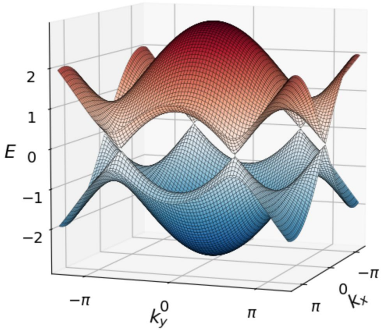
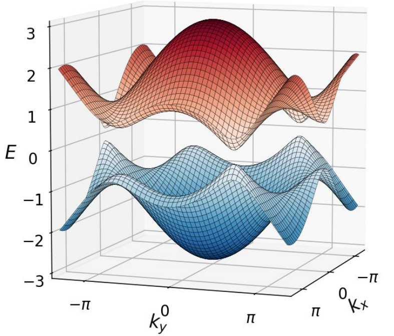
Quantum Spin Hall Effect in Graphene



Authors: C. L. Kane and E. J. Mele

PRL 95, 226801 - Published 23 Nov. 2005



Group 8: Marc Klinger, Surkhab Kaur, Patrick Koch, Nirvaan Khera, Sonali Joshi

Department of Physics ⬢ University of Illinois Urbana-Champaign

PHYS 596

December 4th, 2020

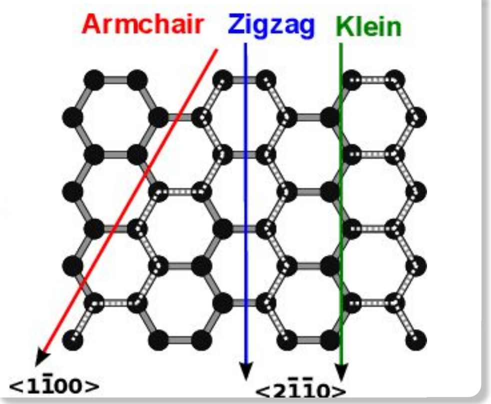
Graphene Importance

Video from Material World with GZA: https://www.redbull.com/us-en/episodes/material-world-liquid-science-s01-e07

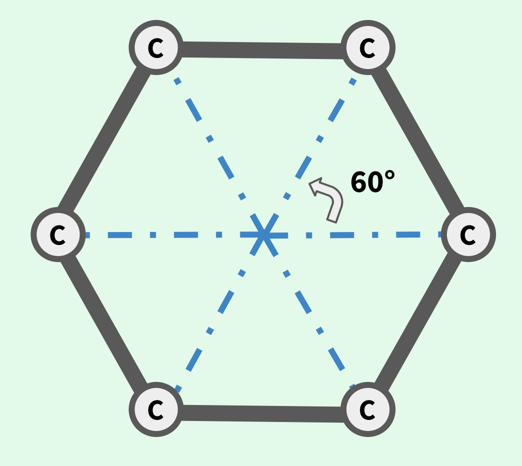
Graphene Symmetries

- 6 Rotational symmetries
- 6 Reflection symmetries
- **Time Reversal symmetry**

Edges of Graphene Sheet



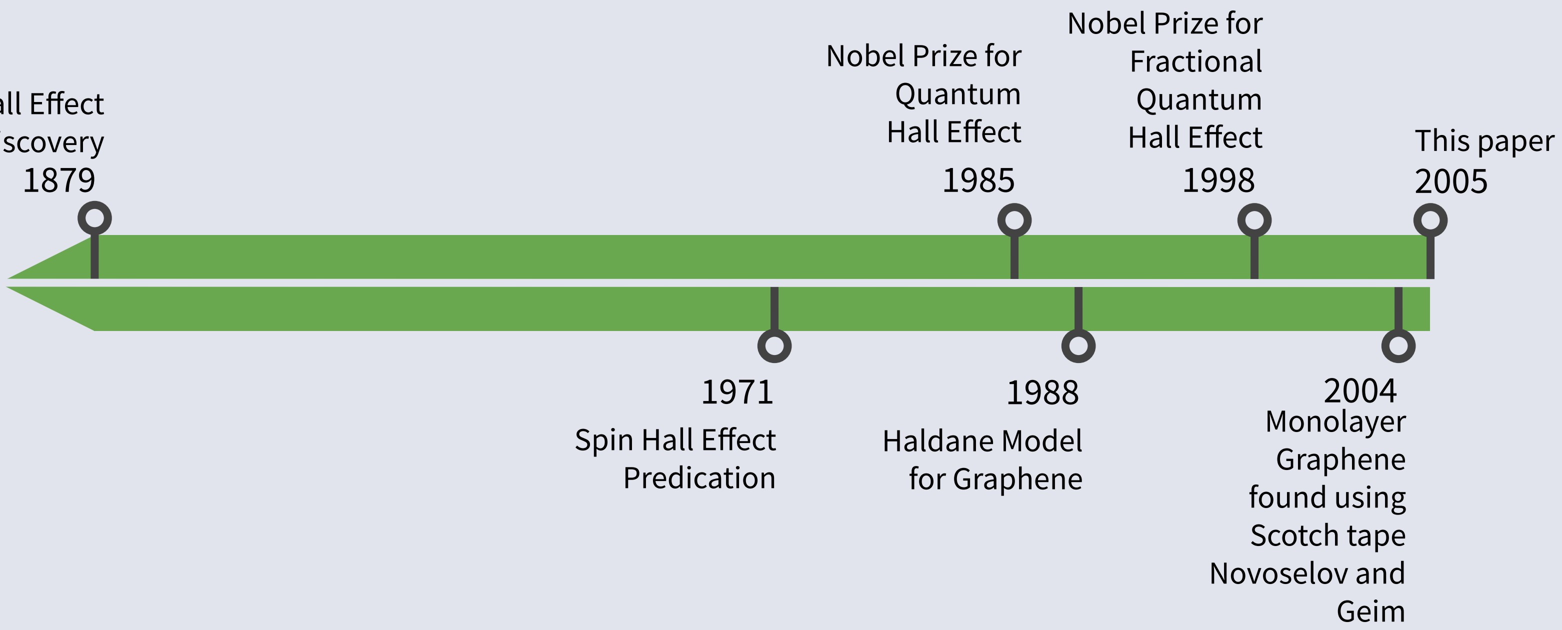
Rotational and Reflection Symmetries



Wagner, P. et al. “Stable hydrogenated graphene edge types: Normal and reconstructed Klein edges.” *Physical Review B* 88 (2013): 1-6.

Hall Effect History

Timeline before this paper



Hall Effect Discovery
1879

1971
Spin Hall Effect Predication

Nobel Prize for Quantum Hall Effect
1985

1988
Haldane Model for Graphene

Nobel Prize for Fractional Quantum Hall Effect
1998

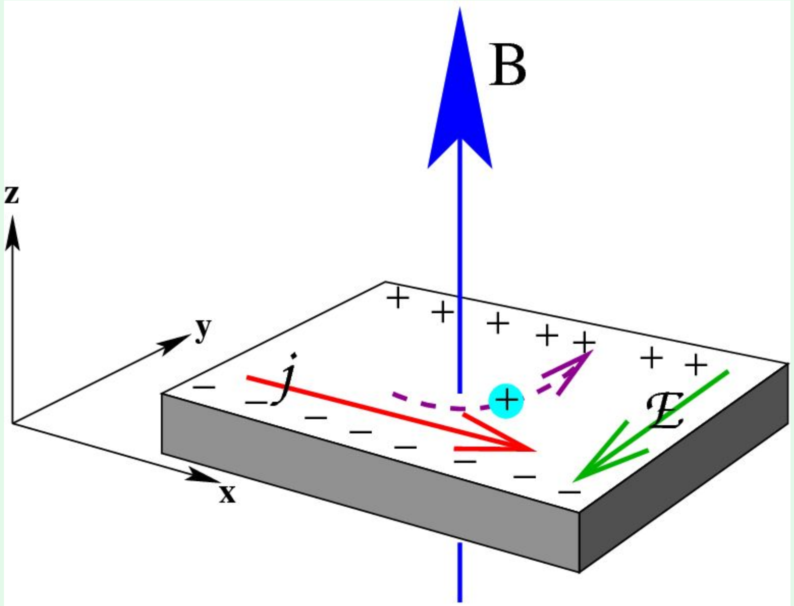
2004
Monolayer Graphene found using Scotch tape Novoselov and Geim

This paper
2005

The Classical Hall Effect

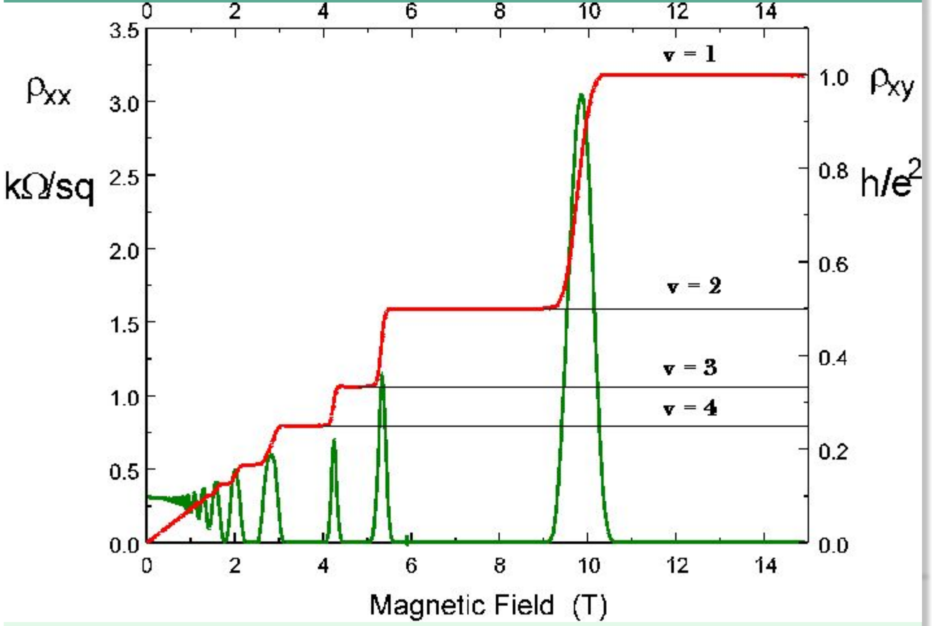
- Magnetic field orthogonal to moving charge
- Accumulation of charge occurs on the edge
- Creates an electric field that counteracts the effect of the magnetic field on the moving charge

https://www.researchgate.net/figure/The-classical-Hall-effect_fig14_243787821



The Quantum Hall Effect (QHE)

- Analog of charged object in the presence of a strong, orthogonal magnetic field: Spin Orbit Coupling.
- We can measure the Hall Conductivity in Quantum Theory and we find the miraculous result: It’s quantized!

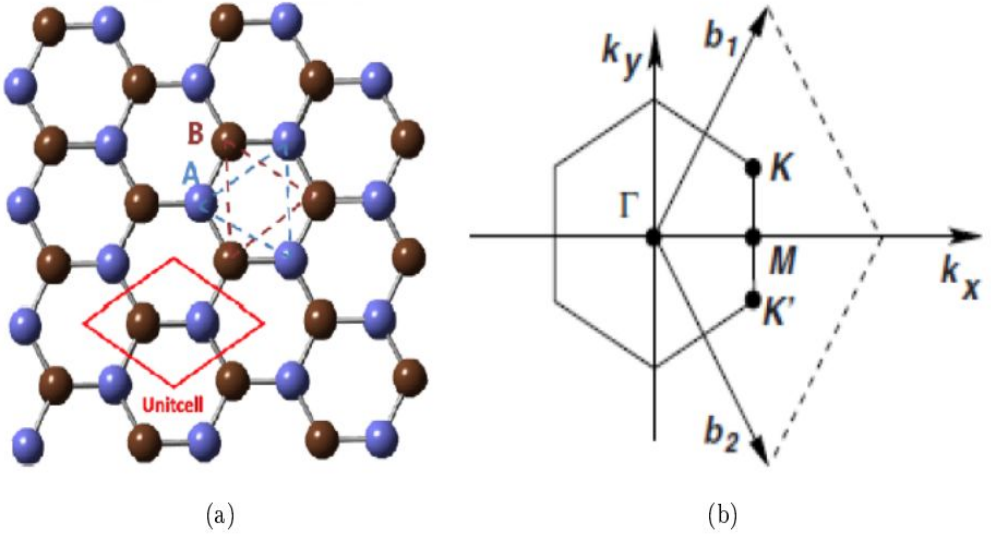


https://www.researchgate.net/figure/8-Typical-measurement-of-the-integer-quantum-Hall-effect-in-a-GaAs-GaAlAs-heterojunction_fig4_258694462

Quantum Spin Hall Effect in Graphene

- Brillouin Zone
    - Smallest repeating structure
    - Hexagon with vertices at unit cell
    - k-space
- Dirac points are transitions b/w conduction and valence bands

Brillouin Zone



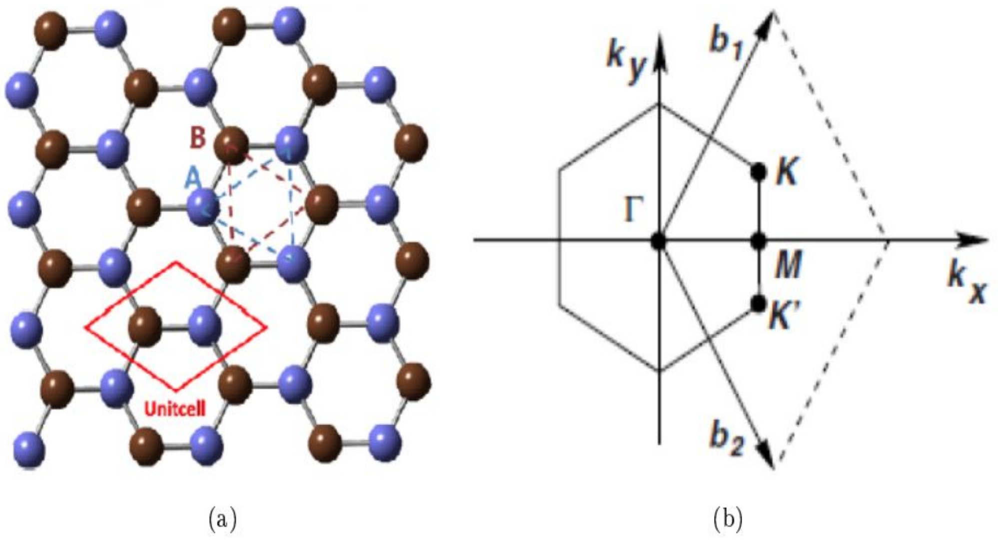
Quantum Spin Hall Effect in Graphene

- Spinless graphene

$$\mathcal{H}_0 = -i\hbar v_F \psi^\dagger(\sigma_x\tau_z\partial_x + \sigma_y\partial_y)\psi.$$

  - $\sigma_z = \pm 1$ - A(B) sublattice states
  - $\tau_z = \pm 1$ - K(K’) states
  - Inversion (A↔B)
  - Time reversal (K→K’)
  - Dirac points protected

Brillouin Zone



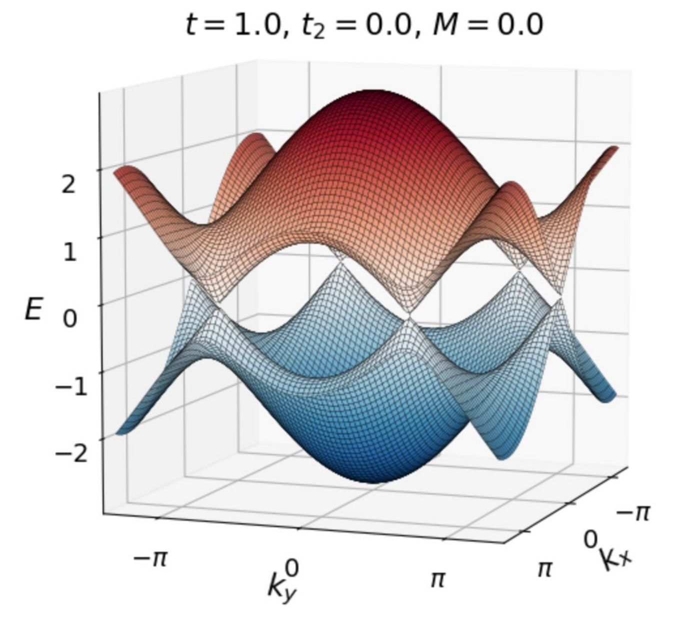
(a) (b)

Quantum Spin Hall Effect in Graphene

- Addition of Spin-orbit coupling at low T
  - $\mathcal{H}_{\mathrm{SO}} = \Delta_{so}\psi^{\dagger}\sigma_z\tau_z s_z\psi.$
  - Invariant under TR+inversion.

- To open gap, TR or inversion need to be broken

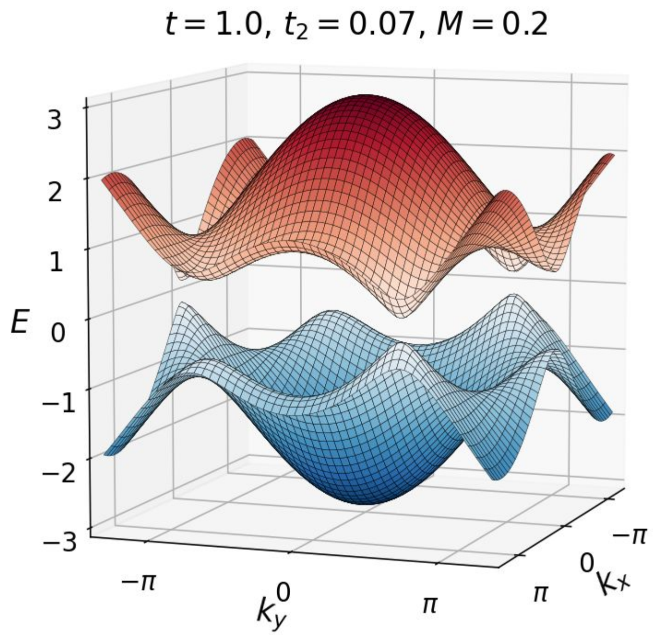
Dirac points touch w/ symmetry - gapless



Quantum Spin Hall Effect in Graphene

- Addition of Spin-orbit coupling at low T
    - $\mathcal{H}_{\text{SO}} = \Delta_{so}\psi^\dagger \sigma_z \tau_z s_z \psi.$
    - Invariant under TR+inversion.
- To open gap, TR or inversion need to be broken
- TRS is broken (Haldane's model) taking Hamiltonians for $s_z = \pm 1$ spins separately

Gapped Bulk

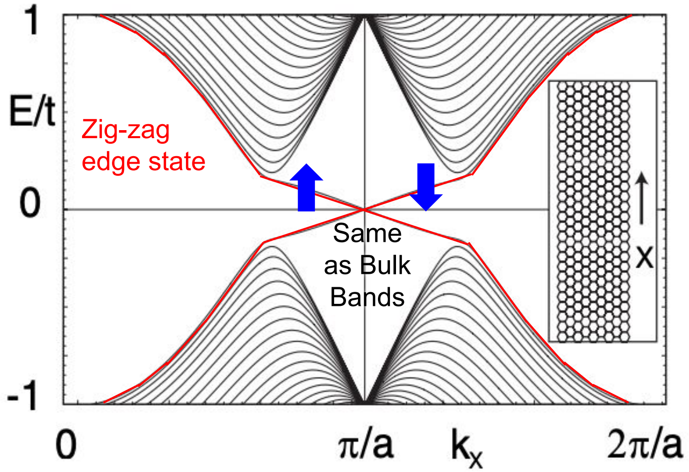


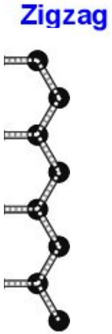
Quantum Spin Hall Effect in Graphene

- Laughlin's argument - Zig-zag edge calculations
    - Bands connect gaps
    - Opposite Dirac points and spins

- Gapless edge states
- Gapped bulk without B-field

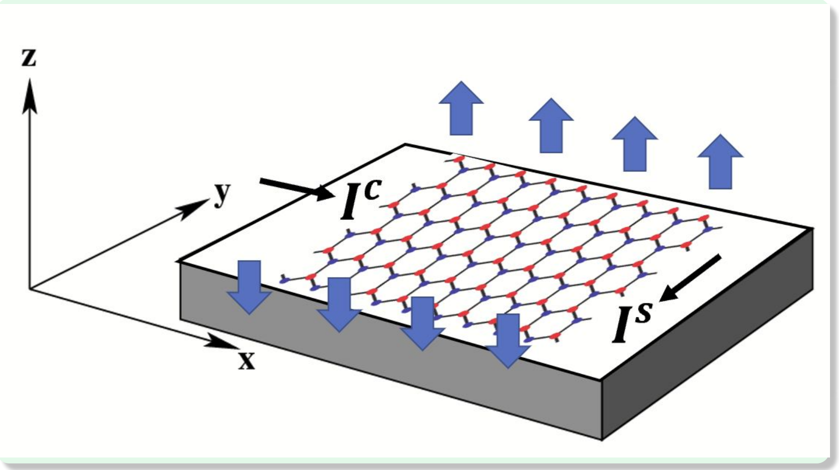
Zig-zag edge states





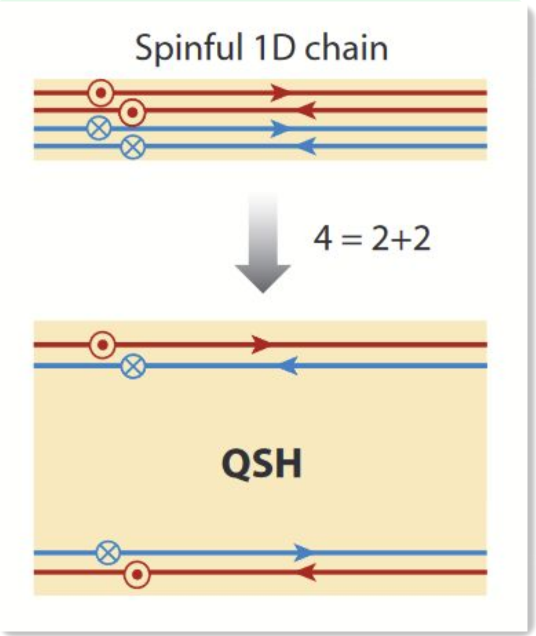
Quantum Spin Hall Effect in Graphene

- Accumulation of spins



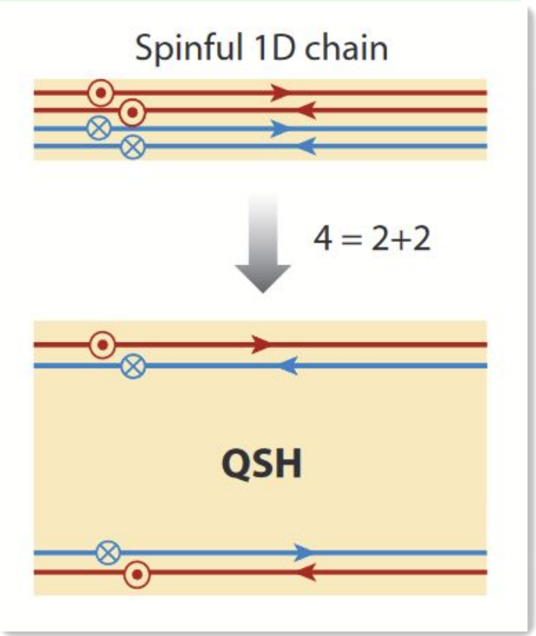
Quantum Spin Hall Effect in Graphene

- Accumulation of spins
- Non-chiral edge states



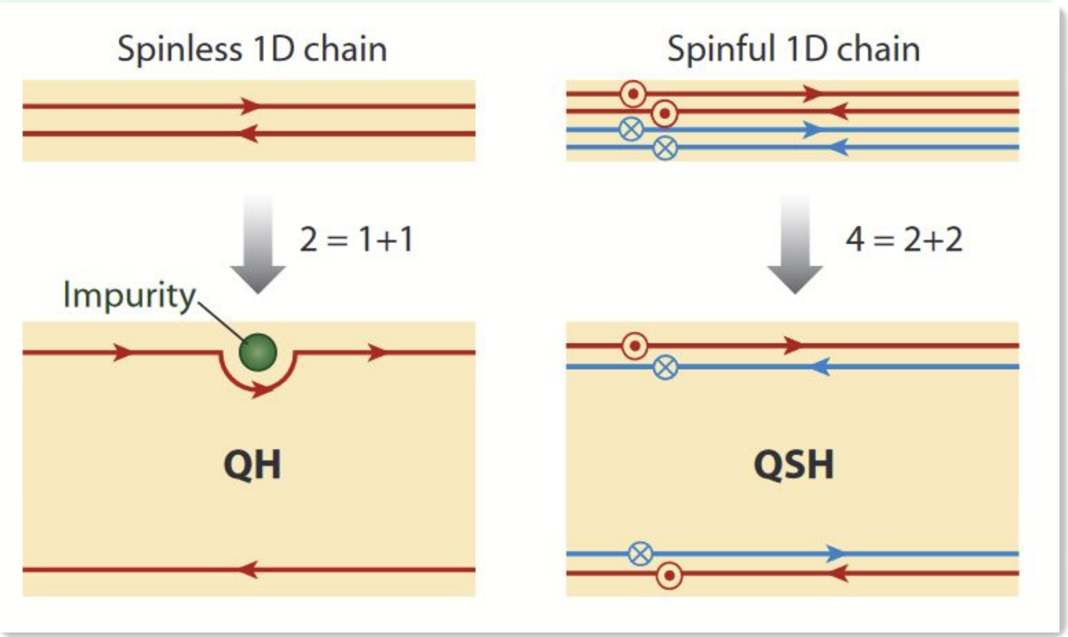
Quantum Spin Hall Effect in Graphene

- Accumulation of spins
- Non-chiral edge states
- New topological state of matter!



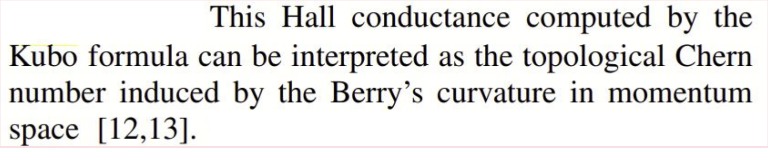
Quantum Spin Hall Effect in Graphene

- Edges obey TRS
    - Robust against weak interactions and disorder
- Dissipationless spin current



Critiques

- The paper glosses over the more subtle points in favor of a more brief report.
- Calculations are often lacking in depth
- These results have not been observed experimentally in pure graphene yet.



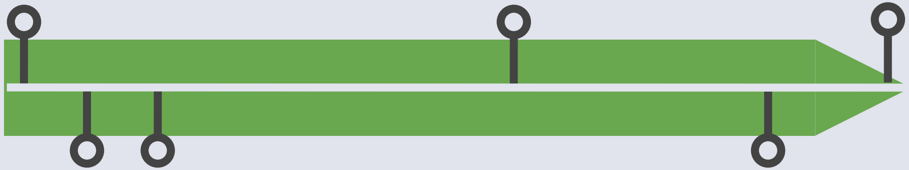
This Hall conductance computed by the Kubo formula can be interpreted as the topological Chern number induced by the Berry's curvature in momentum space [12,13].

- This is an example of a dense sentence, unless you are in the field

Events After the Paper

- Further work on Graphene and QSH Effect
- Prize given to Haldane for his model
- Prize given to Kane and Mele for their work

Timeline after this paper



This Paper
2005

Nobel Prize for Monolayer Graphene to Novoselov and Geim
2010

Breakthrough Prize given to Kane and Mele
2019

2006
Another proposed QSH paper by Bernevig and Zhang

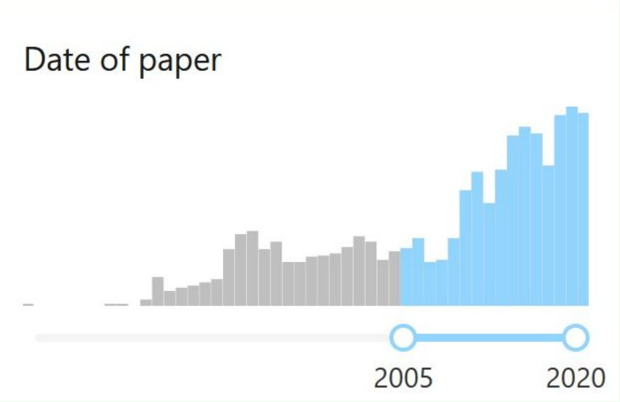
2007
Experimental QSH insulator state found in HgTe

2016
Nobel Prize for Topological Insulators Graphene. Haldane gets awarded one.

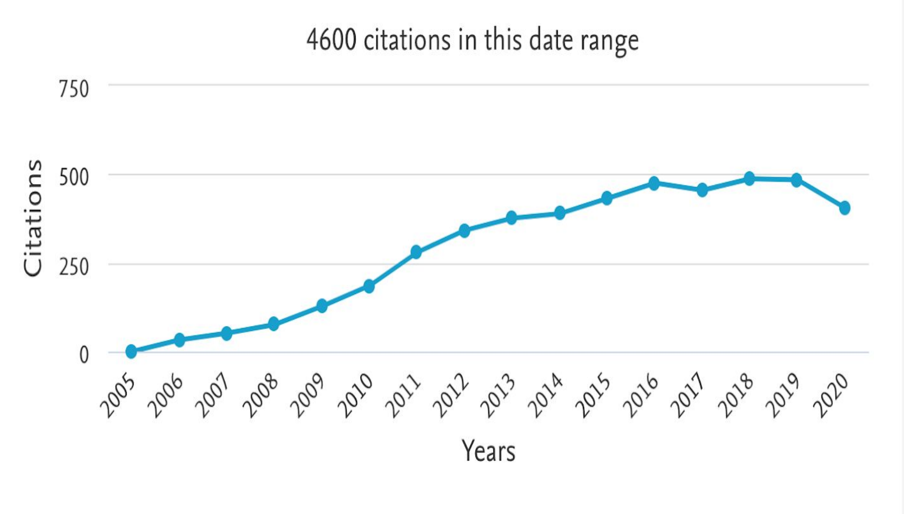
Interest in other sub fields

- Cited 4606 times (Scopus)
- Main field of interest is Condensed Matter
- New interest in High Energy

Graph from Inspire



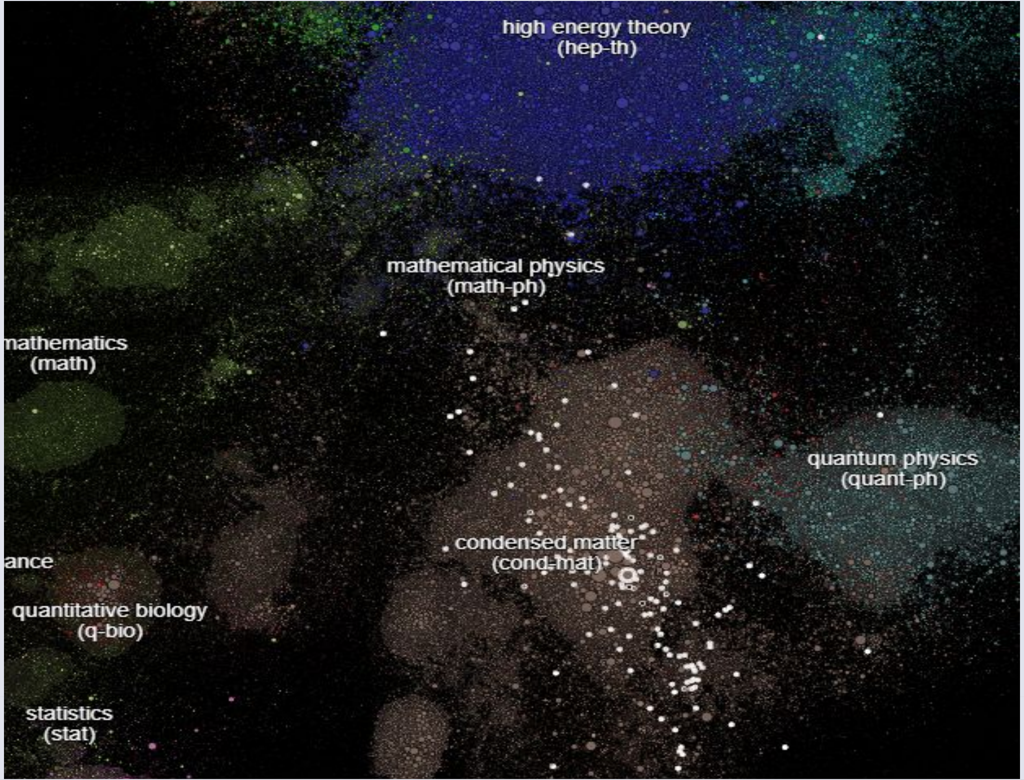
Graph from Scopus



Events After the Paper

- A map of 1,797,782 scientific papers from the arXiv.
- “Spin quantum hall effect”

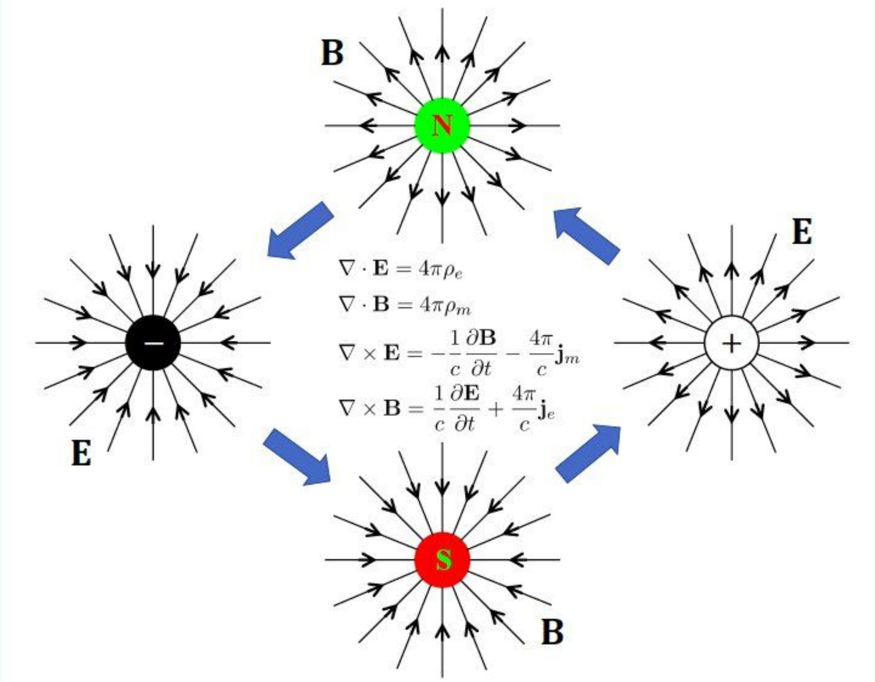
Paperscape.org



The QHE and Duality

- Despite having its origin as a Condensed Matter Phenomenon, it can be fruitfully understood using the tools of Quantum Field Theory.
- This has lead to a revolution in the interaction between different disciplines.
- It is closely related to the idea of duality -- when multiple different theories can be applied to the same physical problem.

Imagine magnetic monopoles existed...



$$\nabla \cdot \mathbf{E} = 4\pi\rho_e$$
$$\nabla \cdot \mathbf{B} = 4\pi\rho_m$$
$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} - \frac{4\pi}{c}\mathbf{j}_m$$
$$\nabla \times \mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{j}_e$$

The QHE Two Ways

- A standard approach to the QHE is the Adiabatic Theorem which utilizes the Berry Connection.

$$\mathcal{A}_\mu = -i\hbar\langle\psi_0(\lambda)|\frac{\partial}{\partial\lambda^\mu}|\psi_0(\lambda)\rangle$$

Berry Connection

Ground State

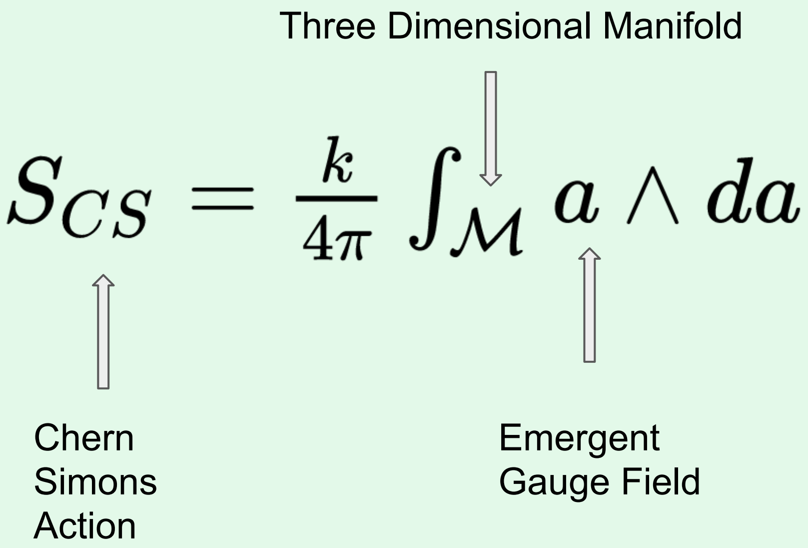
Parametrizes the Hamiltonian

- Roughly, the Berry Connection answers the question: How does the ground state change as we change the Hamiltonian?

The QHE Two Ways

- A standard approach to the QHE is the Adiabatic Theorem which utilizes the Berry Connection.

- This inspired Field Theorists to describe the QHE directly in terms of an emergent Gauge Theory.



Three Dimensional Manifold

$$S_{CS} = \frac{k}{4\pi} \int_{\mathcal{M}} a \wedge da$$

Chern Simons Action

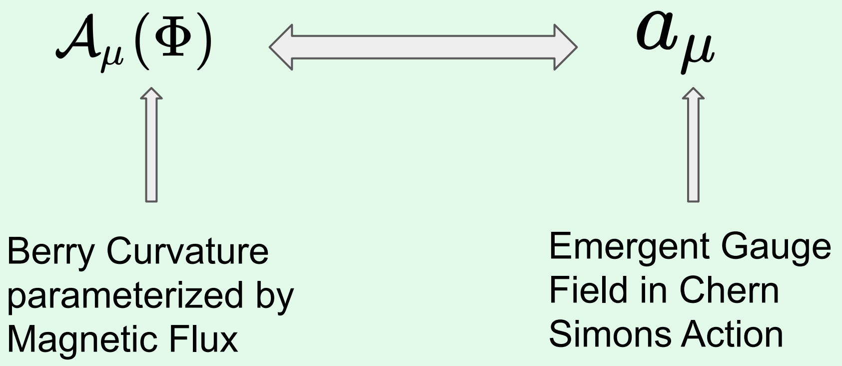
Emergent Gauge Field

- The Chern-Simons action describes a ***Topological*** Quantum Field Theory

The QHE Two Ways

- A standard approach to the QHE is the Adiabatic Theorem which utilizes the Berry Connection.
- This inspired Field Theorists to describe the QHE directly in terms of an emergent Gauge Theory.

Quantum Hall Effect Dictionary:



$$\mathcal{A}_\mu(\Phi) \longleftrightarrow a_\mu$$

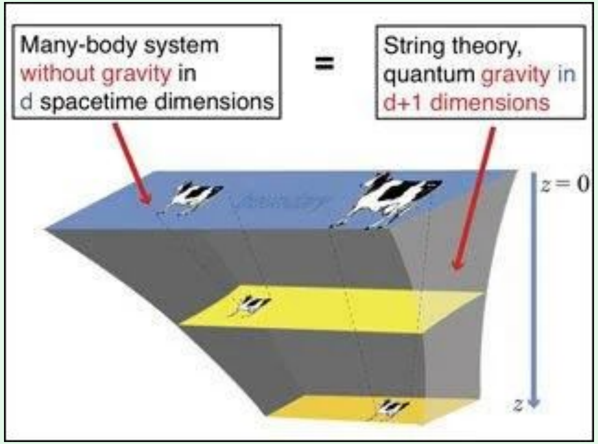
Berry Curvature parameterized by Magnetic Flux

Emergent Gauge Field in Chern Simons Action

The Hall Conductivity is related to the integral of the “Field Strength” associated with each field. It is quantized due to the ***Chern Theorem***

The Power of Duality

- The Chern Theorem is a result from Differential Topology!
- By employing a dual description of the QHE in terms of a topological QFT this insight could be employed to understand the quantization of the Hall Conductivity.
- Duality can map very hard problems onto solvable problems.



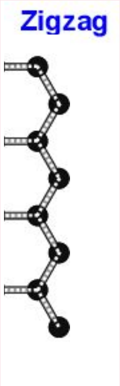
- The AdS/CFT Duality is a powerful result Physicists hope may unravel the mystery of Quantum Gravity!

The Enduring Impact of the QHE

- Encouraged added focus on the role of things like ***topology*** and ***emergence*** in defining novel physics
- Highlighted the benefit of having multiple approaches/perspectives for solving a given problem -- ***Duality***
- Increased intersectionality in physics, especially between Condensed Matter and High Energy.

Main points of the paper

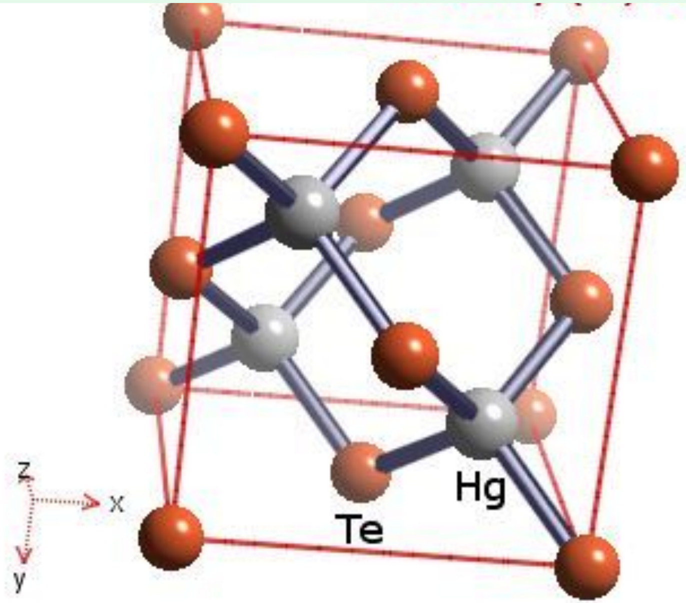
- Bulk states of graphene exhibits anomalous quantum hall effect and has a gap
- Zig-zag Edge states of graphene exhibits quantum **spin** hall effect and are **gapless** without a magnetic field. It has no charge current.
- This shows properties of spin insulation and prevention of disruption via impurities since electrons can not backscatter



Quantum spin hall effect in other materials

- HgTe had empirically been observed to have a high spin orbit interaction compared to Graphene
- **HgTe also experimentally exhibits QSHE and has conducting surface states**

Mercury Telluride



Video from Kane and Mele

https://www.youtube.com/watch?v=ZYuKIGlN5So&feature=youtu.be