ORCs) are widely used in light manipulation. They can confine electromagnetic waves and improve the interaction between light and matter, which also plays an important role in cavity quantum electrodynamics, nonlinear optics and quantum optics. In recent years, the rise of metamaterials, artificial materials composed of subwavelength unit cells, has greatly enriched the design and functionality of ORCs. Here, we review zero-index hyperbolic metamaterials for constructing novel ORCs. First, this paper introduces the classification and implementation of zero-index hyperbolic metamaterials. Second, the distinctive properties of zero-index and hyperbolic cavities are summarized, including geometric invariance, homogeneous/inhomogeneous field distribution, topological protection (anomalous scaling law, size independence, continuum of higher-order modes, and dispersionless modes) for zero-index (hyperbolic) metacavities. Finally, the paper introduces some typical applications of zero-index and hyperbolic metacavities, and advances the research of metacavities.

Keywords: metamaterials, effective medium theory, photonic topological insulators, cavity modes

(Some figures may appear in colour only in the online journal)

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[*] Author to whom any correspondence should be addressed.

+21 more

Metamaterials for LiFi

- Light Fidelity (LiFi) requires media that can control wave amplitude, phase, polarization, and impedance arbitrarily

- Helps to have a better understanding of metamaterials!

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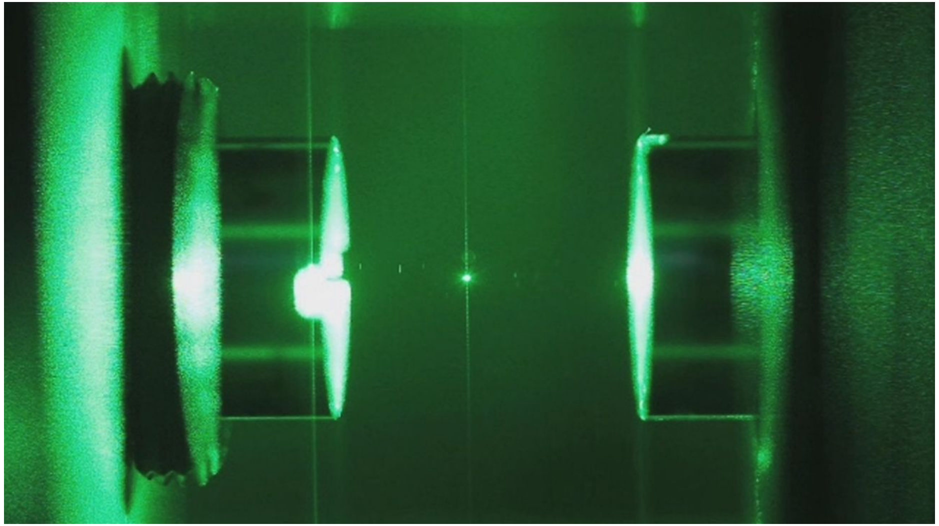
Trapping Light With Metamaterials



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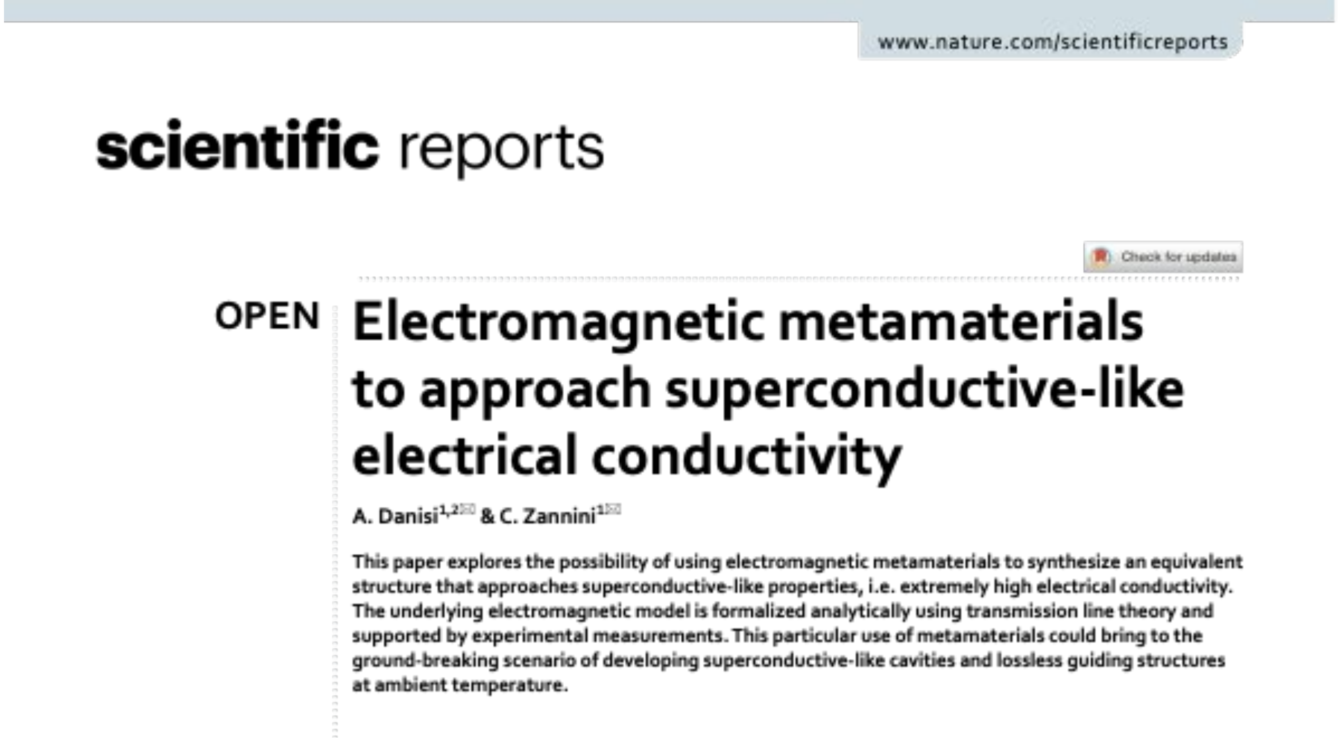
Metamaterials can be the new Superconductors?



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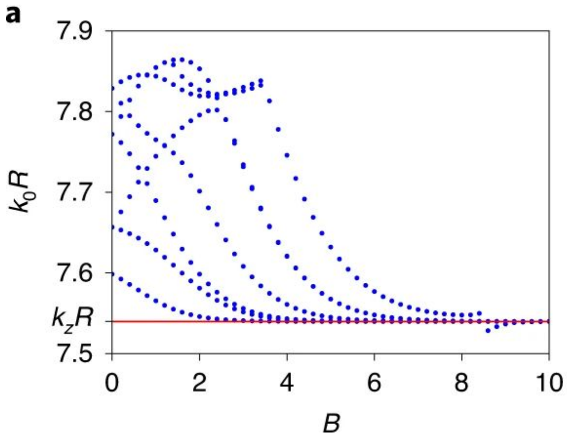
Electromagnetic metamaterials to approach superconductive-like electrical conductivity

A. Danisi[1,2]✉ & C. Zannini[1]✉

This paper explores the possibility of using electromagnetic metamaterials to synthesize an equivalent structure that approaches superconductive-like properties, i.e. extremely high electrical conductivity. The underlying electromagnetic model is formalized analytically using transmission line theory and supported by experimental measurements. This particular use of metamaterials could bring to the ground-breaking scenario of developing superconductive-like cavities and lossless guiding structures at ambient temperature.

Our comments about the paper



- Some figures are ambiguous.

- The abstract is well written.
    - The structure makes it easy to understand the main ideas of the research.
    - Highlight the interesting parts of the paper

- The paper provides different examples to highlight interesting effects due to zero refractive index which enhance our physical understanding of the calculations

Conclusions

- A general way to calculate the directional dependent refractive index and the condition for the zero index in given direction
- When the zero index direction is complex valued a material supports waves that can propagate in one sense
- Extending the concept of the refractive index

The End!

Backup slides

Extra notes

Magnitude of refractive index → magnitude of the inverse of the eigenvalues

Sign of refractive index → sign of $\boldsymbol{k} \cdot \mathbf{S}$ = $\operatorname{sign}\left[F_m^{\dagger}\chi F_m\right]$

A general linear EM material can be characterized

Maxwell's equations in 6-vector form:

$$\mathbf{D} = \epsilon_0 \epsilon \cdot \mathbf{E} + \frac{1}{c} \xi \cdot \mathbf{H}$$

$$\mathbf{B} = \mu_0 \mu \cdot \mathbf{H} + \frac{1}{c} \xi^{\dagger} \cdot \mathbf{E}$$

Results in **6 eigenvalues**

- 2 eigenvalues = 0
- Others: 2 pairs of equal magnitude (polarizations), opposite sign (direction)

Condition for zero-index from 2D Dirac equation solutions

At the **transition point** between the 2 behavior, $k^2 \rightarrow 0$.

Equivalent to zero refractive index, but with an unusual form.

- Ψ_b obeys the equation:

$$2\frac{\partial \psi_b}{\partial Z} = 0$$

Waves that obey *Cauchy-Riemann equations* **propagate in only 1 sense** around the origin.

- The solution represents **clockwise propagating waves**

CAN Media Support One-Way Waves!

$$\begin{aligned} \langle P_{\circlearrowleft} \rangle &= \frac{\eta_0 \mu_{zz} k_0 |j_0|^2}{16a^2} (a-b)^2 \\ \langle P_{\circlearrowright} \rangle &= \frac{\eta_0 \mu_{zz} k_0 |j_0|^2}{16a^2} (a+b)^2 \end{aligned}$$

- Sending a -> ± b sends n -> 0 along one direction
- Circulation dependent power!

Simple Eigenvalue Problem

Set up matrix for our complex valued permittivity

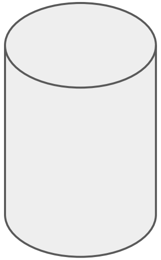
$$\epsilon = \begin{pmatrix} a & -ib \\ ib & a \end{pmatrix} = a\mathbb{1}_3 + ib\hat{\mathbf{z}}\times$$

Sending a -> ± b sends n -> 0 along one direction

$$\begin{aligned} \langle P_{\circlearrowleft} \rangle &= \frac{\eta_0 \mu_{zz} k_0 |j_0|^2}{16a^2}(a-b)^2 \\ \langle P_{\circlearrowright} \rangle &= \frac{\eta_0 \mu_{zz} k_0 |j_0|^2}{16a^2}(a+b)^2 \end{aligned}$$

Circulation dependent power!

Scattering from bianisotropic CAN medium



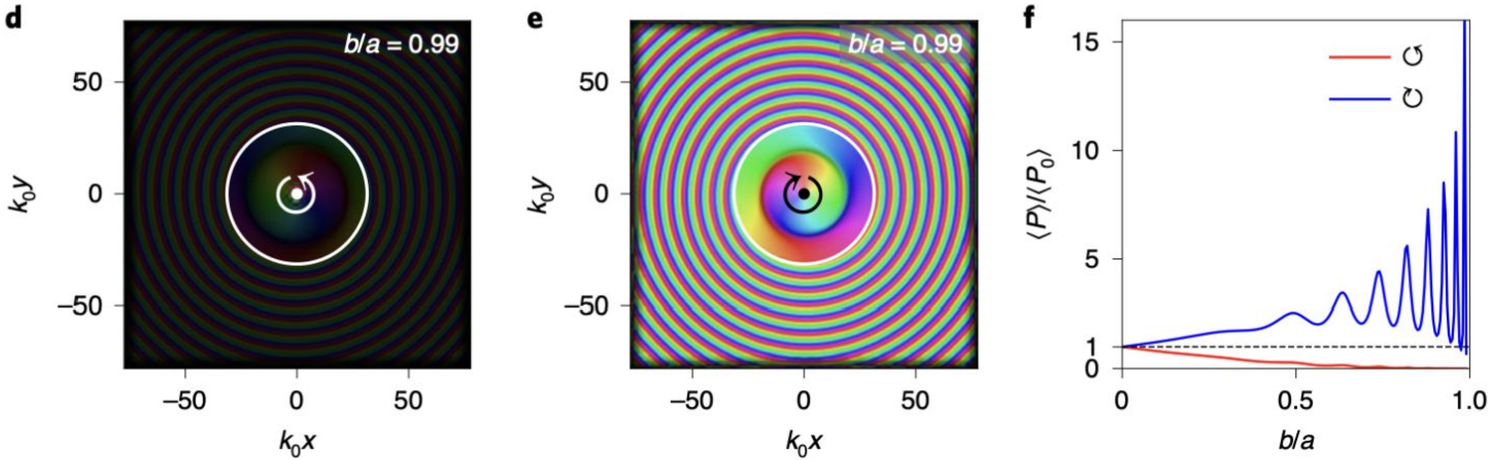
$$\Psi = E_z + \eta_0 H_z \qquad \mathbf{m} = \hat{\mathbf{x}} + i\hat{\mathbf{y}}$$

$$2\frac{\partial}{\partial \mathcal{Z}^\star}(E_z + \eta_0 H_z) = k_0\left[\mathbf{m}\cdot(\epsilon - \xi^\dagger)\cdot\mathbf{E} - \mathbf{m}\cdot(\mu - \xi)\cdot\eta_0\mathbf{H}\right] = 0 \tag{18}$$

$$\mathbf{m}\cdot\left(\epsilon - \xi^\dagger\right) = 0, \quad \mathbf{m}\cdot(\mu - \xi) = 0 \tag{19}$$

$$\epsilon = 1_3,\ \mu = 1_3,\ \xi = \lambda\begin{pmatrix} 0 & i & 0 \\ -i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{20}$$

Emission from CAN Cylinder



Comparison to other relevant work

- Horsley, S. A. R. Unidirectional wave propagation in media with complex principal axes. *Phys. Rev. A* **97**, 023834 (2018)
    - Same use of maxwell's equations
    - Focused on designing materials where waves can be trapped at an interface with a perfect conductor
- https://seas.harvard.edu/news/2017/10/zero-index-waveguide (2017)
    - Developed first on chip metamaterial with zero refractive index
    - A **zero-index waveguide**
- Horstmeyer, R., Ruan, H. & Yang, C. Guidestar-assisted wavefront-shaping methods for focusing light into biological tissue. *Nature Photon* 9, 563–571 (2015). https://doi.org/10.1038/nphoton.2015.140
    - Use **wavefront shaping** to focus light onto biological material (application of zero index)
- Davoyan, A. R. & Engheta, N. Phys. Rev. Lett. 111, 257401 (2013)
    - Showed (analytically) that the transparency of the medium can be altered with the magnetization
    - Predicted that one-way photonic surface states may exist at the interface
- Silveirinha, M. G. Phys. Rev. B 92, 125153 (2015)
    - Developed theoretical methods to topologically classify a wide class of bianisotropic continuous media

Highly Degenerate Bound states

- Zero-index condition affects bound states in electromagnetic materials, resulting in highly degenerate state
- The effect is identical to degeneracy in ground state of spin half particle in magnetic field

2D Dirac with Magnetic field

$$\left[-\mathrm{i}\boldsymbol{\sigma}\cdot(\boldsymbol{\nabla}-\mathrm{i}e\hbar^{-1}\boldsymbol{A})+\frac{mc}{\hbar}\sigma_z\right]|\psi\rangle=\frac{\omega}{c}|\psi\rangle$$

$$\mathcal{D}^\dagger\mathcal{D}|\psi\rangle=\frac{2m\Omega}{\hbar}|\psi\rangle \qquad \mathcal{D}=-\mathrm{i}\left(\frac{\partial}{\partial x}-\mathrm{i}e\hbar^{-1}A_x\right)+\left(\frac{\partial}{\partial y}-\mathrm{i}e\hbar^{-1}A_y\right)$$

We are interested in ground states where they get annihilated by D

$$\begin{aligned}\mathcal{D}|\psi\rangle&=\left[-\mathrm{i}\left(\frac{\partial}{\partial x}-\mathrm{i}e\hbar^{-1}A_x\right)+\left(\frac{\partial}{\partial y}-\mathrm{i}e\hbar^{-1}A_y\right)\right]|\psi\rangle\\&=-2\mathrm{i}\left[\frac{\partial}{\partial z^\star}+e\hbar^{-1}\frac{\partial\phi}{\partial z^\star}\right]|\psi\rangle\\&=0\end{aligned}$$

Example Materials which have

$$\epsilon=\frac{1}{\alpha}\begin{pmatrix}\mathbf{1}_2 & i\mathbf{a}^{\mathrm{T}}\\ -i\mathbf{a} & \beta\end{pmatrix} \qquad \mu=\alpha\begin{pmatrix}\mathbf{1}_2 & i\mathbf{a}^{\mathrm{T}}\\ -i\mathbf{a} & \beta\end{pmatrix}$$

In polar basis where $\Psi=E_z+i\alpha\eta_0H_z$ $\quad\mathbf{\Phi}=\mathbf{E}_{\parallel}+i\alpha\eta_0\mathbf{H}_{\parallel}$

With the assumption that Ψ=0 as the field is entirely in-plane

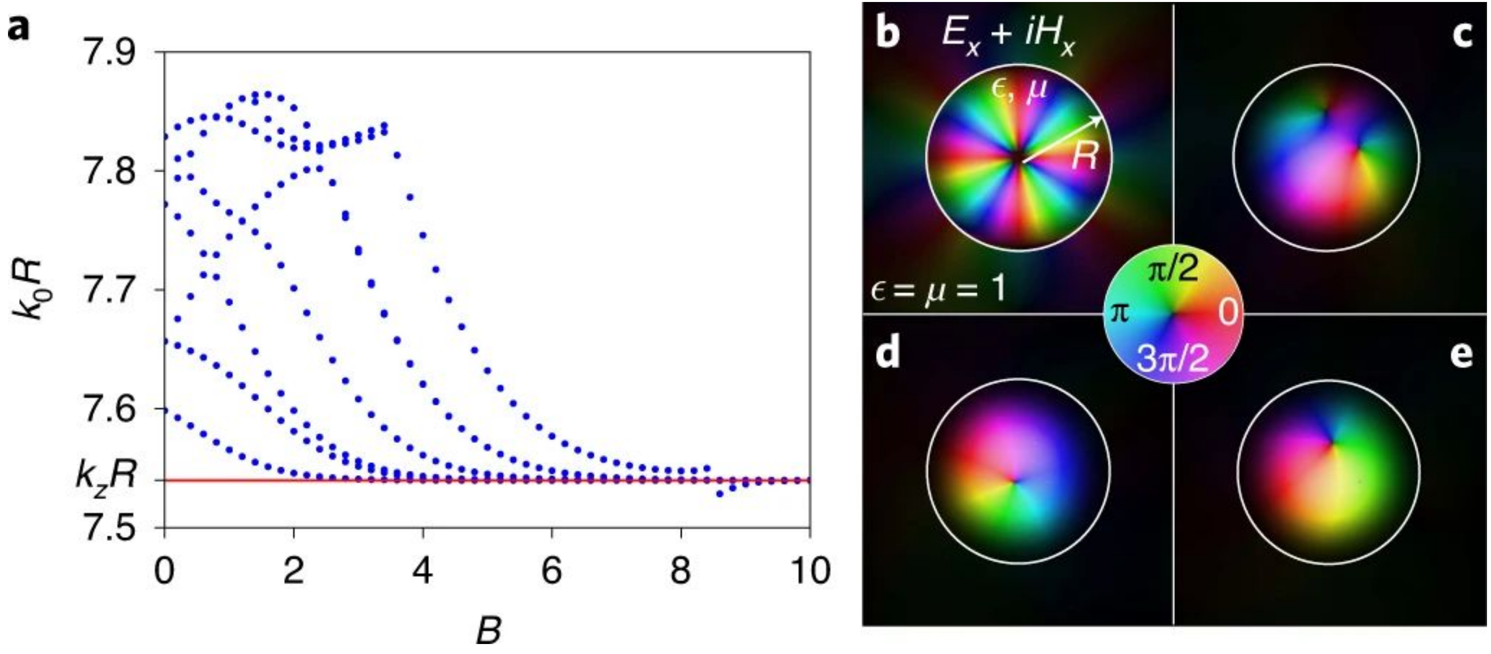
$$\begin{aligned}\left(\mathbf{1}_2-i\tfrac{k}{k_0}\hat{\mathbf{z}}\times\right)\cdot\mathbf{\Phi}&=0\\ \hat{\mathbf{z}}\cdot\boldsymbol{\nabla}\times\mathbf{\Phi}+ik_0\,\mathbf{a}\cdot\mathbf{\Phi}&=0\end{aligned}$$

In the zero refractive inc limit k->k0

$$\mathbf{\Phi}=(\hat{\mathbf{x}}+i\hat{\mathbf{y}})\Phi.$$

$$|\psi\rangle=\mathrm{e}^{-\frac{e\phi}{\hbar}}f(x+\mathrm{i}y). \qquad \Phi=f(x+iy)\exp(-k_0\psi-ik_0\phi) \qquad \left(\frac{\partial}{\partial x}+i\frac{\partial}{\partial y}\right)\Phi+k_0\left(\frac{\partial\psi}{\partial x}+i\frac{\partial\psi}{\partial y}\right)\Phi+ik_0\left(\frac{\partial\phi}{\partial x}+i\frac{\partial\phi}{\partial y}\right)\Phi=0$$

Bound states in a CAN medium with a pseudo magnetic field.



Outcomes and Looking Ahead

- Interface of quantum mechanics, complex analysis, and crystal optics
- New way to analyze and design one-way media
- Helps explain observed phenomena