Ergodic Dynamics and Thermalization in Isolated Quantum Systems

C. Neill *et al*

DOI: 10.1038/NPHYS3830

Ameya Patwardhan

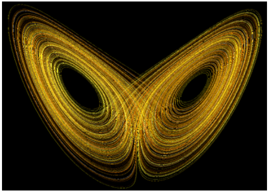
Ashwith Prabhu

Cheeranjeev Purmessur

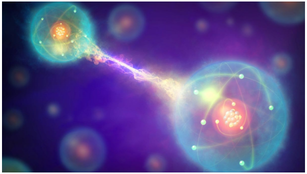
Frederick Pardoe

Goal of Paper

- Paper demonstrates strong correlation between classical chaos and entanglement using a coupled system with 3 superconducting qubits.



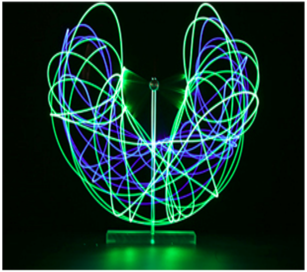
Chaos, *Butterfly effect, Wikimedia Commons*

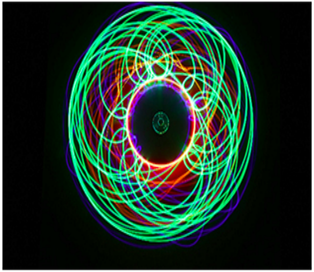


Entanglement, *Livescience.com, 2021*

Definition of Chaos

- Classical Chaos: Non linear classical systems highly sensitive to initial conditions traverse every attainable configuration over large period of time.

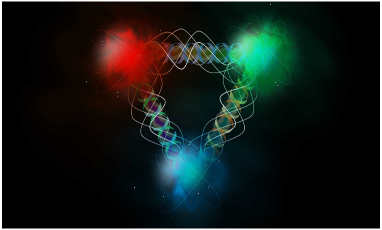




Chaotic Double Pendulum, *Wikimedia Commons*

Definition of Entanglement and Entanglement Entropy

- Entanglement: Particles are entangled when their quantum states are described with reference to one another
- Entanglement entropy: A measure of entanglement.The higher the entanglement of two systems, more “mixed” or “random” each system is



Quantum Entanglement, *Science News*

Definition of Ergodicity and Thermalization

- Ergodic classical systems:
  - System explores all microstates over time
  - $\langle O \rangle_{\text{time}} = \langle O \rangle_{\text{states}}$
  - Chaos drives ergodicity in classical systems

Definition of Ergodicity and Thermalization

- Ergodic classical systems:
  - System explores all microstates over time
  - $\langle O \rangle_{\text{time}} = \langle O \rangle_{\text{states}}$
  - Chaos drives ergodicity in classical systems
- Thermalization: It is caused by interaction of a system with the "environment" (many degrees of freedom).

Definition of Ergodicity and Thermalization

- Ergodic classical systems:
  - System explores all microstates over time
  - $\langle O \rangle_{\text{time}} = \langle O \rangle_{\text{states}}$
  - Chaos drives ergodicity in classical systems
- Thermalization: It is caused by interaction of a system with the "environment" (many degrees of freedom).
- There is no direct analogue chaos in quantum system since the Schrodinger's equation is linear

Some Unanswered Questions in Quantum Dynamics

- Is there a quantum version of classical chaos?

Some Unanswered Questions in Quantum Dynamics

- Is there a quantum version of classical chaos?
- Do quantum systems display ergodic behaviour?

Some Unanswered Questions in Quantum Dynamics

- Is there a quantum version of classical chaos?
- Do quantum systems display ergodic behaviour?
- Can interactions between purely quantum systems lead to thermalization-like behaviour?

Experiments in Quantum Chaos and Entanglement

- Past experiments have demonstrated quantum thermalization [1, 2]
  - using highly correlated atoms in optical lattice
  - doesn't compare with classical chaotic case

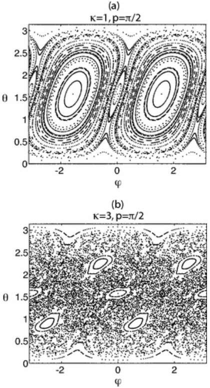
Experiments in Quantum Chaos and Entanglement

- Past experiments have demonstrated quantum thermalization [1, 2]
  - using highly correlated atoms in optical lattice
  - doesn't compare with classical chaotic case
- Some have also shown a link between entanglement and chaos [3]
  - However, entanglement was due to environment decoherence and not from within the quantum system

[1] Trotzky, S. et al, Nature Phys. 8, 325–330 (2012)
[2] Langen, T, Nature Phys. 9, 640–643 (2013)
[3] Lemos, G, Nature Commun. 3, 1211 (2012).

The “Top” Model

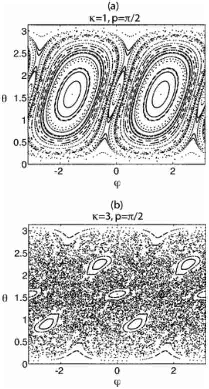
- Popular model for investigating the link between entanglement and quantum chaos [1].
- The system is a collection of interacting qubits that are periodically rotated in steps.



[1] Haake et al, Z. Phys. B 65, 381–395 (1986)

The “Top” Model

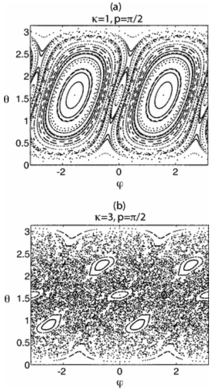
- Popular model for investigating the link between entanglement and quantum chaos [1].
- The system is a collection of interacting qubits that are periodically rotated in steps.
- Has a clear classical limit similar to a top.



[1] Haake et al, Z. Phys. B 65, 381–395 (1986)

The “Top” Model

- Popular model for investigating the link between entanglement and quantum chaos [1].
- The system is a collection of interacting qubits that are periodically rotated in steps.
- Has a clear classical limit similar to a top.
- Chaos tunable by a parameter κ.



[1] Haake et al, Z. Phys. B 65, 381–395 (1986)

The “Top” Model Continued

- Several theoretical studies have been conducted with this model [1-3].
- These studies showed that entanglement should be greater when κ is larger, and in more chaotic regions of phase space.

[1] Wang et al, Phys. Rev. E 70, (2004)
[2] Ghose et al, Phys. Rev. A 78, (2008)
[3] Lombardi & Matzkin, Phys. Rev. E 83, (2011).
[4] Chaudhury, Nature 461, 768–771 (2009).

The “Top” Model Continued

- Several theoretical studies have been conducted with this model [1-3].
- These studies showed that entanglement should be greater when κ is larger, and in more chaotic regions of phase space.
- There have been few experimental studies prior to ours.
- One such experiment confirmed the experimental correspondence between entanglement and chaos in phase space [4].

[1] Wang et al, Phys. Rev. E 70, (2004)
[2] Ghose et al, Phys. Rev. A 78, (2008)
[3] Lombardi & Matzkin, Phys. Rev. E 83, (2011).
[4] Chaudhury, Nature 461, 768–771 (2009).

Hamiltonian for time evolution of 3-qubit system

Three qubit system subjected to

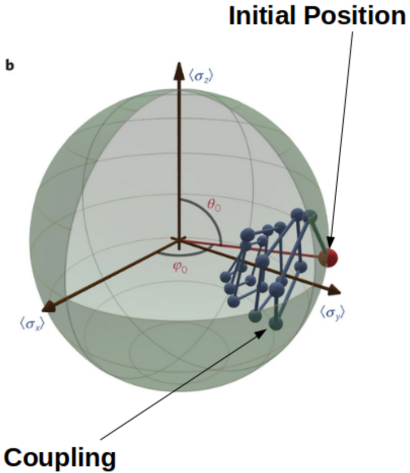
$$H(t) = \frac{\pi}{2\tau} J_y + \frac{\kappa}{2j} J_z^2 \sum_{n=1}^{N} \delta(t - n\tau)$$

Rotates each qubit about Y axis

Induces interactions among qubits N times

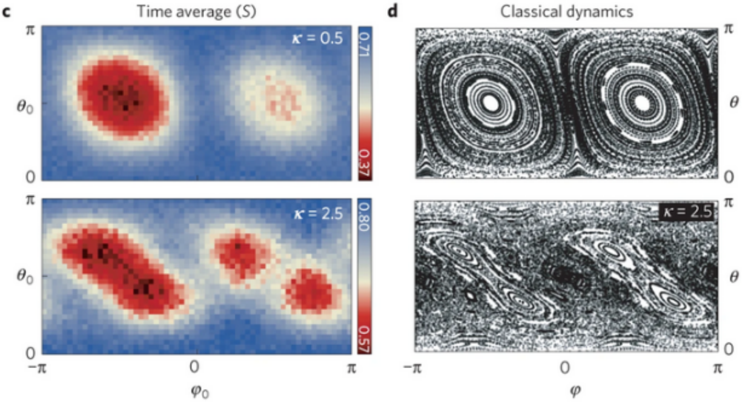
Experimentally realized using three-qubit ring of planar transmons with tunable inter-qubit coupling

Time Evolution of Single Qubit



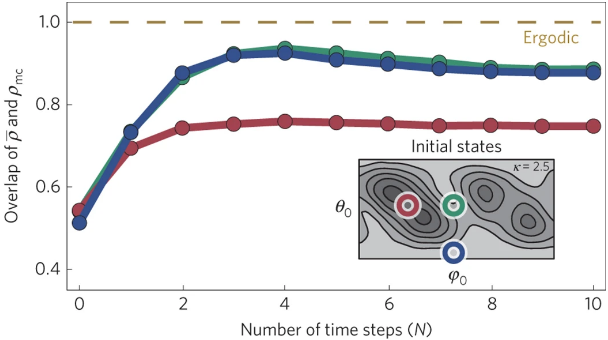
- Each qubit initially in state $|\theta_0, \phi_0\rangle = \cos(\theta_0)\,|\sigma_z\rangle + \sin(\theta_0)e^{-i\phi_0}\,|-\sigma_z\rangle$

Entanglement Entropy and Classical Chaos



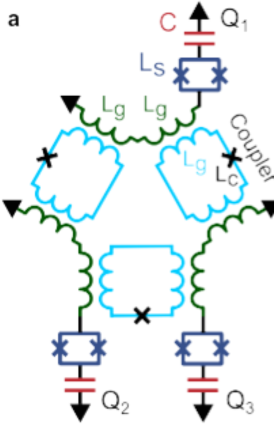
- Low entropy $\rightarrow$ regular classical dynamics
- High entropy $\rightarrow$ classical chaotic regime

Checking for Ergodicity



- Graph of overlap between qubit state & ergodic state
- Indicates: Higher entanglement entropy $\rightarrow$ Higher ergodicity

Experimental Details



Interaction between qubits is controlled by microwave pulses

Conclusions

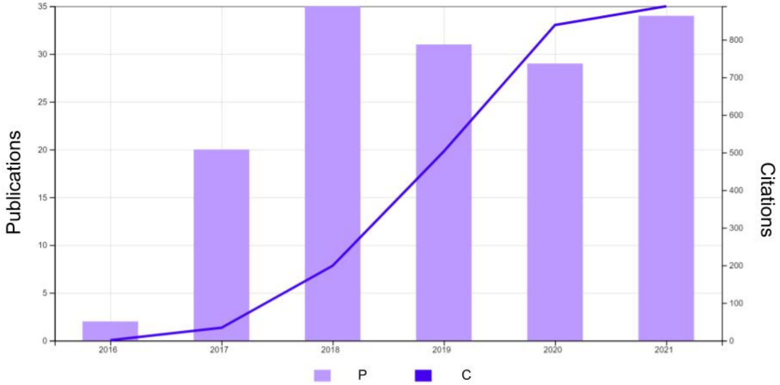
- Unexpected correspondence between classical chaos and entanglement entropy for a truly quantum system
- Entanglement entropy consistent with thermalization of single qubit
- Strong correlation between high entanglement entropy and ergodic behaviour

Critical Analysis

- Use of unnecessarily confusing color scheme makes it harder to interpret plots.
- Unclear distinction between experimental results and simulated results.
- Absence of discussion of the experimental aspects within the main text.

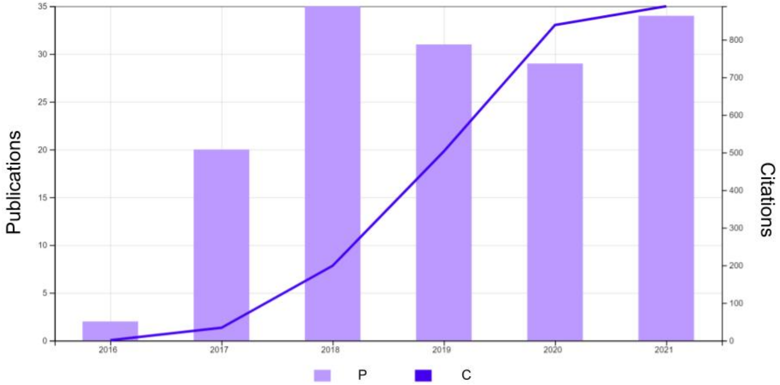
Citations

- Web of Science: Shows 151 citing papers
- Ads: Shows 155 citing papers
- Scopus: Shows 152 citing papers



Citations

- Web of Science: Shows 151 citing papers
- Ads: Shows 155 citing papers
- Scopus: Shows 152 citing papers
  - 98th Percentile of cited papers
  - Field weighted citation impact: 7.89 (1.00 is average)



What makes this paper so important?

- This is generally regarded as an important recent experiment within the field.
  - Many review papers reference it for the point of saying this.
  - Builds upon earlier top model experiments.
  - Employs a new experimental method to achieve its results.

What makes this paper so important?

- This is generally regarded as an important recent experiment within the field.
  - Many review papers reference it for the point of saying this.
  - Builds upon earlier top model experiments.
  - Employs a new experimental method to achieve its results.
- At least one paper compared the results of a numerical simulation to these results.

What makes this paper so important?

- This is generally regarded as an important recent experiment within the field.
  - Many review papers reference it for the point of saying this.
  - Builds upon earlier top model experiments.
  - Employs a new experimental method to achieve its results.
- At least one paper compared the results of a numerical simulation to these results.
- It is also important because it shows that even a small system of 3 objects behaves like a system with many components.

Checking for Environment-Qubit Interaction

- Higher entropy=higher correlations
- Qubit-Qubit entanglement $\gg$ Qubit-Environment Entanglement