# Fast Rotating Relativistic Stars: Spectra and Stability without Approximation[1]

Presented by:

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PHYS 596, December 4, 2020

[1]C. J. Krüger and K. D. Kokkotas, “Fast rotating relativistic stars: Spectra and stability without approximation,”Phys. Rev.Lett., vol. 125, p. 111106, Sep 2020.

# What is a Neutron Star and Why is it special?



Characteristics of Neutron Stars (very small, very dense, and spin very fast)

Why do we care? Because neutron stars are *stellar* sources of gravitational radiation

LIGO! Observed a neutron star merger

(Abbott, B. P.; et al., Physical Review Letters. 119 (16): 161101)

Are there other times in a neutron stars life when we might look for gravitational radiation? Yes! oscillations and instabilities.

# A Brief History of Oscillations and Instabilities in Neutron Stars

Background & Motivation
Methods
Results
Critiques
Conclusion & Questions

Oscillations and instabilities were first looked at by Kip Thorne et al. in the 1960s ( K. S. Thorne and A. Campolattaro, Astrophys. J. 149, 591 (1967); K. S. Thorne, Astrophys. J. 158, 1 (1969).)

Later on, ***C***handrasekhar, ***F***riedman, and ***S***chutz fleshed out the secular ***CFS*** instability (a *stellar* source of gravitational radiation)

( S. Chandrasekhar, Phys. Rev. Lett. 24, 611 (1970); J. L. Friedman and B. F. Schutz, Astrophys. J. 199, L157 (1975); J. L. Friedman and B. F. Schutz, Astrophys. J. 222, 281 (1978).)

What is CFS instability? When a star becomes unstable to gravitational radiation as a result of counter-rotating perturbations in its oscillation spectrum

# What does this paper do?



This paper proposes onset values of the CFS instability

- focus is on quadrupolar f-modes of neutron stars with relatively high spin values ($\Omega \geq 0.8\Omega_K$)
- these conditions are likely to occur in the post merger phase

This paper also presents asteroseismological relations

- allows observers to place constraints on the mass and radius of the neutron star such by observing the $\ell=|m|=2$ f-modes
- can also constrain the equation of state

AND it does it all without the Cowling or Slow Rotating Star Approx.

# Methods - Einstein's Field Equations



## Einstein's Field Equations (EFEs):

- Relate the geometry of space-time to the distribution of matter and energy.
- non-linear, coupled, partial differential equations
- Requires assumptions to solve for exactly
- Example Solution: Tolman-Oppenheimer-Volkoff Eqn:

$$\frac{dP}{dr} = -\frac{Gm}{r^2}\rho\left(1+\frac{P}{\rho c^2}\right)\left(1+\frac{4\pi r^3 P}{mc^2}\right)\left(1-\frac{2Gm}{rc^2}\right)^{-1}$$

## Rotating Neutron Star (RNS) Code:

Prewritten code which takes in ***Equations of State*** and assumptions and returns bulk parameters from EFEs (ie Ω, M, R, σ, ...)

## This Paper's Assumptions:

1. The neutron star is a perfect fluid (i.e. has 0 viscosity)
2. Temperature of the star is 0.

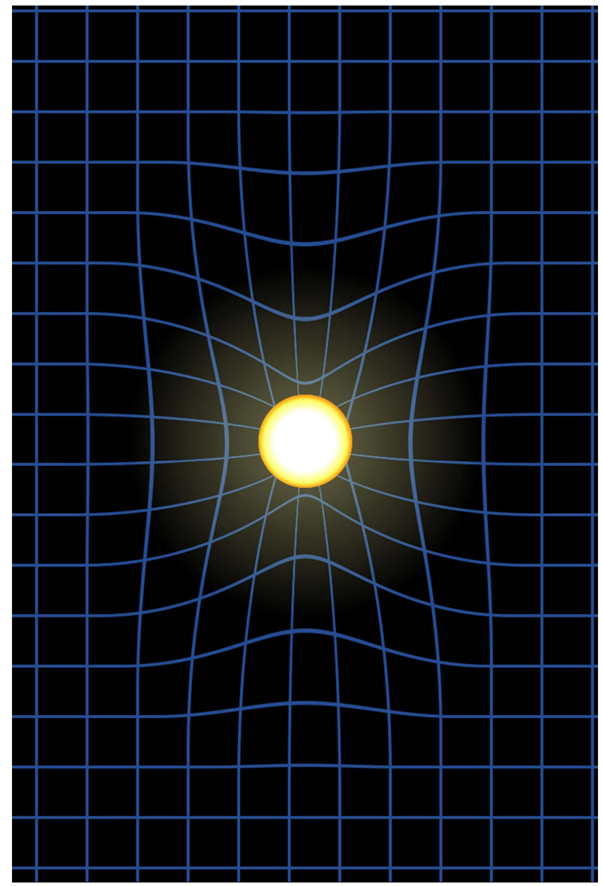


***Image Taken From:***

https://wtamu.edu/~cbaird/sq/2015/06/09/does-the-influence-of-gravity-extend-out-forever/

# Methods - Polytropic Equations of State (EoS)



## Thermodynamic EoS:

1. $PV = k_B NT$ (Ideal Gas Law)
2. $(P + \frac{a}{V_m{}^2})(V_m - b) = k_B NT$ (Van der Waals Equation)

## What are Polytropic EoS?

- A nuclear equation of state has state variables such as the following:

$f(P, T, R, \rho, \mathcal{E}, n, Y_e, \ldots) = 0$

- Neutron star EoS's assume dependence on only density/energy density and pressure.

- Can be an arbitrary equation or, usually, a polytropic equations.

$P = K\rho^{\alpha}$ α is set to [0.6849, 0.7463, and 1]

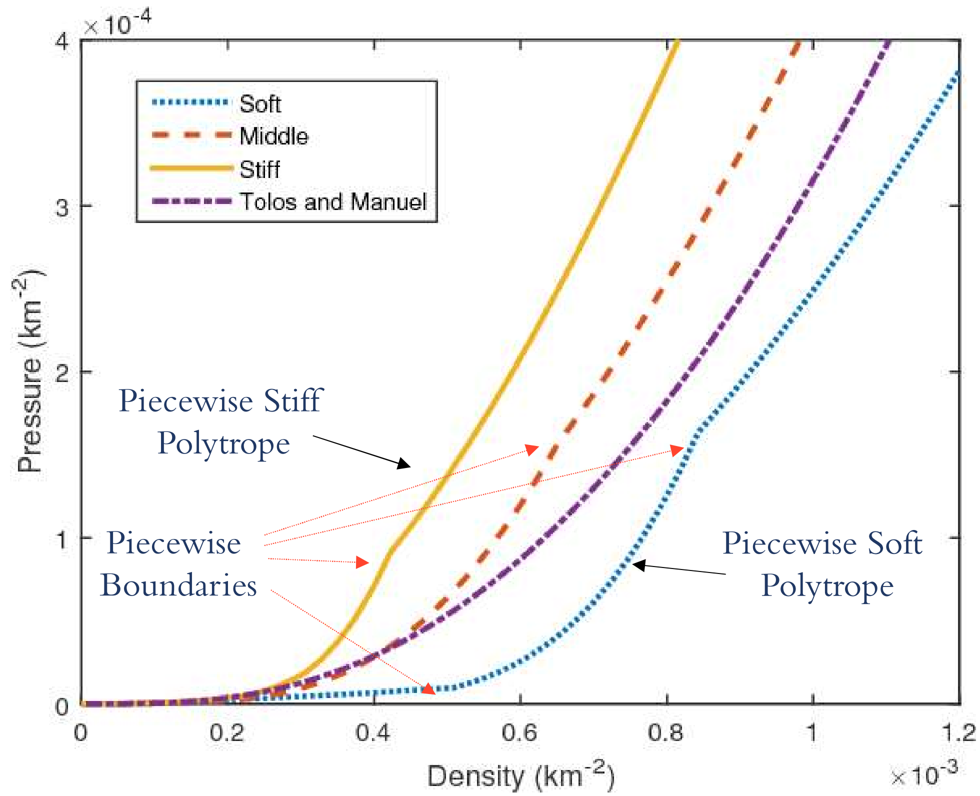



*Image Taken From:*

M. A. Resco, A. Cruz-Dombriz, F. Llanes-Estrada, and V. Z. Castrillo, "On neutron stars in $f(r)$ theories: Small radii,large masses and large energy emitted in a merger,"Physics of the Dark Universe, vol. 13, pp. 147–161, 2016.

# Methods - Realistic Equations of State (EoS)

Background & Motivation

Methods

Results

Critiques

Conclusion & Questions

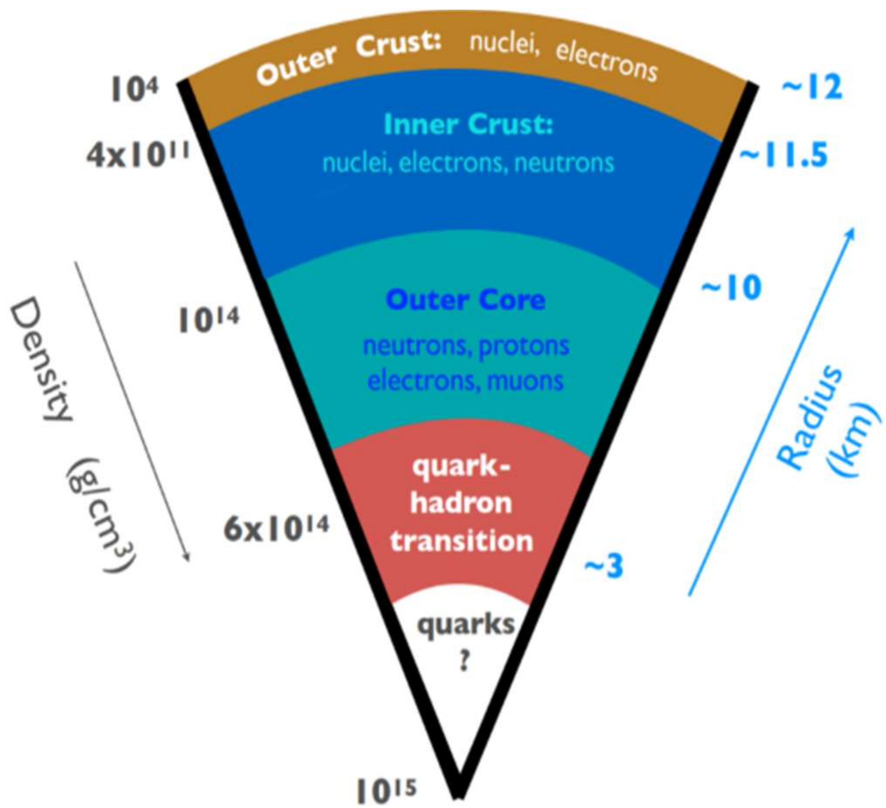



*Image Taken From:*

https://n3as.berkeley.edu/p/can-gravitational-waves-reveal-phase-transitions-in-the-cores-of-neutron-stars/

## What are Realistic EoS?

- Realistically, EoS should incorporate interactions between particles.
- This becomes more important at higher densities.
- ***These EoS are still ONLY dependent on pressure and density.***

## The Realistic EoS in this Paper:

***APR4 / WFF1 / SLy:*** Assumes particle makeup is p, n, e, and muons.

***H4:*** Assumes particle makeup is p, n, e, muons, and hyperons.

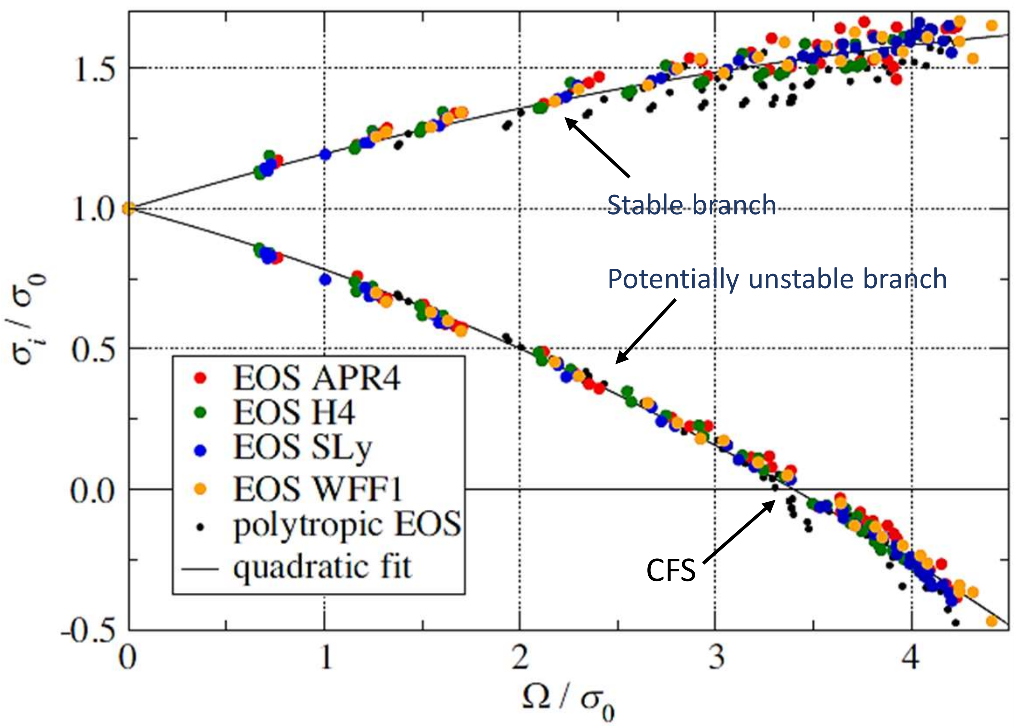
# Results: CFS Instability in the Inertial Frame



- The data shows sequences (21 total: 3 per EOS with 3 polytropic and 4 realistic) of constant central energy density.
- Least squares fitting:

$$\frac{\sigma_i}{\sigma_0} = 1 + a_1\left(\frac{\Omega}{\sigma_0}\right) + a_2\left(\frac{\Omega}{\sigma_0}\right)^2$$

- Potentially unstable branch RMSE = 0.024
- Stable branch RMSE = 0.048
- CFS instability presents itself in the unstable branch at $\Omega \approx 3.4\ \sigma_0$.



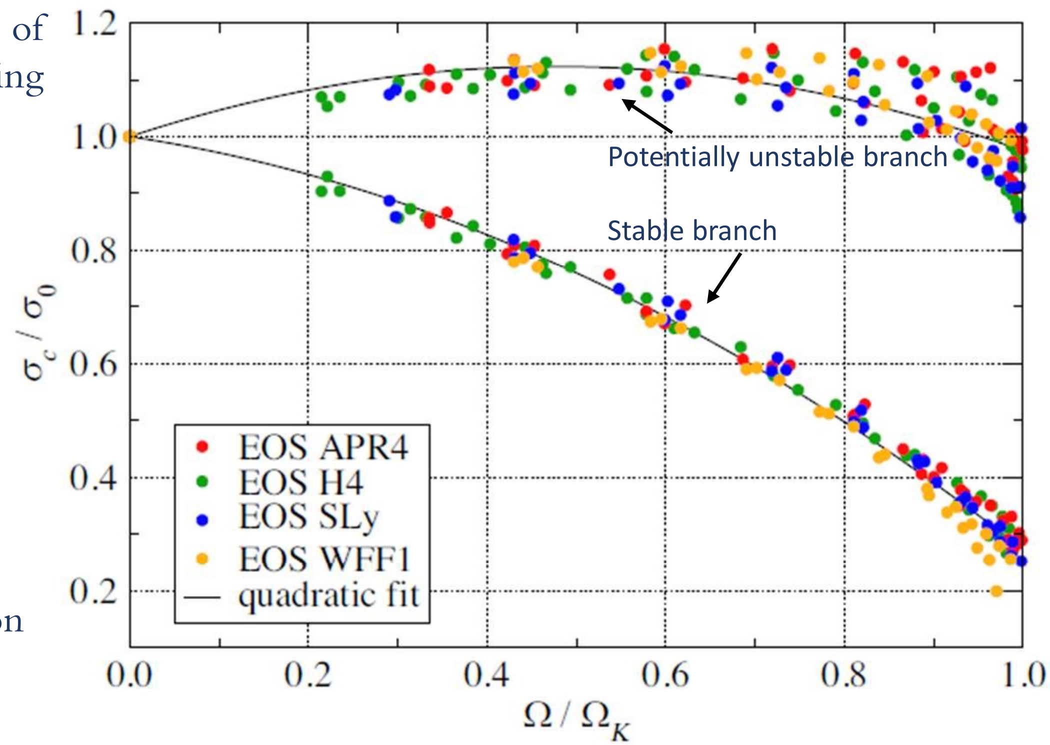
# Results: Comoving Frame



- The data shows sequences (12 total: 3 per EOS) of constant baryon mass as observed in the comoving frame.
- Least squares fitting:

$$\frac{\sigma_c}{\sigma_0} = 1 + b_1\left(\frac{\Omega}{\Omega_k}\right) + b_2\left(\frac{\Omega}{\Omega_k}\right)^2$$

- Potentially unstable branch RMSE = 0.051
- Stable branch RMSE = 0.024
- Using previous studies that fitted curves for $\sigma_0(M_0, R_0)$, the mass, radius, and angular rotation are connected.



# Results: Cowling Approximation



- The data shows lines of constant $\hat{\Omega}$ for the potentially unstable branch.

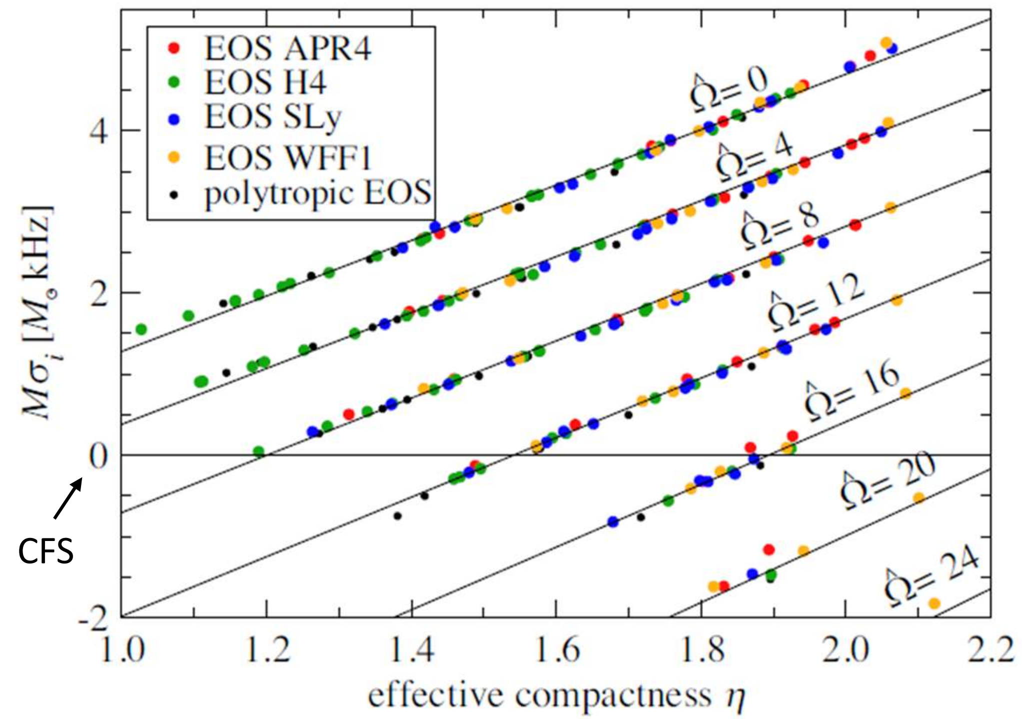
- Least squares fitting:

$$\hat{\sigma}_i = \left(c_1 + c_2\hat{\Omega} + c_3\hat{\Omega}^2\right) + \left(d_1 + d_2\hat{\Omega} + d_3\hat{\Omega}^2\right)\eta$$

where $\eta$ is a function of M and R.

- CFS instability appears to occur more readily in stars with low effective compactness. However, their rotations are considerably closer to $\Omega_k$.

- Although this includes more significant approximations, this does not depend on $\sigma_0$.



# General Critiques



- Little discussion about possible sources of error
- Not much motivation for the form of fitting equations and the significance of parameters
- Assumptions not always explicitly stated
- Not much qualitative or physical discussion of results

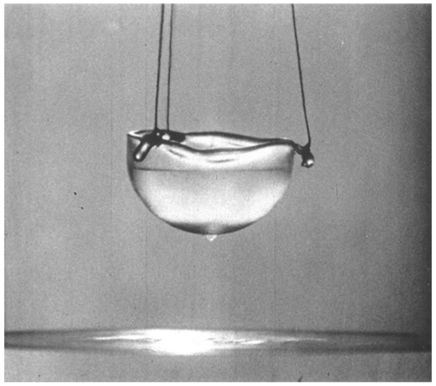
# Criticisms on Procedure and Results



- Fit parameters cut out entirely to yield a better fit with little explanation

- Fit equations taken in different frames (inertial and comoving), with little reasoning

- Title of the paper claims to make no assumptions but…
  - assumes that the temperature of the star is negligible, and the star's composition is a perfect fluid



# Brief overview of Citations

Background & Motivation
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- Published on 11 September 2020 (only a couple months old!)
- Both papers which cited the article were written by the same author + collaborators
- Little social media presence (1 tweet) and readers (6)
- Difficult to know impact of the article

Fast Rotating Relativistic Stars: Spectra and Stability without Approximation
(2020) Physical Review Letters, 125(11), 111106,

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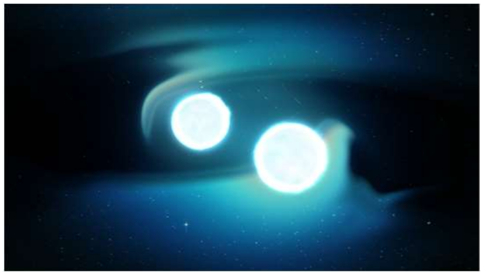
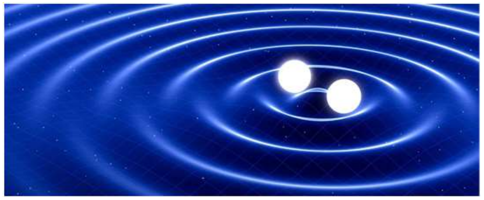
# Future Prospects

- Background & Motivation
- Methods
- Results
- Critiques
- Conclusion & Questions

- Krüger and Kokkotas only considered EOSs for the $\ell = |m| = 2$ f-mode.
  - Extend to higher multipole moments ($\ell > 2$) - similar fitting formulas are already derivable[1]
  - Analyze lower p-, g- and w-modes



- Extend to differentially rotating neutron stars (e.g. rotation differs at different points on star), and hot EOSs



- Future research could elucidate more on nascent neutron stars, mergers, and gravitational waves

[1]D. Doneva and K. D. Kokkotas, Phys. Rev. D 92, 124004 (2015)

# A brief summary of conclusions



- Krüger and Kokkotas extracted the frequencies of the $\ell = |m| = 2$ f-mode for general relativistic, rapidly rotating neutron stars without assuming small $\Omega$, or using the Cowling approximation
- Provided universal relations for frequencies for such stars independent of the EOS
- Determine an accurate estimate for the onset of the CFS instability