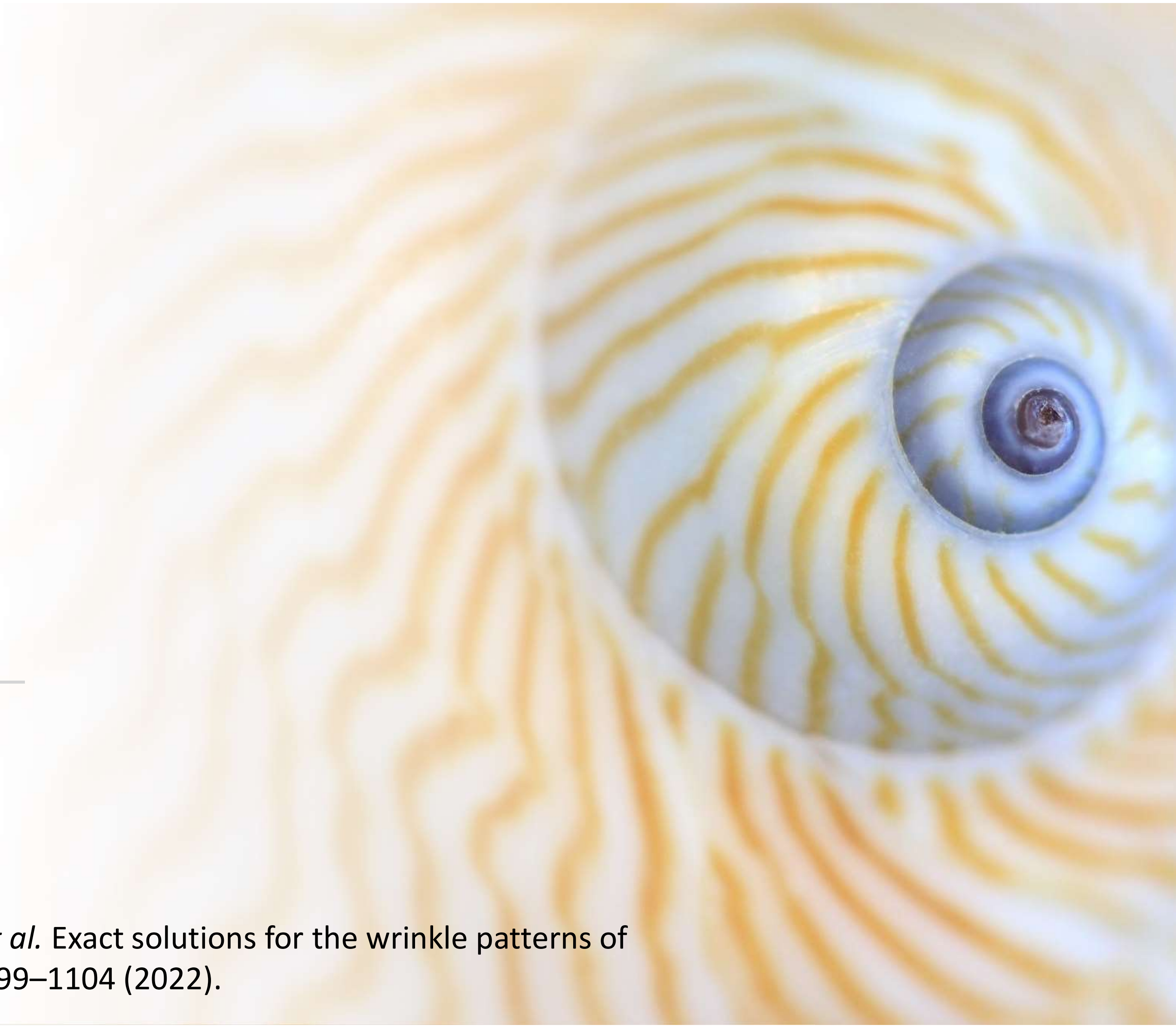
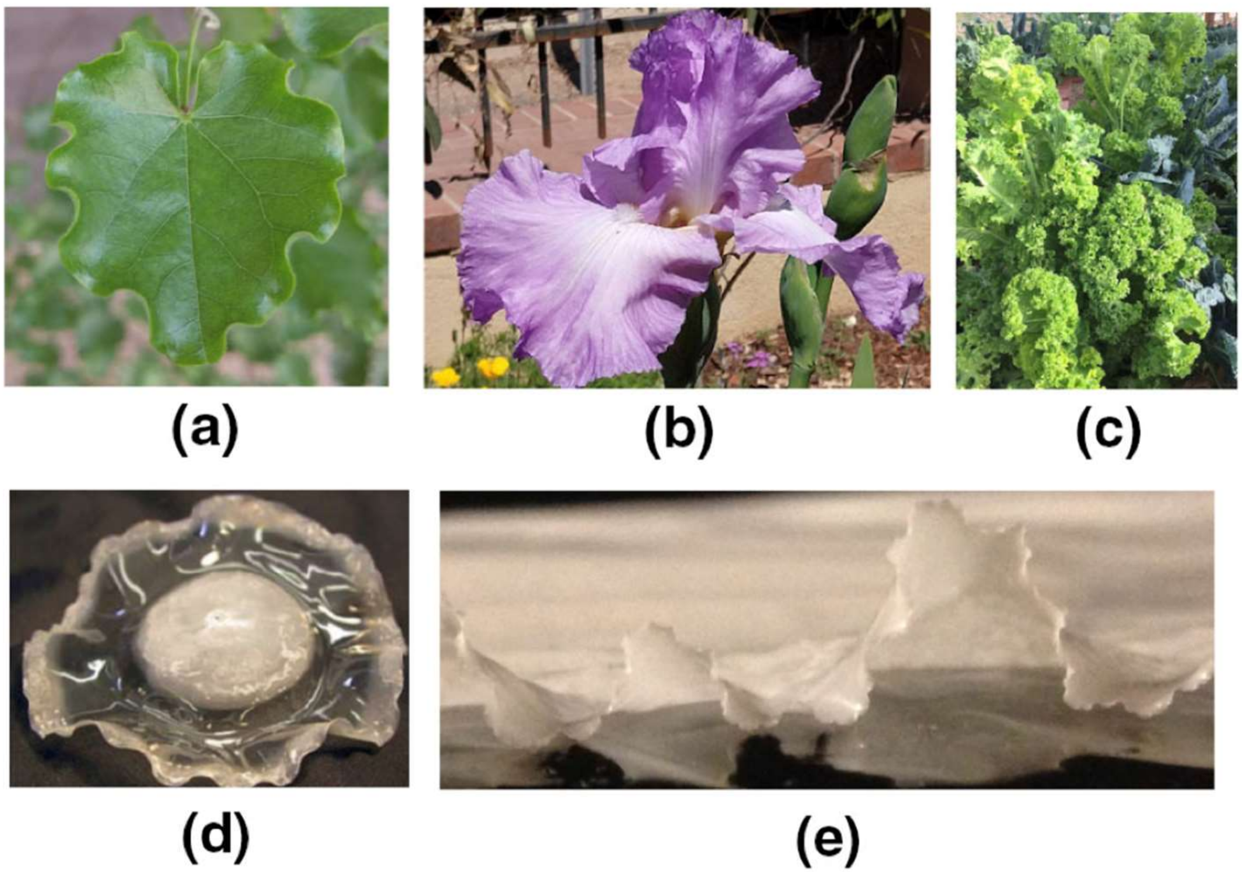
"Exact Solutions for the Wrinkle Patterns of Confined Elastic Shells" by Tobasco *et al.*

By Sounak Sinha, Chris Vairogs, Hao Zhang, Katie Zine, and Jack Zwettler



Tobasco, I., Timounay, Y., Todorova, D. *et al.* Exact solutions for the wrinkle patterns of confined elastic shells. *Nat. Phys.* **18**, 1099–1104 (2022).

Wrinkling is ubiquitous



Wrinkling can be observed in...

- Flowers and leaves
- Hydrogel disks
- Torn trash bags
- Weather balloons
- Foldable mirrors in space missions

- It is often important to know when and where wrinkles will develop

Yamamoto, K.K., Shearman, T.L., Struckmeyer, E.J. *et al.* Nature's forms are frilly, flexible, and functional. *Eur. Phys. J. E* **44**, 95 (2021).

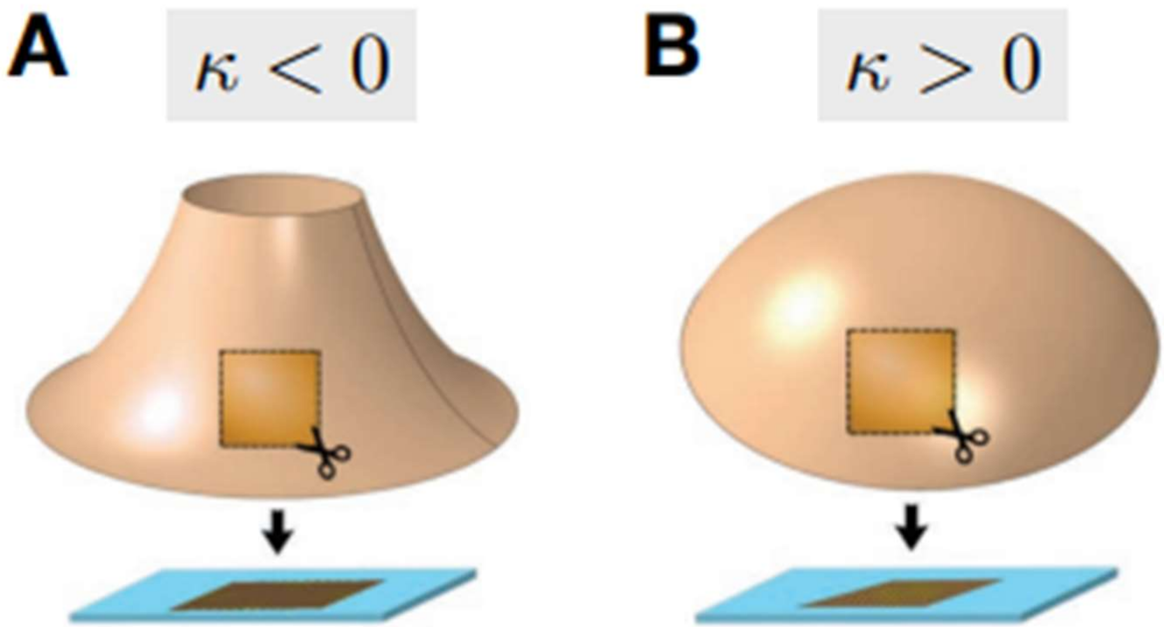
Controlling complex wrinkled surfaces is important

- Theoretically important in understanding geometric non-linearities in elasticity.
- Has practical applications in engineering such as lithography free microplating.
- No general predictive theory of wrinkles.

Article Overview

- Wrinkling patterns of thin sheets are investigated experimentally and in simulation.
- A model using Lagrange multipliers is developed that predicts the nature of wrinkling in sheets with defined curvature on a liquid substrate.

The experimental setup

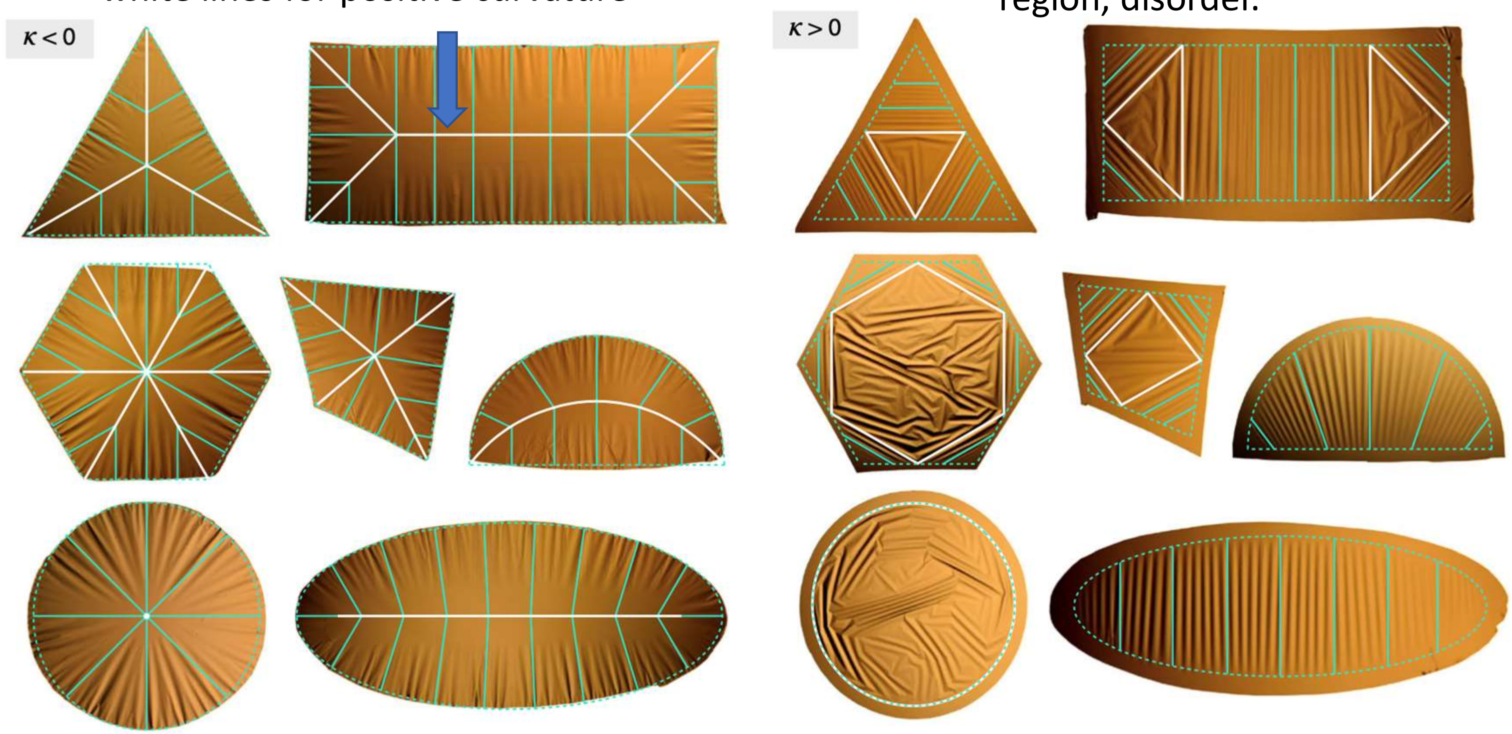


A square is cut out from a sphere (A) and a saddle (B) and placed on a planar liquid bath. κ is the Gaussian curvature of the initial surface.

The curvature determines the wrinkling behavior

Wrinkles perpendicular to
white lines for positive curvature

Wrinkles don't cover entire
region, disorder.



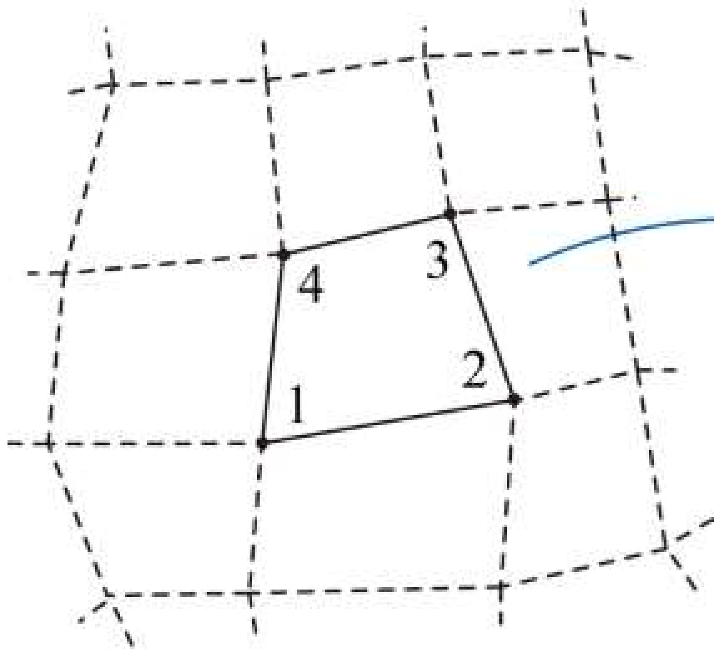
Wrinkles align with the green lines, and white lines designate boundaries between regions with different wrinkling regimes.

Films with negative Gaussian curvature (κ) behave differently than films with positive Gaussian curvature.

Upon performing the experiment these images were obtained.

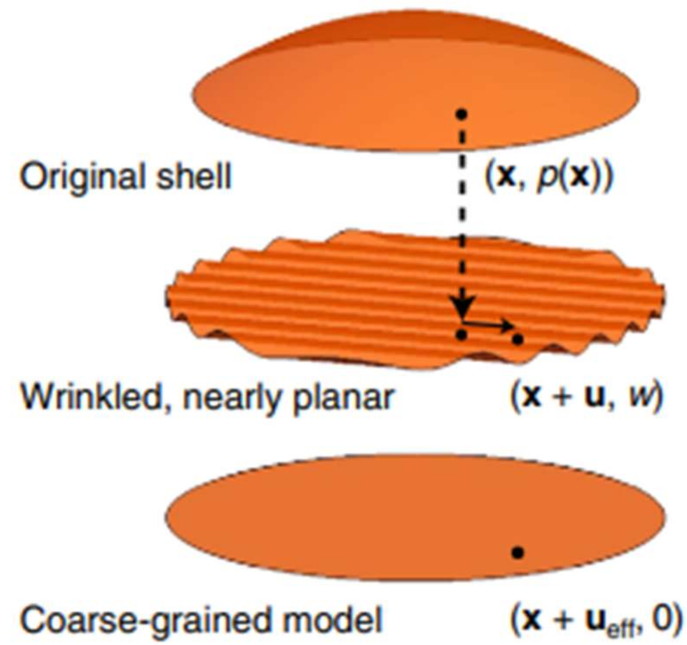
Simulating Wrinkles Using Finite Element Method

- Membranes modeled using the ABAQUS **finite element** software package.
- Membranes represented as mesh of 4-node elements.
- Deformation of the membranes as they adapt to a planar substrate using free boundary conditions.
- Results consistent across hundreds of simulations.



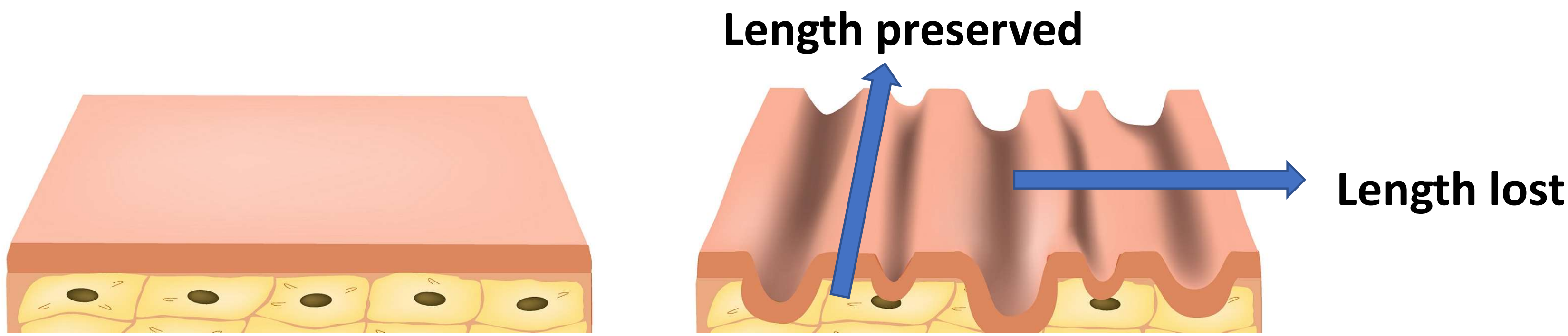
Strain Tensor Describes Local Behavior of Deformations

- **Strain tensor** describes how much the displacement of points in a material varies locally as the material is deformed.
- **Effective displacement** = deformed shape of membrane with the wrinkles course-grained away.
- **Effective strain tensor** = strain tensor for the effective displacement of the initial membrane.



Strain Eigenvalue Sign Determines Length Preservation

- Effective strain tensor has **nonpositive** eigenvalues.
- Eigenvectors of effective strain tensor with eigenvalue **zero** = directions in which length **preserved** = direction of peaks and troughs of linear **wrinkles**.
- Eigenvectors of effective strain tensor with **negative** eigenvalue = directions in which length is **lost**.



Behavior of Effective Strain Tensor

- Tobasco et al. Showed existence of nonnegative scalar field α and vector field **T** that satisfy

$$\alpha \varepsilon_{\text{eff}}(\mathbf{T}) = 0$$

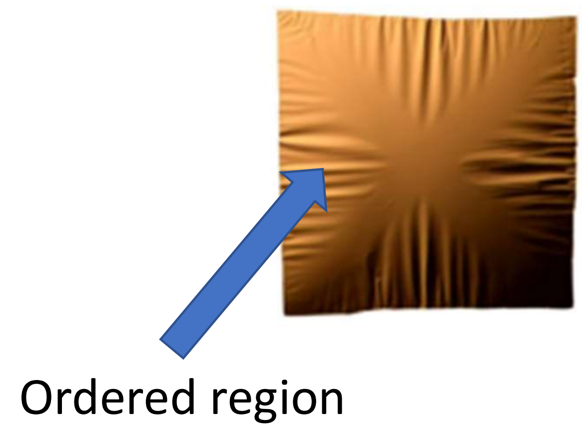
- Note that α>0 implies

$$\varepsilon_{\text{eff}}(\mathbf{T}) = 0.$$

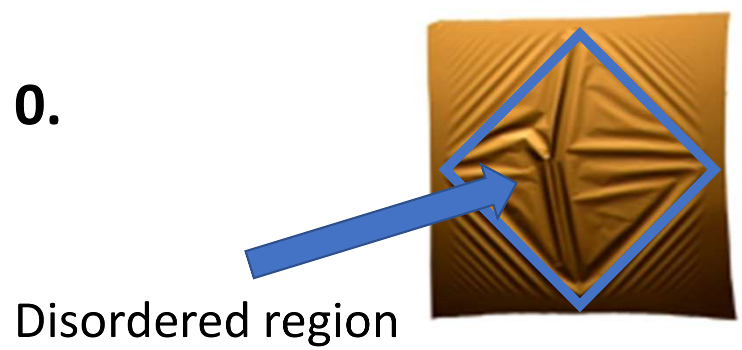
- Thus, wrinkles form in direction of **T** in regions with α>0.
- Note that α=0 implies nothing about eigenvalues of effective strain tensor.
- Thus, can't make predictions about wrinkles in regions with α=0.

Disordered Regions and Wrinkle Direction

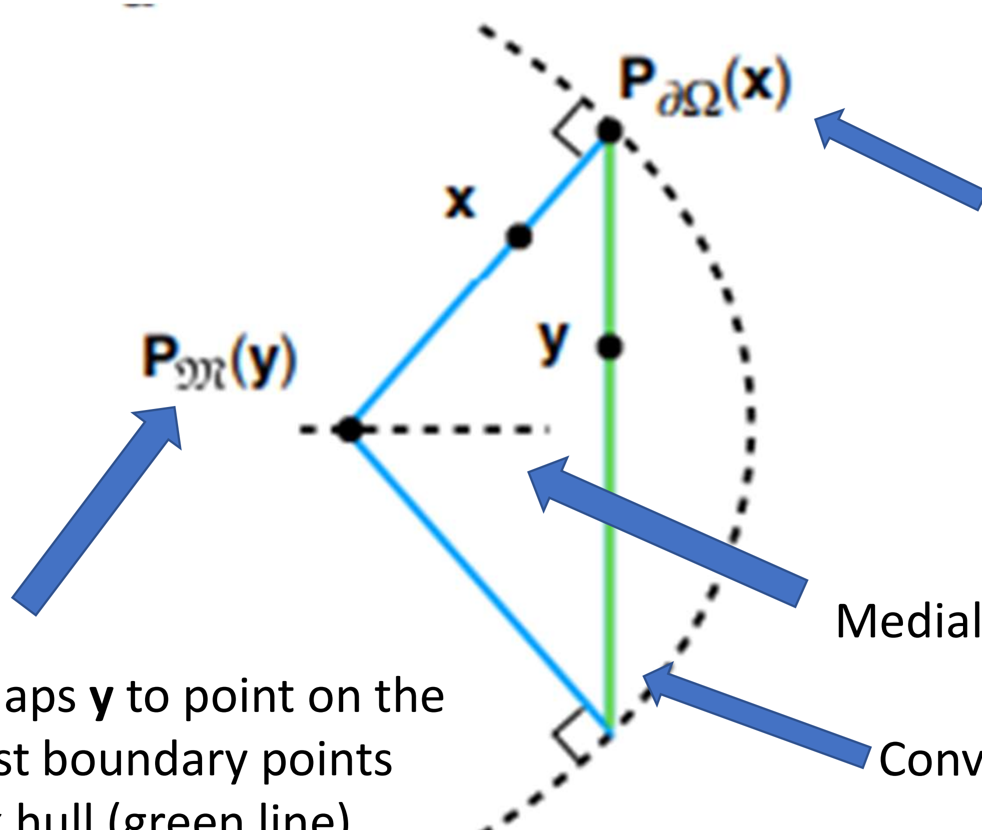
- **Ordered** regions = regions where **α > 0**.



- **Disordered** regions = regions where **α = 0.**
- In ordered regions, wrinkles form in direction of **T.**



Defining Fields for Wrinkle Formation Calculation



$\mathrm{P}_{\partial\Omega}$ is a vector field that maps **x** to its nearest boundary point

Medial axis

Convex Hull

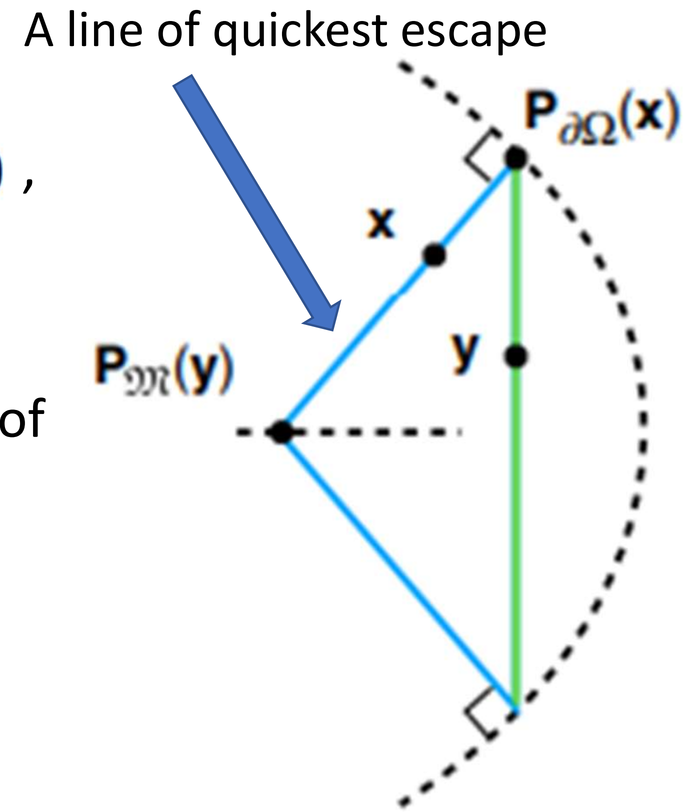
Pm is vector field that maps **y** to point on the medial axis whose closest boundary points contain **y** in their convex hull (green line)

Rules of Wrinkle Formation for Initial Negatively Curved Membranes

- Tobasco et al. showed using a Lagrange multiplier method that whenever Gaussian curvature $\kappa < 0$,

$$\mathbf{T} \cdot \nabla \mathbf{P}_{\partial\Omega} = \mathbf{0}.$$

- Thus, the "rate of change" of $\mathrm{P}_{\partial\Omega}$ in the direction of **T** is zero.
- But since $\mathrm{P}_{\partial\Omega}$ is constant along the lines of quickest escape, we must have that **T** is in the direction of quickest escape in ordered regions.
- **Conclusion**: wrinkles form along lines of **quickest escape** when $\kappa < 0$.

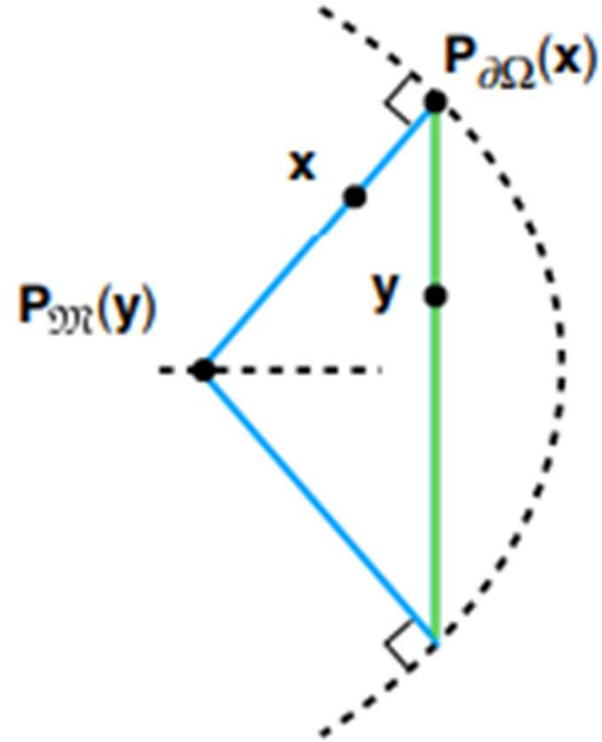


Rules of Wrinkle Formation for Initially Positively Curved Membranes

- Similarly, Tobasco et al. also showed that whenever $\kappa > 0$, we have

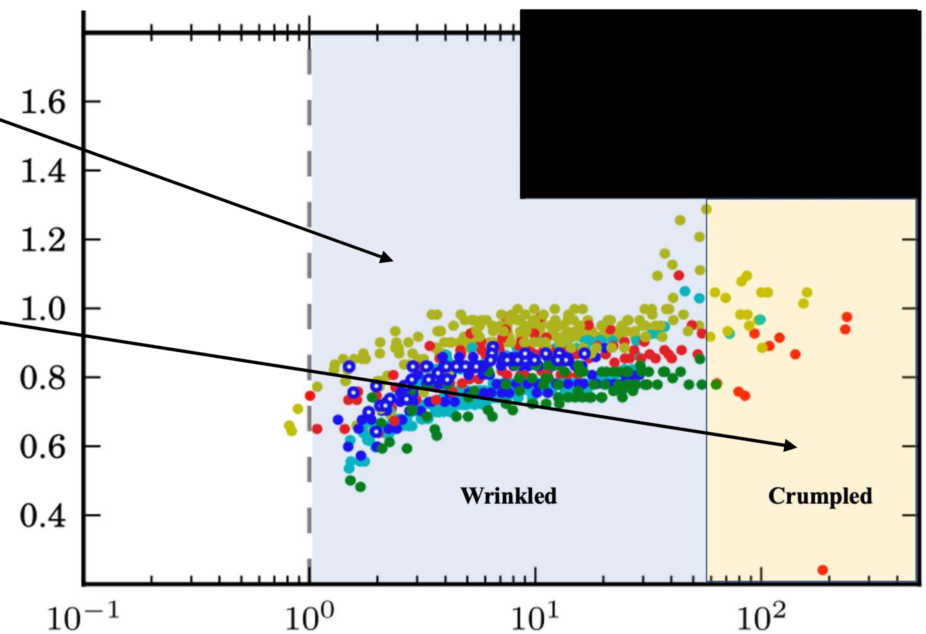
$$\mathbf{T} \cdot \nabla \mathbf{P}_{\mathfrak{M}} = \mathbf{0}.$$

- This can be used to explain the wrinkle formation rules for surfaces with positive Gaussian curvature.



Wrinkling is an ordered phenomena

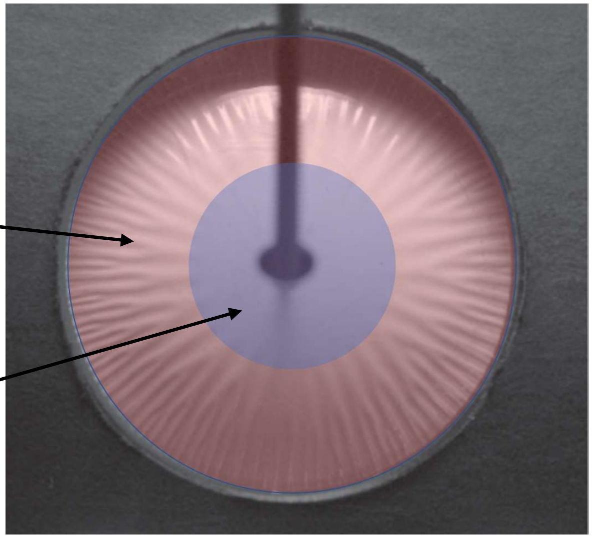
- At low confinement, ordered wrinkling persists (blue-shaded region)
- At high confinement, disordered crumpling dominates (tan-colored region)
- The inset is an image of disordered crumpling.



King, H., Schroll, R. D., Davidovitch, B. & Menon, N. Elastic sheet on a liquid drop reveals wrinkling and crumpling as distinct symmetry-breaking instabilities. *Proc. Natl Acad. Sci. USA* **109**, 9716–9720 (2012).

Wrinkles are driven by compression

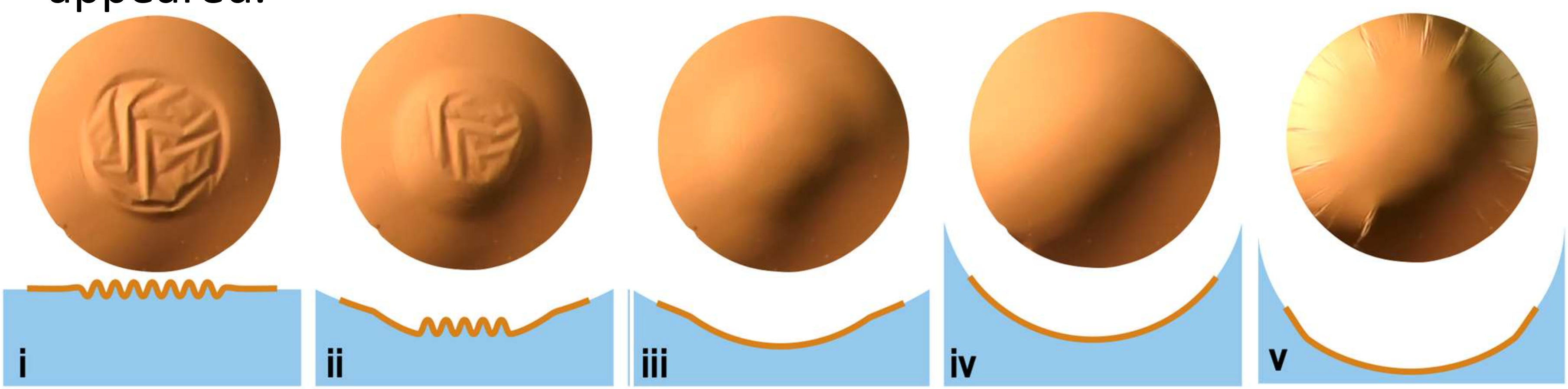
- Wrinkles form when there is local compressive force.
- This circular sheet pressed onto a sphere (effectively $\kappa < 0$) could have wrinkles extending to its center according to the paper's rules.
- However, the magnitude of the curvature confines wrinkles to the red-shaded region, where local compression is present.
- The blue-shaded region is devoid of wrinkles because the local force is tensile.



Hure, J., Roman, B. & Bico, J. Stamping and wrinkling of elastic plates. *Phys. Rev. Lett.* **109**, 054302 (2012).

The sign of the curvature determines the wrinkling regime

- Another group used a curved sheet and put it on a flat liquid interface
- At low applied pressure, the relative curvature of the sheet was positive, leading to disordered wrinkling
- At higher applied pressure, the relative curvature decreased to roughly 0
- Eventually, the relative curvature became negative, and ordered wrinkles appeared.



Timounay, Yousra et. al. Sculpting Liquids with Ultrathin Shells. *Phys. Rev. Lett.* **127**, (2021).

Conclusions of the paper

Several simple rules are discovered by the paper:

- Minimize the total energy, including the energy of bending and stretching the shell and gravitational potential energy of the bath plus its liquid surface energy.
- In the studied case, the confined shell is tension free.
- Introduce an “effective stress” to pair with the effective strain, which is mathematically a Lagrangian multiplier.

Conclusions of the paper

- **These rules imply a string of predictions about the nature of confinement-driven wrinkling,**

•A typical shell exhibits ordered wrinkle domains. The existence of disordered wrinkle domains.

•The arrangement of these domains is tied to the shell's medial axis, a distinguished locus of points from geometry.

•The wrinkled topography depends only on the sign of its Gaussian curvature.

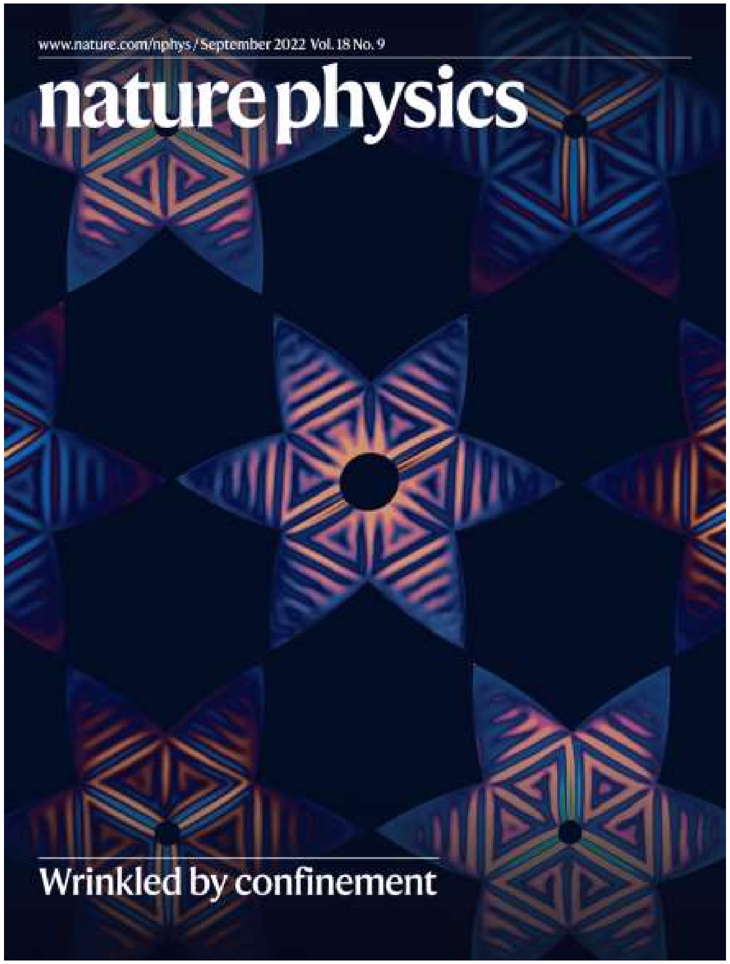
•The wrinkle domains of oppositely curved shells are reciprocally related so that the response of a given shell can be deduced from another.

Our Conclusions

- The rules the authors summarized are concise and simple, but worked very well and explains many properties of the wrinkles.
- One of the key rules that make this paper outstanding is their conclusion on the role of the sign of the Gaussian curvature.
- Since ordered wrinkles follow specific rules, it makes sense that they are following formulas, which the paper derives.

Citation Evaluation

- Published September 2022 in Nature Physics
- Published on arXiv in April 2020 as "Principles of Pattern Selection for Confined Elastic Shells"
  - Updated in December 2020
  - New title
- No citations in SCOPUS
- 9 citations in NASA ADS
  - 7 from arXiv version
  - 2 from Nature version
- Via Nature Physics:
  - 0 citations in Web of Science
  - 0 citations in CrossRef



Nature Physics. **18** (2022)

Evolution of the Field

3 main areas of evolution:

- Changes in configuration and conditions of a shell producing wrinkles
- Wrinkles in nature
- Further work by Tobasco and reviews

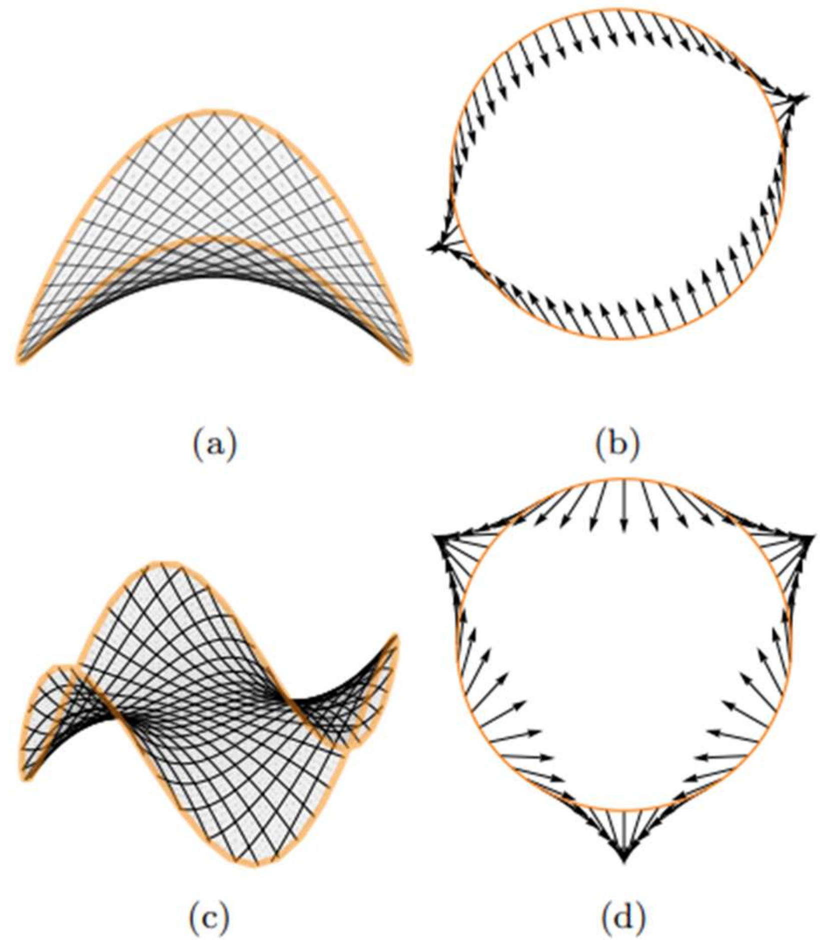
Evolution of Field: Changes of Configuration and Conditions-Alben

- Study on the effects of friction on wrinkles.
- The study focuses on elastic circles which are made increasingly small, leading to effects from friction
- These different amount of friction cause different wrinkles and amounts of elastic energy

Alben, S. Packing of elastic rings with friction. *Proc. R. Soc. A.* **478**, (2022).

Evolution of Field: Wrinkles in Nature- Yamamoto *et al.*

- In nature, many materials have wrinkles (leaves, etc.)
- "Buckling" occurs based on what happens at the edges of the material
- Introduces "branch point" defect for hyperbolic surfaces
  - "branch points" are where more than four rhombuses meet when making a 3D shape



Yamamoto, K.K., Shearman, T.L., Struckmeyer, E.J. *et al.* Nature's forms are frilly, flexible, and functional. *Eur. Phys. J. E* **44**, 95 (2021).

Review Article on Paper in *Physics Today*

- An article reviewing recent work, Tobasco *et al.*'s paper being one of the focuses
- Both the paper in *Nature* and another paper in 2021 as well as some experimental results by other scientists
- Describes Tobasco *et al.*'s rules as well the idea that the idea from his 2021 paper that the sheet will extend over the maximum area possible
- Describes experiments by Paulsen, Legget, and Timounay where they tested wrinkle formation after putting a sheet over curved glass

Lopatka, Alex. The behavior of thin curved sheets is ironed out, *Physics Today* **75**, 19-20 (2022)

Our Thoughts and Critiques

- The math described in the paper was too technical and complicated for a *Nature* paper
    - Doesn't add much beyond the geometrical rules for the average reader

- Predictive power is low

- The theory presented is not generally applicable

- Some assumptions are not adequately explained