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Quantum Mechanics of Gravitational Waves

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PHYSICAL REVIEW LETTERS **127,** 081602 (2021)

Editors' Suggestion

Quantum Mechanics of Gravitational Waves

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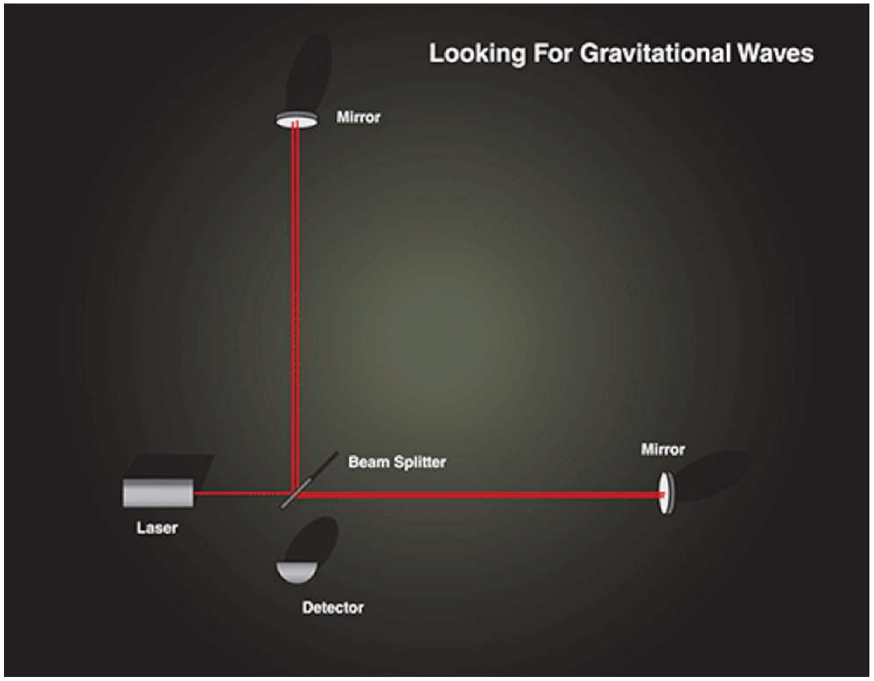
Classical Gravity

Einstein's Theory of General Relativity:
Massive objects distort (or bend) spacetime around them.

- Gravitational waves: ripples in spacetime caused by moving objects.



Classical Hill Equation

$$\underbrace{\ddot{\xi}}_{} = \frac{1}{2}\ddot{h}\xi$$

Metric perturbation

Geodesic separation

The Quantization of Gravity

- <u>Graviton:</u> massless spin-2 particle that mediates gravitational interactions.

$$\boxed{\text{Einstein's gravitational theory}} \underset{\text{classical limit}}{\overset{\text{quantization}}{\rightleftarrows}} \boxed{\text{Gravitons}}$$

- Previous attempts to quantify gravity: string theory, loop quantum gravity

- Main idea: If gravity is quantized, how could we observe this?

<u>Question:</u>
How does this equation of motion change if the spacetime metric is a quantum field?

$$\ddot{\xi} = \frac{1}{2}\underbrace{\ddot{h}}_{\text{Metric perturbation}}\underbrace{\xi}_{\text{Geodesic separation}}$$

Summary of results

- By treating the gravitational field quantum mechanically, we find that

$$\ddot{\xi} = \tfrac{1}{2}\ddot{h}\xi \;\Longrightarrow\; \ddot{\xi} = \frac{1}{2}\left(\underbrace{\ddot{h}}_{\text{Classical}} + \underbrace{\ddot{N}_\Psi}_{\text{Quantum noise}} - \underbrace{\frac{m_0 G}{c^5}\frac{d^5}{dt^5}\xi^2}_{\text{Radiation Reaction}}\right)\xi$$

- Characteristics of N depend on quantum state of the gravitational field
  - Calculated explicitly for coherent, thermal, and squeezed states
- Detection of noise $\Longrightarrow$ quantization of gravity, existence of gravitons!

Proposed method for detection of quantized gravity

- Detector is modeled as two free-falling masses
  - Separation is influenced by perturbations of metric (i.e., gravitational waves)
- Hill Equation:
  - acceleration of geodesic separation in presence of gravitational waves

$$\ddot{\xi} = \frac{1}{2}\ddot{h}\xi$$

Metric perturbation

Geodesic separation

<u>Question:</u>

How does this equation of motion change if the spacetime metric is a quantum field?

Coupling of falling masses with the gravitational field

- Action of the system
  - Einstein-Hilbert action coupled with free-falling masses;
  - Assumptions: small perturbation, weak field, one mass much smaller.
- Simplification: for a single mode and single polarization,

$$S_\omega = \int dt \left(\frac{1}{2} m(\dot{q}^2 - \overbrace{\omega}^{\text{Frequency of Fourier modes}}{}^2 \underbrace{q}_{\text{Amplitude of Fourier modes}}{}^2) + \frac{1}{2} m_0 \dot{\xi}^2 - \overbrace{g}^{\text{Coupling constant}} \dot{q} \dot{\xi} \xi \right) .$$

Like harmonic oscillator coupled with a free particle

Gravitational Field Mode

Energy = ħω

Quantization!

Probability of transitioning between detector states

- How does ξ evolve in time, for any $|f\rangle$?
- Transition probability:

$$P_{\psi_\omega}(\phi_A \to \phi_B) = \sum_{|f\rangle} |\langle \overbrace{f}^{\text{Final mode}}, \underbrace{\phi_B}_{\text{Final ξ}} | \underbrace{\hat{U}(T,0)}_{\text{Time Evolution from action } S_\omega} | \overset{\text{Initial gravitational field mode}}{\psi_\omega}, \overset{\text{Initial state of ξ}}{\phi_A} \rangle|^2 \, .$$

Probability of transitioning between detector states

- Harmonic oscillator is the gravitational field mode
  - Energy = ħω.
- How does ξ evolve in time, for any $|f\rangle$?
- Transition probability:

$$P_{\psi_\omega}(\phi_A \to \phi_B) = \sum_{|f\rangle} |\langle \overbrace{f}^{\text{Final mode}}, \underbrace{\phi_B}_{\text{Final } \xi} | \underbrace{\hat{U}(T,0)}_{\text{Time Evolution from action } S_\omega} | \overset{\text{Initial gravitational field mode}}{\psi_\omega}, \overset{\text{Initial state of } \xi}{\phi_A} \rangle|^2 .$$

Transition probability for all gravitational field modes

- Path integral to calculate transition probability:

$$P_{\psi_\omega}(\phi_A \to \phi_B) \sim \int \underbrace{\mathcal{D}\xi \mathcal{D}\xi'}_{\text{Path integral}} e^{\frac{i}{\hbar}\int_0^T dt \frac{1}{2} m_0 (\dot{\xi}^2 - \dot{\xi}'^2)} \overbrace{F_{\psi_\omega}}^{\text{Feynman-Vernon Influence Functional}}[\xi, \xi']$$

- Several approximations allow us to evaluate this probability for different types of states.
- Generally,

$$P_\Psi(\phi_A \to \phi_B) \sim \int \mathcal{D}\boxed{N_\Psi} e^{-\frac{1}{2}\int A_\Psi^{-1} N_\Psi^2} \left| \int \mathcal{D}\xi e^{\frac{i}{\hbar}\int_0^T dt \left(\frac{1}{2} m_0 \dot{\xi}^2 + \frac{1}{4} m_0 (\ddot{h} + \ddot{N}_\Psi)\xi^2\right)} \right|^2$$

Additional **fundamental** fluctuation on the detector.

Quantum Geodesic Deviation Equation

$$\ddot{\xi} = \frac{1}{2}\left(\underbrace{\ddot{h}}_{\text{Classical}} + \overbrace{\ddot{N}_{\Psi}}^{\text{Quantum Noise}} \underbrace{- \frac{m_0 G}{c^5}\frac{d^5}{dt^5}\xi^2}_{\text{Radiation Reaction}}\right)\xi$$

Quantum Noise Spectra

- Coherent State: Minimum Uncertainty / Classical Behavior

$$S = 4G\hbar\omega/c^5$$

- Thermal State: Temperature Effects / Hawking Radiation

$$S = 4G\hbar\omega/c^5 \coth\left(\hbar\omega/2k_B T\right)$$

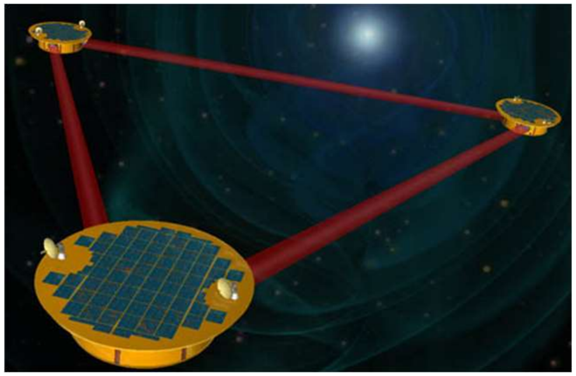
- Squeezed State: Concentrated Uncertainty / Early Universe Inflation

$$S = \sqrt{\cosh 2r}\, 4G\hbar\omega/c^5$$

Detecting Quantum Noise

- Coherent State:
  - Planck length / 10^{-35} m
  - 17 orders of magnitude beyond current LIGO sensitivity
- Thermal State:
  - 13 orders of magnitude beyond LIGO
  - Just 10 beyond planned LISA sensitivity
- Squeezed State:
  - Hard to detect
  - Authors unsure if there are realistic physical sources with sufficient squeezing

Image Credit: NASA



Paper summary

$$\ddot{\xi} = \frac{1}{2}\ddot{h}\xi$$

Metric perturbation

Geodesic separation

Question:
How does this equation of motion change if the spacetime metric is a quantum field?

$$\ddot{\xi} = \frac{1}{2}\left(\ddot{h} + \ddot{N}_\Psi - \frac{m_0 G}{c^5}\frac{d^5}{dt^5}\xi^2\right)\xi$$

118th numbered equation of the extended paper!

Quantum Noise

Detection?

Cited by…

Note: Previous essay announcing results (2020) cited by 15

Companion paper

Signatures of the quantization of gravity at gravitational wave detectors

Maulik Parikh, Frank Wilczek, and George Zahariade
Phys. Rev. D **104**, 046021 – Published 19 August 2021

Two independent citations

Physical Review D • *Open Access* • Volume 104, Issue 8 • 15 October 2021 • Article number A21

Indirect detection of gravitons through quantum entanglement

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Proposed experiment

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First post-Newtonian generation of gravitational waves in Einstein-Cartan theory

Battista E.[a], De Falco V.[b, c]
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Germany
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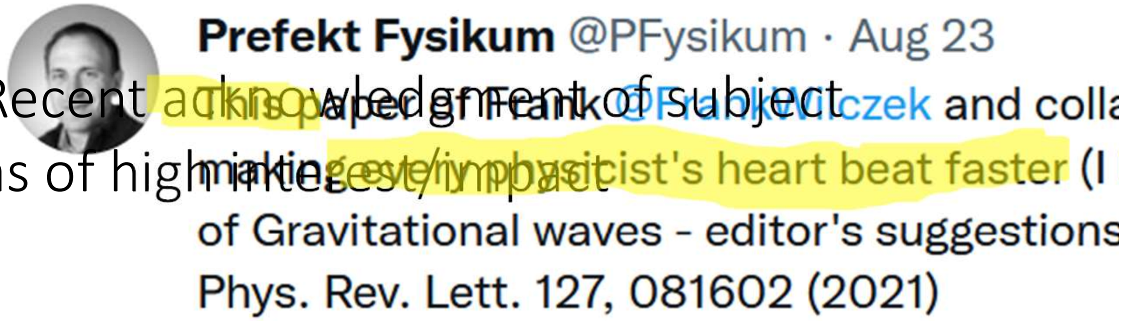
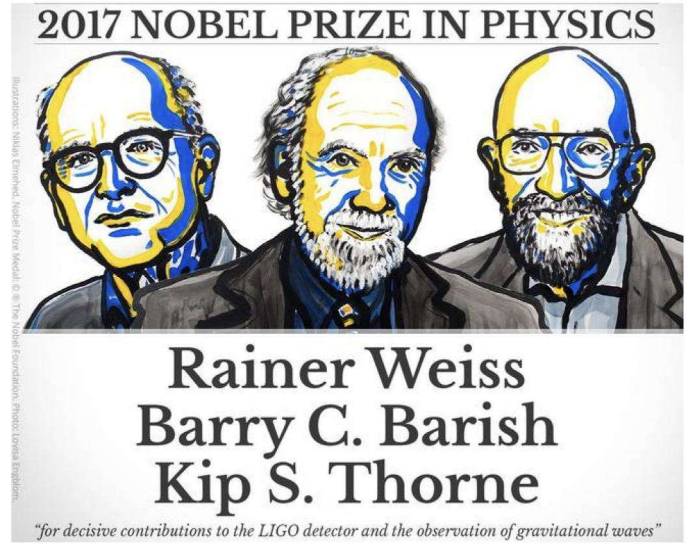
II, Via Cintia, N
[c] Istituto Nazionale di Fisica N … Na … esso … rio di Monte S.
Angelo, Via Cintia Edificio 6, Napoli, 801

Estimation of spin contributions to gravitational wave signal

Paper impact

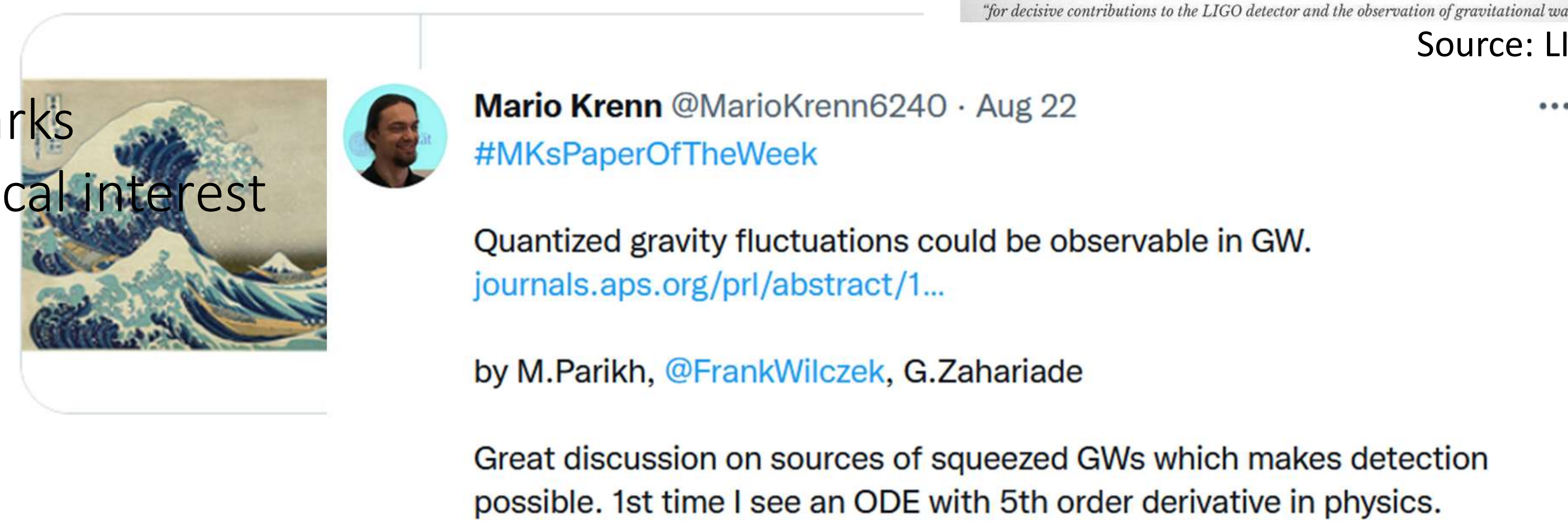
Recent acknowledgement of subject as of high interest/impact





Source: LIGO

Also sparks theoretical interest



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International Journal of Modern Physics D | Vol. 29, No. 14, 2042001 (2020) | Award-Winning Essay

The noise of gravitons

Maulik Parikh, Frank Wilczek and George Zahariade

https://doi.org/10.1142/S0218271820420018 |

Thank You!

The Proposed Experiment

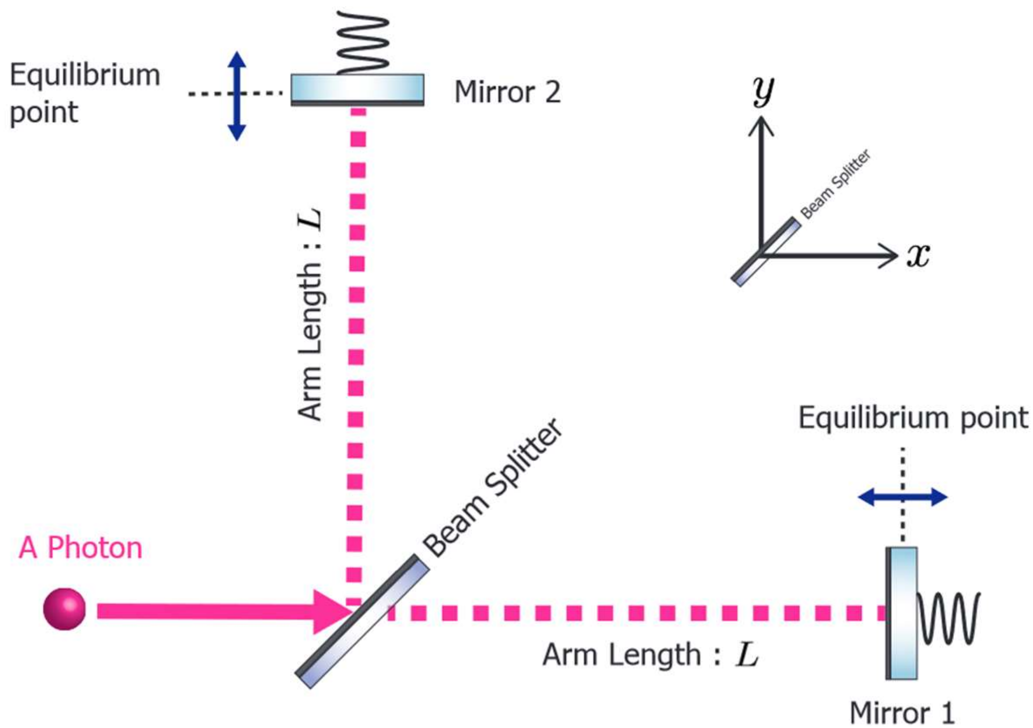

FIG. 1. The proposed setup: an equal-arm Michelson interferometer for a single photon where there is a macroscopic suspended mirror at the end of each arm

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[b] Department of Physics, Kobe University, Kobe, 657-8501, Japan

- **Indirect detection makes use of decoherence** caused by noise of gravitons.
- Cites the theory of this paper.
- Reasonable because length of mirrors required are **only one order of magnitude larger than LIGO**.