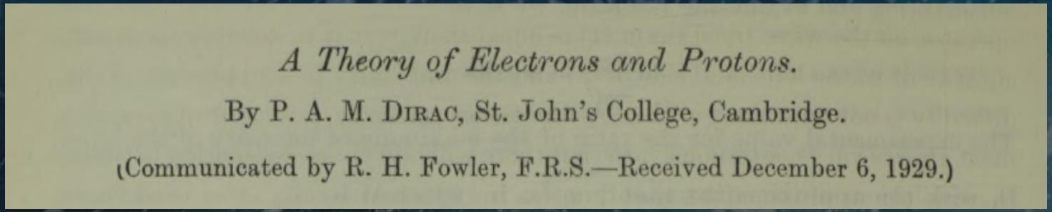
A Theory of Electrons and Protons.

By P. A. M. Dirac, St. John's College, Cambridge.

(Communicated by R. H. Fowler, F.R.S.—Received December 6, 1929.)

Team 4:

Becket Hill, Paul Harmston, Yuntao Guan,
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A Voyage to the Dirac Sea

A tale of two particles

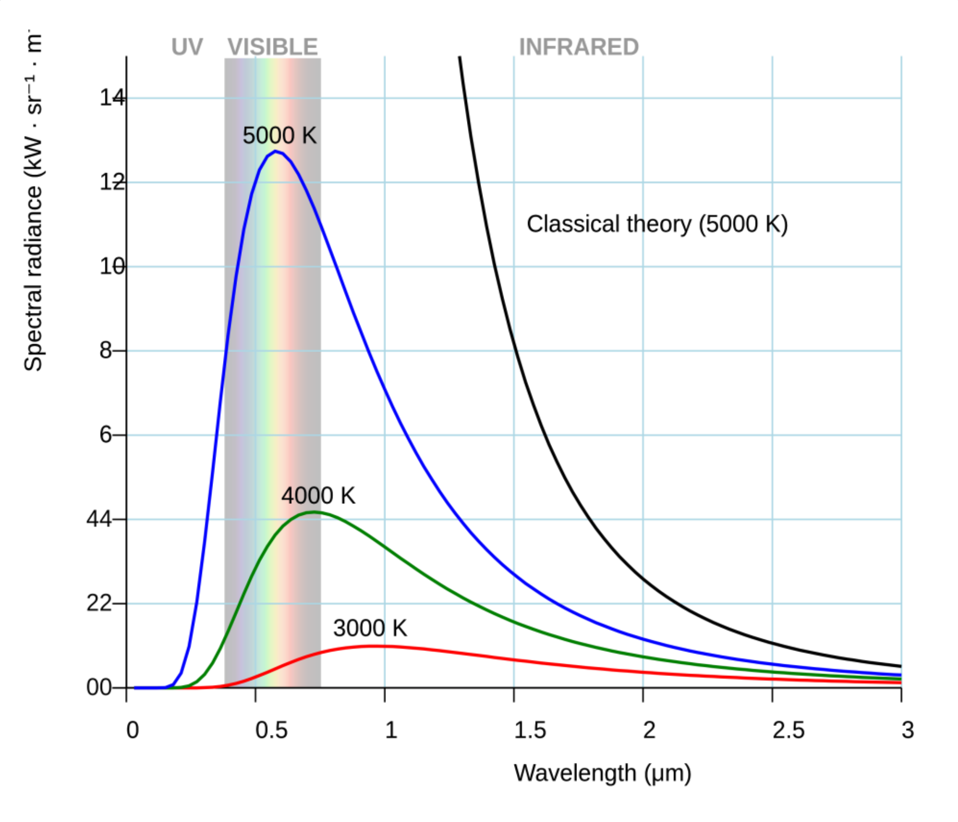
Glimpses of a World Beyond Classical: Black-Body Radiation (Plank, 1900)

- EM radiation of body at finite temperature
- **Classical:** arbitrary energy allowed at each frequency
- **Equipartition:** leads to UV divergence...

- **Solution:** Plank used a mathematical "trick"

$$E(\nu) = nh\nu$$

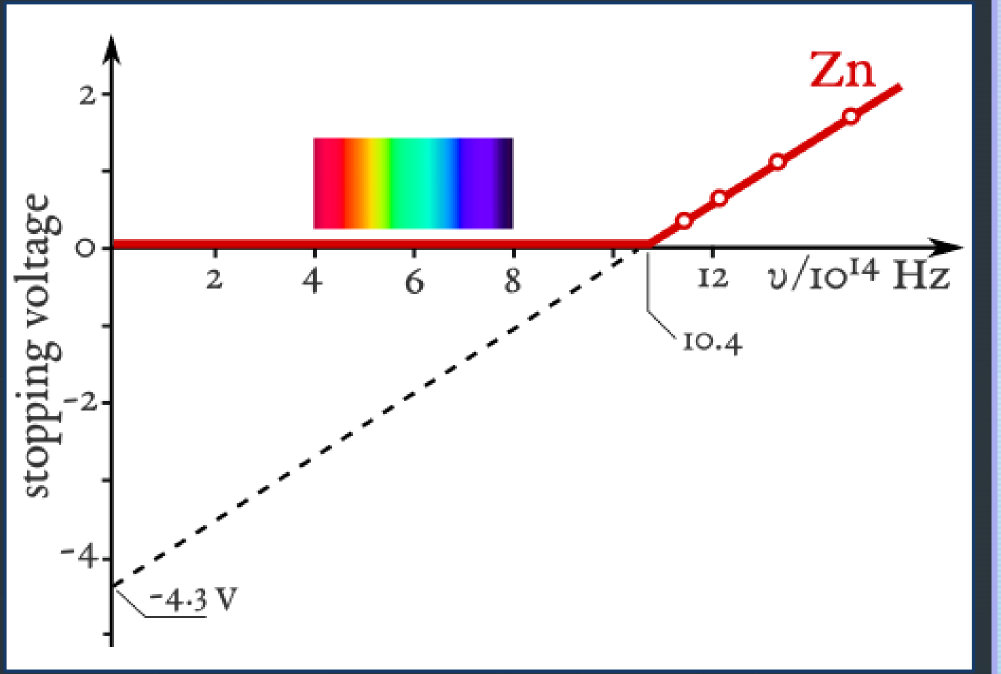
discrete quanta of energy
in each frequency mode



Glimpses of a World Beyond Classical: Photoelectric Effect (Einstein, 1905)

- Electrons emitted from material due to incident light
- **Classical:** Energy is proportional to intensity of light
- **Experiment:** Minimum frequency required, regardless of intensity...

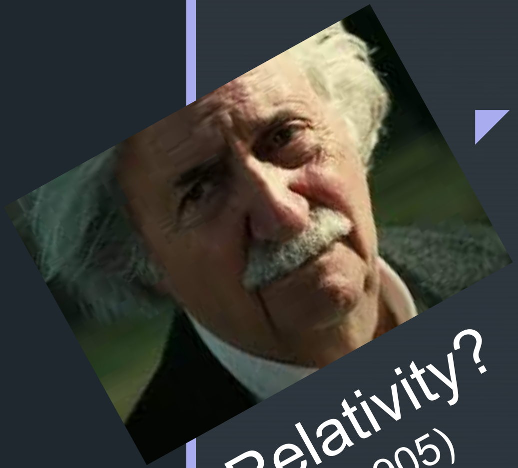
- **Solution:** Einstein takes Plank's idea seriously... **photons**!



$$E = h\nu$$

Each electron only absorbs one photon

Seems to be a pattern...



A Cool Cat Called Schrödinger

- In 1926, Schrödinger put the pieces together
- We can motivate the Schrödinger equation by considering the classical energy of a ***non-relativistic*** particle

$$E = T + V$$

non-relativistic dispersion

$$E \to \hat{E} = i\hbar\frac{\partial}{\partial t}, \quad T \to \hat{T} = \frac{\hat{p}^2}{2m}, \quad \hat{p} = -i\hbar\frac{\partial}{\partial x}$$

put hats on things

$$\hat{E}\psi = (\hat{T} + \hat{V})\psi$$

and...

$$\boxed{i\hbar\frac{\partial}{\partial t}\psi = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right)\psi}$$

Schrodinger Eq!

The Klein-Gordon Equation: A Relativistic Quantum Theory

- In 1926, Klein and Gordon developed a quantum theory which is manifestly Lorentz invariant
- We can do so by quantizing the ***relativistic*** dispersion relation

$$E^2 = (pc)^2 + (mc^2)^2$$

relativistic dispersion

$$E \to \hat{E} = i\hbar\frac{\partial}{\partial t}, \quad p \to \hat{p} = -i\hbar\frac{\partial}{\partial x}$$

put hats on it... again

$$\boxed{\left[\frac{1}{c^2}\underbrace{\frac{\partial^2}{\partial t^2}}_{} - \underbrace{\frac{\partial^2}{\partial x^2}}_{} + \frac{m^2c^2}{\hbar^2}\right]\psi = 0}$$

Klein-Gordon Eq!

Lorentz invariant!

Spin?

But problems persist...

A Problem in KG Equation: Probability Density is not Positive-Definite

- Continuity Equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0 \qquad \rho = i\hbar(\psi^* \frac{\partial \psi}{\partial t} - \psi \frac{\partial \psi^*}{\partial t})$$

- One more initial condition can give us negative density:

$\frac{\partial^2 \psi}{\partial t^2}$ in KG Equation $\Longrightarrow$ $\psi|_{t=0}$ & $\boxed{\frac{\partial \psi}{\partial t}|_{t=0}}$ are needed

$\Downarrow$

An extra degree of freedom

- Choose eigenstate as an example:

If $\psi = \Phi(\mathbf{r})e^{-iEt/\hbar} \Longrightarrow \rho = 2E|\Phi|^2 < 0$ for $E < 0$

How to Construct the Dirac Equation

Taking the square root of the KG Equation

- First-order in both space and time

$$(A\partial_x + B\partial_y + C\partial_z + \frac{i}{c}D\partial_t)\psi = \zeta\psi$$

- Recover KG equation to get relativistic E-P relation again

$$(A\partial_x + B\partial_y + C\partial_z + \frac{i}{c}D\partial_t)^2 = \nabla^2 - \frac{1}{c^2}\partial_t^2 \qquad \zeta^2 = (\frac{mc}{\hbar})^2$$

- Four numbers are not enough, we need four matrices!

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} I_{4\times 4} \qquad \boxed{(i\hbar\gamma^\mu\partial_\mu - mc)\psi = 0}$$

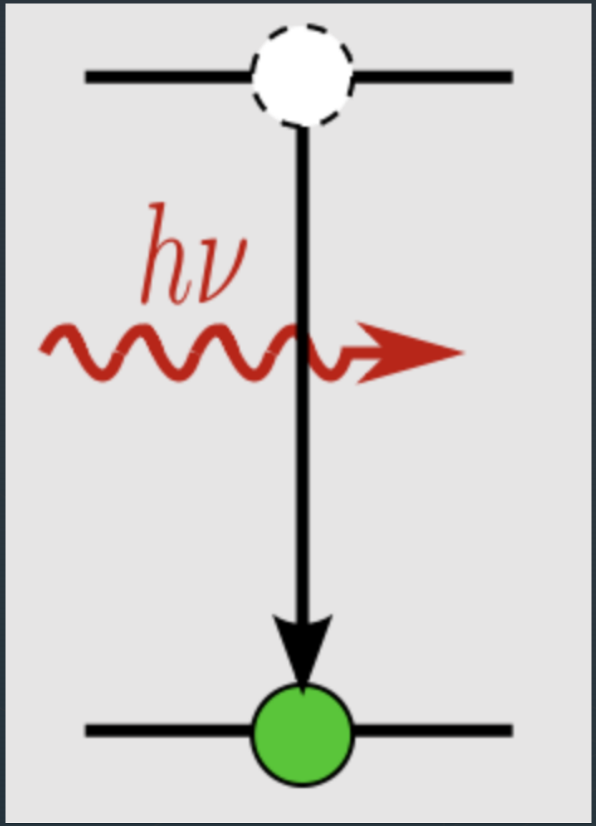
Spinor!

- Probability density is positive-definite again

$$\frac{\partial}{\partial t}(\psi^\dagger\psi) + \nabla\cdot(\psi^\dagger\boldsymbol{\alpha}\psi) = 0 \qquad \rho = \psi^\dagger\psi \geq 0$$

But, how to explain the NEGATIVE energy solution?

A Theory of Electrons and Protons: Accounting for Negative Energies

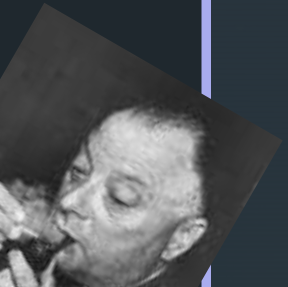


Problem:

- Dirac equation implies negative energy solutions -> Electron could emit infinite photons

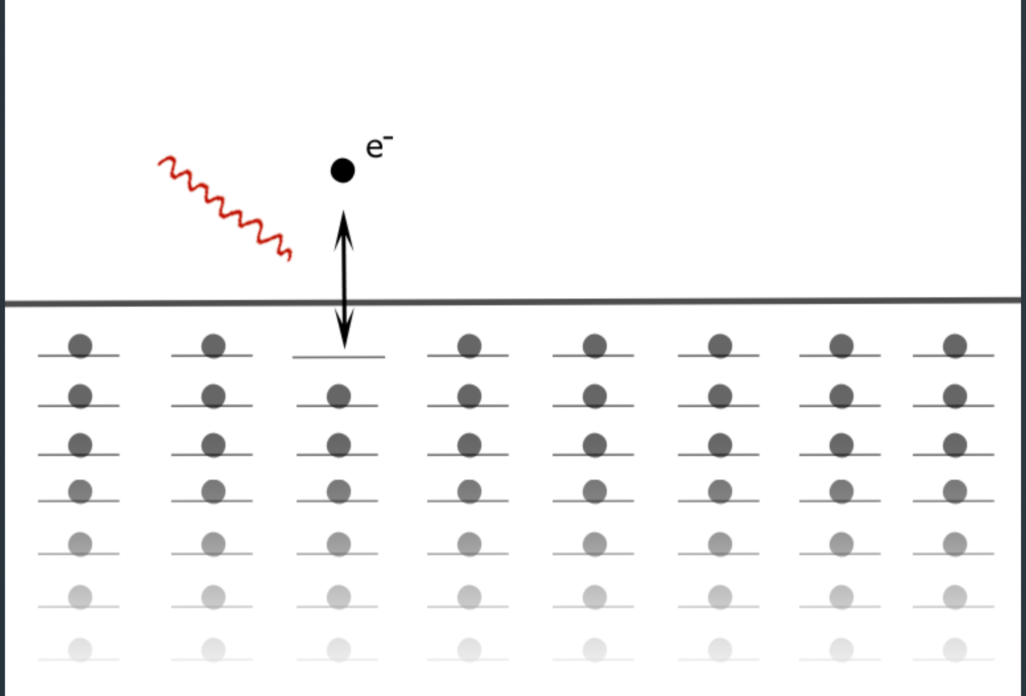
Solution:

- Negative energy states are all filled in a giant electron "sea" -> Pauli Exclusion applies



A Theory of Electrons and Protons: When a Vacuum isn't a Vacuum!

- So, the vacuum is not actually a vacuum??
- It's just an infinite negative charge!
- With an occasional "anti-electron" hole
  - Dirac theorized this to be the proton



Oppenheimer first reading Dirac's "A Theory of Electrons and Protons" (colorized)

Stormy Seas: Oppenheimer's Response

- The proton can't be the anti-electron, since they need to annihilate each other
- If the proton and electron annihilated each other, the universe would be nothing but radiation
- There must be an infinite density of positive electricity to prevent the divergence of the E field to be infinite everywhere

How to detect a positron: instrumentation

1911 Wilson Cloud Chamber

Directly observe particle tracks

Piston + film allowed rapid observations

1927 Dmitri Skobeltsyn

Discovers ionizing cosmic rays

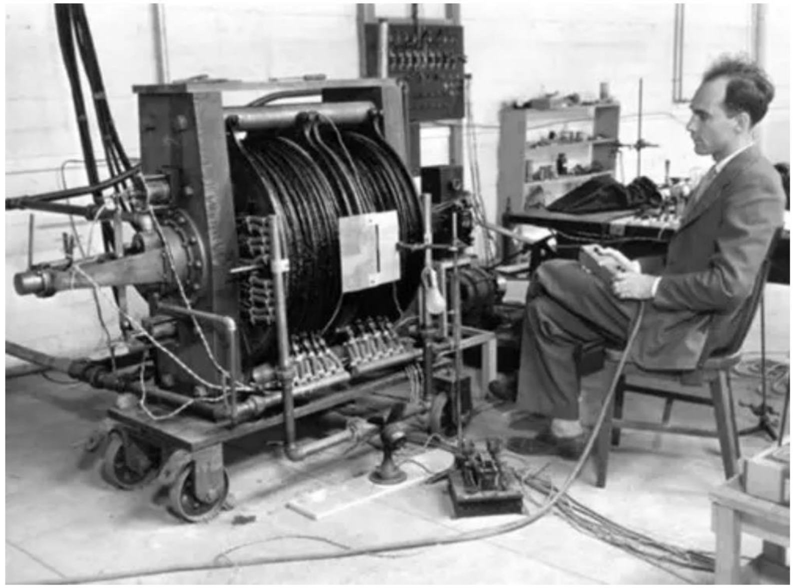
Adds magnetic field to cloud chamber

1929 Chung-Yao Chao

Grad student at Caltech

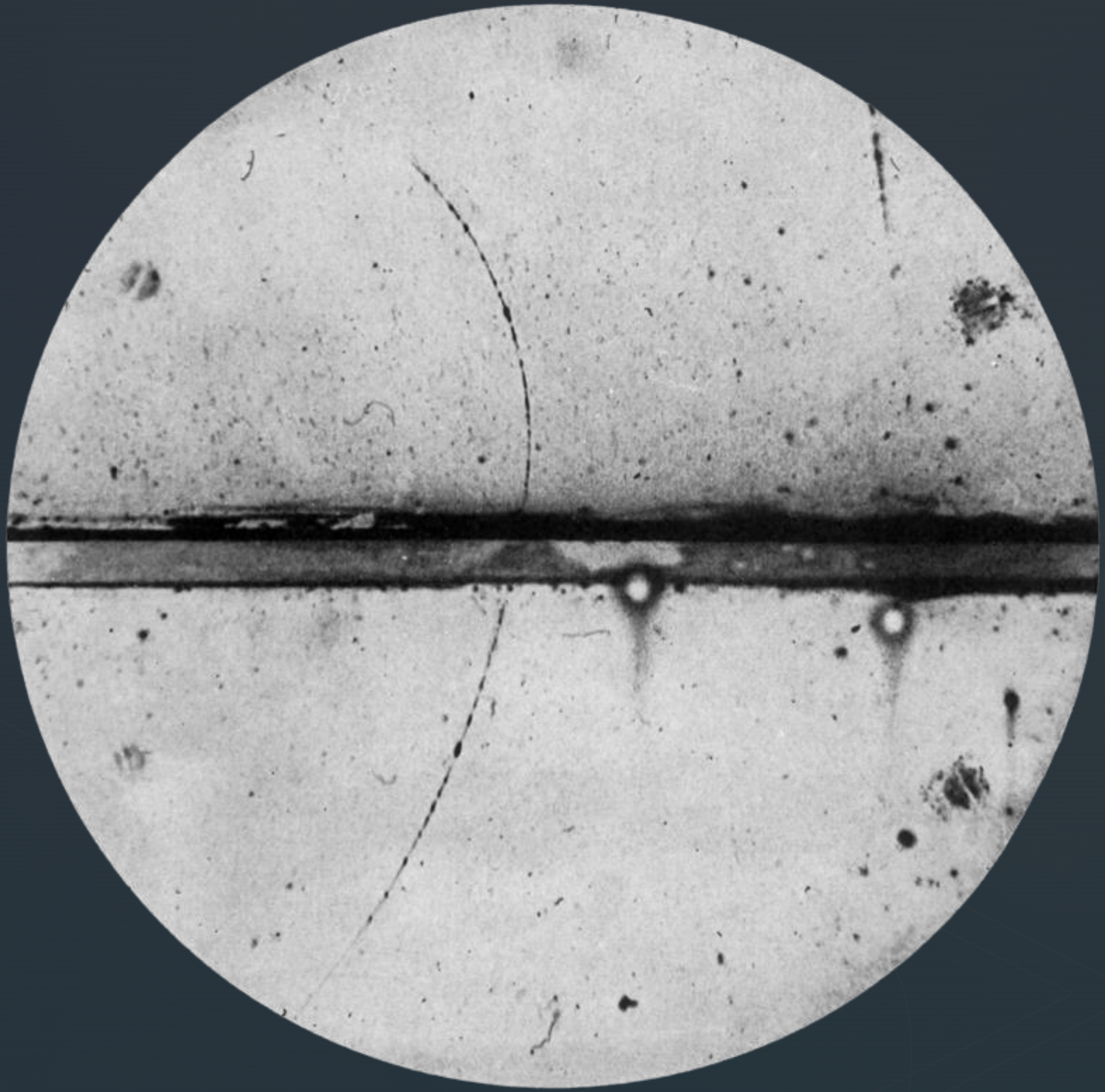
First noticed positive charge particle tracks in cloud chamber

1933 Anderson's Discovery of the Positron!

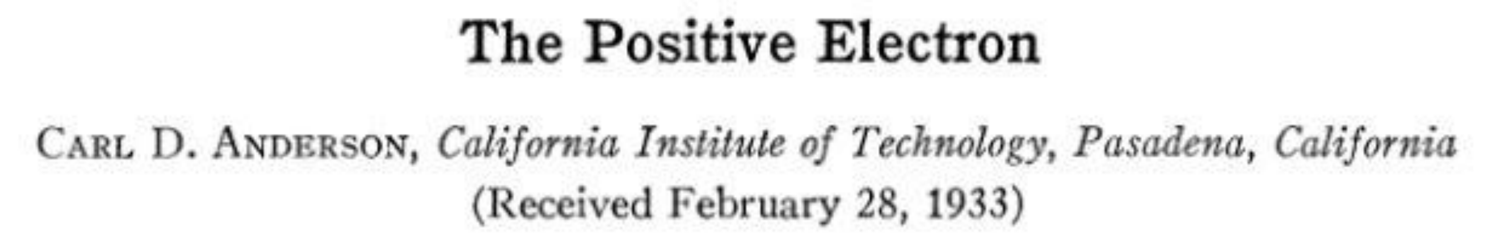


Carl Anderson with his cloud chamber surrounded by an electromagnet, Caltech, 1932 (Caltech.edu)

- 1300 photographs, 15 positron tracks
- lead plate used to illustrate trajectory direction and determine sign of charge
- Close to electron charge, protons eliminated due to curvature



Implications of positron discovery



The Positive Electron

Carl D. Anderson, *California Institute of Technology, Pasadena, California*

(Received February 28, 1933)

Anderson notes the following conclusions from the positron discovery.

"The positron should prove a stimulus to search for evidence of the existence of negative Protons."

"[the charge of the] positron is very probably equal to that of a free negative electron"

"If the neutron should prove to be a fundamental particle … the proton will then in all probability be represented as a complex particle consisting of a neutron and positron."

Where is the Dirac Equation Now?

From the Sea to QED

Feynman in 1959
after discovering
QED (colorized)

Used in modern quantum electrodynamics

$$S_{\text{QED}} = \int d^4x \left[-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \boxed{\bar{\psi} \left(i\gamma^\mu D_\mu - m \right) \psi} \right]$$

ψ is a Dirac field!

- Modern quantum field theory describing EM interactions
- The electron (ψ) still obeys a Dirac equation
- The modern "theory of electrons and p(r)hotons"

Dirac Equation and the Standard Model

Dirac kinetic term in the action for each fermion (ψ)

$$i\bar{\psi}\gamma^\mu\partial_\mu\psi \longrightarrow i\gamma^\mu\partial_\mu\psi$$

e.g. The electroweak interaction

$$\mathcal{L}_{\mathrm{EW}} = \sum_\psi \bar{\psi}\gamma^\mu\left(\boxed{i\partial_\mu} - g'\frac{1}{2}Y_{\mathrm{W}}B_\mu - g\frac{1}{2}\boldsymbol{\tau}\mathbf{W}_\mu\right)\psi$$

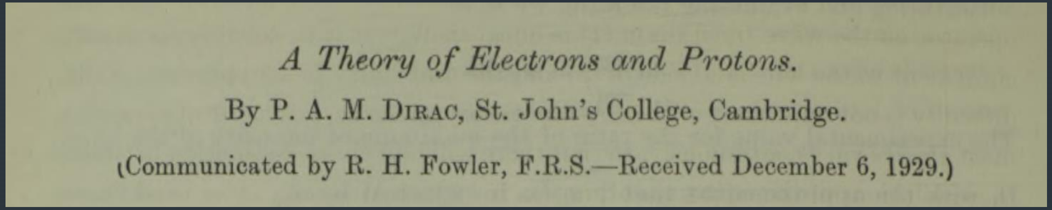
...and quantum chromodynamics

$$\mathcal{L}_{\mathrm{QCD}} = i\overline{U}\left(\boxed{\partial_\mu} - ig_sG^a_\mu T^a\right)\gamma^\mu U + i\overline{D}\left(\boxed{\partial_\mu} - ig_sG^a_\mu T^a\right)\gamma^\mu D.$$

Up quark

Down quark

Only 1154 Citations



A Theory of Electrons and Protons.

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- The Dirac Sea was a landmark paper that predicted the existence of antimatter
- Dirac's interpretation was made mostly obsolete by the development of modern QED
- Even though it wasn't as timeless as its predecessor (which has 6000+ citations), it was a fascinating look into Dirac's mind and the interesting conclusions he reached with the limited knowledge of his time.

Thanks for listening!!!
Dirac Sea you later!!