

# Negative representation and instability in democratic elections

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PHYS 596  
Team 1

# Motivation: Mathematical Description of Elections

- Large-scale behaviours (e.g sound waves) can be understood without understanding the small details (atoms).
- What does a mathematical formulation for democratic elections tell us about the relationship between electorate opinions and election outcomes?

## Methods

- All political opinions embedded in a 1D spectrum,  $f(x)$
- $y$ : Opinion of the elected official.
- Election:  $y[f(x)]$ , a functional that maps the distribution of electorate opinions to election outcome.

# Representation and Dual Candidate Elections

- Representation: Quantifying the impact of a change in opinion on the election outcome.
- $\delta y$ : Change in  $y$  as  $x \rightarrow x + c$
- For a large population, in the limit  $c \rightarrow 0$ ,

$$r(f, x) = \frac{d}{dx} \frac{\delta y}{\delta f(x)}$$

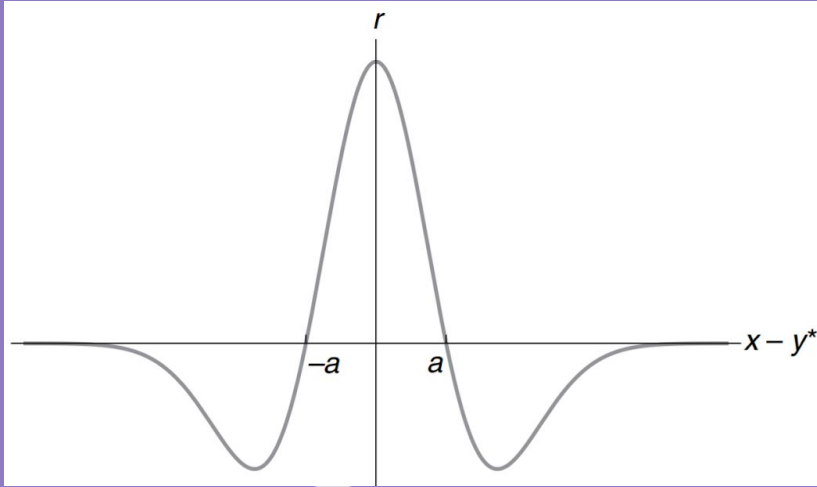
- Probability of candidate  $y$  winning the election:

$$\operatorname{argmax}_{y \in \mathbb{R}} \int_{-\infty}^{\infty} u_x(y) f(x) dx$$

- Utility function: Probability of someone with opinion  $x$  preferring  $y$
- In case of voter alienation:

$$u_x(y) = u(y - x) = e^{-\frac{(y-x)^2}{2a^2}}$$

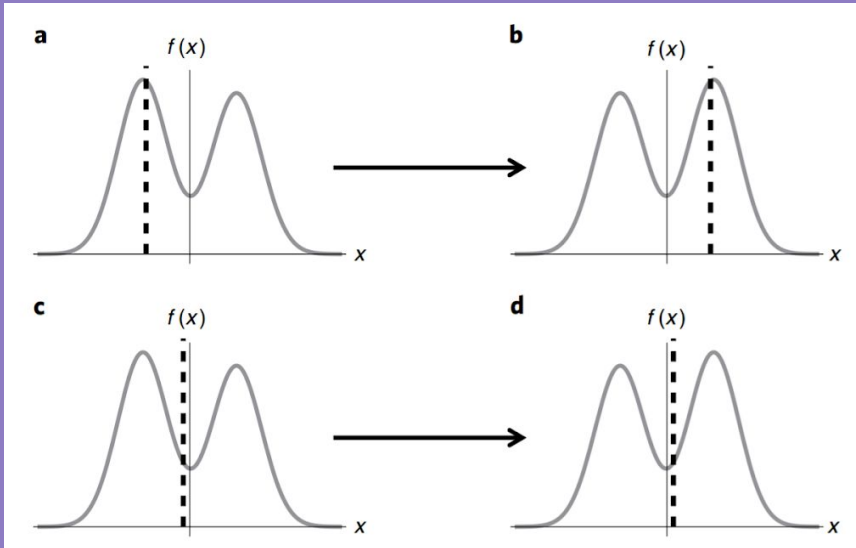
# Negative Representation due to Voter Alienation



Representation ( $r$ ) of opinions ( $x$ ) as a function of their distance from the election outcome ( $y^*$ )

- With voter alienation, representation is found to be negative for opinions far from the election outcome
- For certain population, a change in opinion in one direction, shifts the outcome in the other direction

# Election Stability and Negative Representation



Dashed line: Outcome

- a, b -> Unstable Election
- c, d -> Stable Election

- Stable election: small change in electorate opinion causes small change in election outcome.
- Unstable Election: small change in electorate opinion causes large change in election outcome.
- **All unstable elections contain negatively represented opinions!**

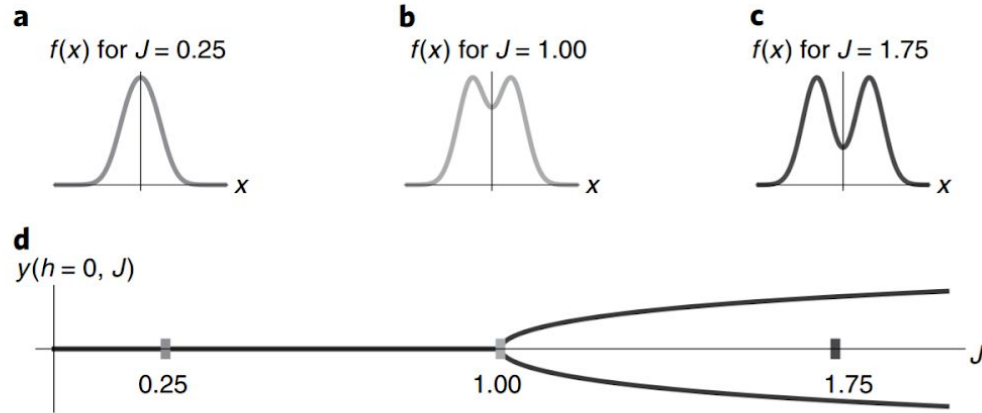
# Polarization and Election Stability

- Opinion Distribution for a polarized electorate:

$$f(x) = w_1 e^{-\frac{(x+\Delta)^2}{2\sigma^2}} + w_2 e^{-\frac{(x-\Delta)^2}{2\sigma^2}}$$

- Measure of polarization:

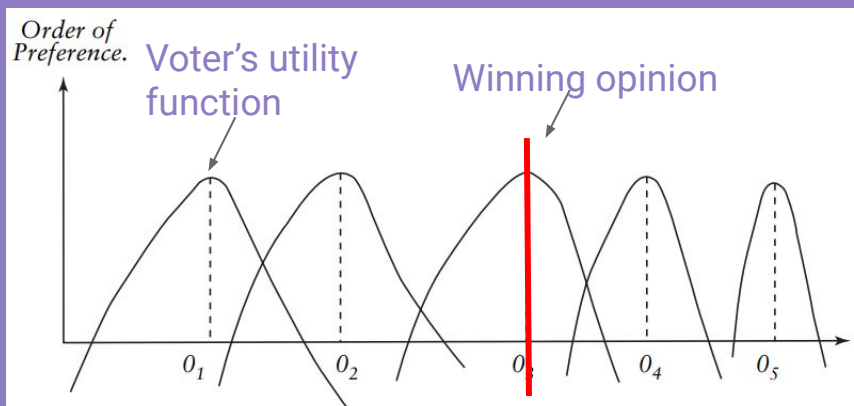
$$J = \Delta^2 / (a^2 + \sigma^2)$$



- $J \leq 1$ , One probable outcome (stable)
- $J > 1$ , Two probable outcomes (unstable)

# Median Voter Theorem

- Winning opinion is that of the median voter
  - Condorcet winner, specifically
  - Assumes each voter chooses according to a unimodal utility function
    - “Economically rational” voting
    - Deterministic voting



Black, 1948

# Linear Utility Difference model

$$P(x_A, x_B) = P(u(x_A) - u(x_B))$$

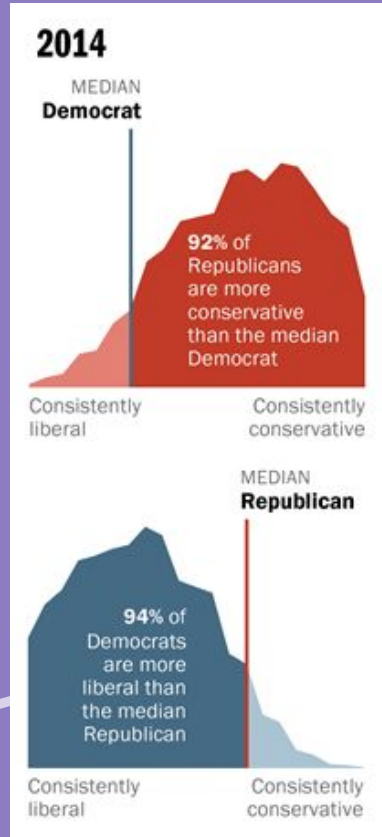
Probability of  
voting for A

Voter's utility function

- Utility difference model allows nondeterministic voting
  - Includes rational voting as limiting case
- *Linear* utility difference model assumes  $P$  linear
  - No longer capable of describing rational voting
  - A strange modeling choice

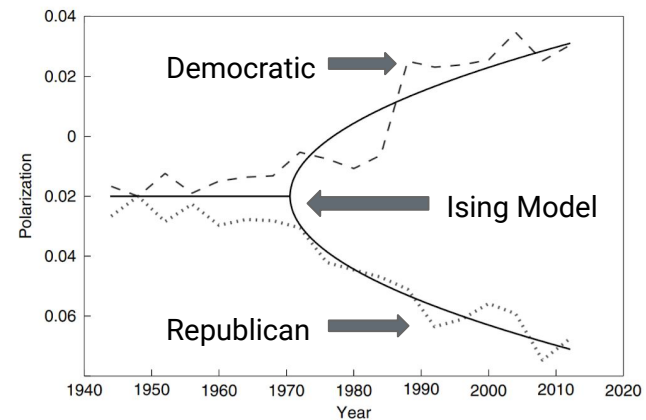


# Polarization

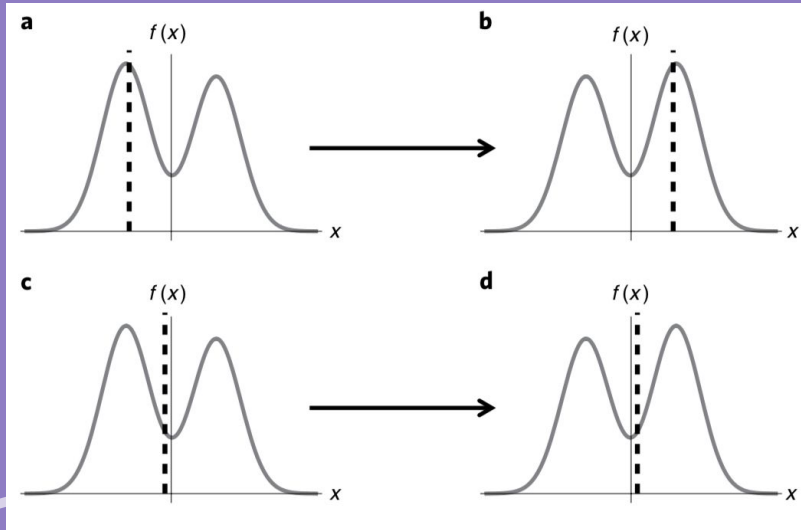


Pew research, 2014

- Lots of work on increasing polarization in U.S. politics
  - Tricky to define meaningful numerical measure
- Authors calculate theoretical polarization using distribution of voter opinions
- Compare to data on language in party platforms



# Critical Analysis of Results



This paper proposed that the election functional was translationally invariant,

$$y[f(x + c)] + c = y[f(x)]$$

Paper's main results:

- Unstable elections always lead to negative representation
- Low voter turnout leads to negative representation
  - The authors do not specify how many voters need to not turn out in order for this effect to take place.

# Critical Analysis of Results, cont.

A main weakness of this paper is the crudeness of the model.

It does not account for:

- Voters rank ordering their opinions
- Elections with more than two candidates

The model assumes that voter turnout is only a function of voter opinion.

- Doesn't discuss how voter turnout can affect electoral college outcome in US elections

The paper offers some solutions to election instability that seem outside the considerations in the model presented in the paper.

# Model Conclusions

- Physical models can be a useful tool in understanding social and political science.
  - Study of collective behaviour
  - Ising model parallel to elections
- The model marks the 1970s as a key transition phase in US election history
  - Before: Election results captured political opinions moderately well
  - After: **Instability**- Small changes in preferences have led to extreme swings in election outcomes

# Model Conclusions

- Negative Representation
  - Overall shift to left leads to a right shift in election or vice versa
- Unstable elections necessarily contain negative representation
- Possible Solutions:
  - Increase voter turnout
  - Ranked voting systems

# Our Conclusions

- This is a relatively new paper, so it is unclear how much impact it will have in the field.
- The authors do not always provide clear definitions on some key terms such as “low voter turnout”.
- The model is quite simple and assumes certain generalisations about the voter base that are not always held in reality.

# This Paper is Young, But it Made a Splash Online

↑ FermiParadox14 5 points · 10 months ago  
↓ This paper makes me ashamed to have studied physics.

↑ My\_Bored\_Brain 1 point · 10 months ago  
↓ Haven't read it. That bad?



**Martin Holub**

@martinholub\_

Phase transitions, now available for elections as well

13 Jan 2020

## Online attention



33 tweeters

10 news outlets

45 Mendeley

2 blogs

1 Redditors

1 Facebook pages

The paper was published on Feb 1 of 2020.

It has 2 citations

"This model is an excellent heuristic for understanding some critical phenomena, like how slow-moving concepts like partisanship can still yield large-scale effects in aggregate outcomes." -

@daniel\_bilar

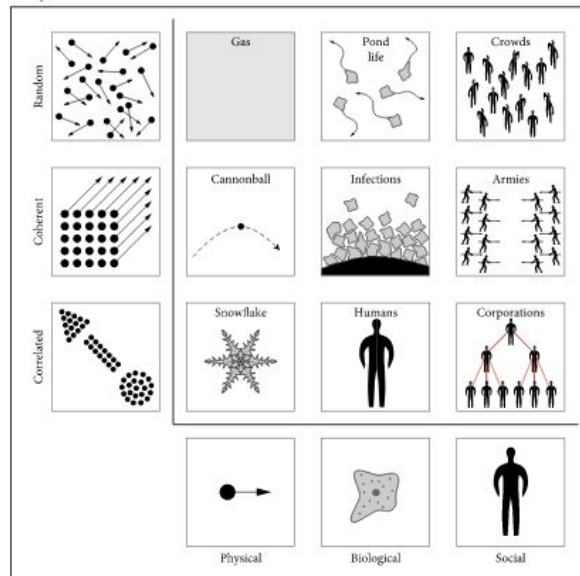
# Our Authors



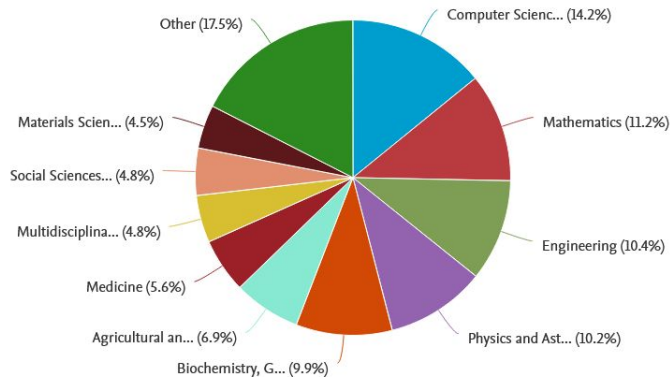
## An Introduction to Complex Systems Science and Its Applications

Siegenfeld, A.F.<sup>a,b</sup> ✉, Bar-Yam, Y.<sup>b</sup> ✉ 

Examples of Behaviors



## Bar-Yam Citations in 2017

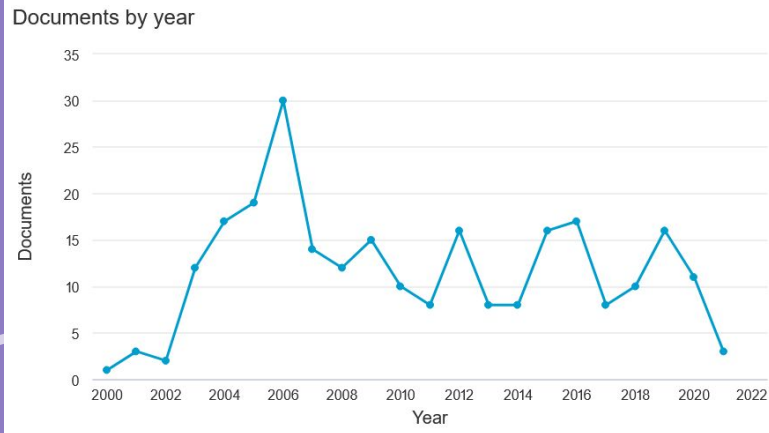


“Complex systems science considers systems with many components. ... Systems may differ from each other not because of differences in their parts but because of differences in how these parts depend on and affect one another.”



# The Subfield of Sociophysics

The field emerged in the 1970s in a “hostile” environment of physicists (see reddit thread).



Serge Galam - The role of inflexible minorities in the breaking of democratic opinion dynamics

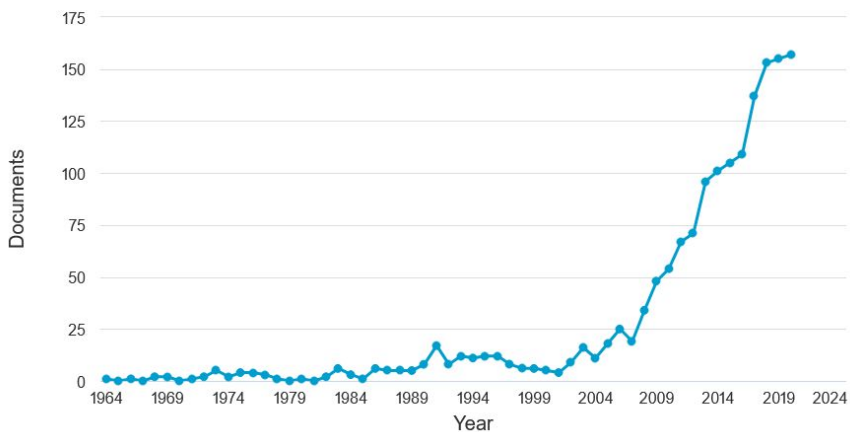


Katarzyna Sznajd-Weron - Opinion evolution in closed community

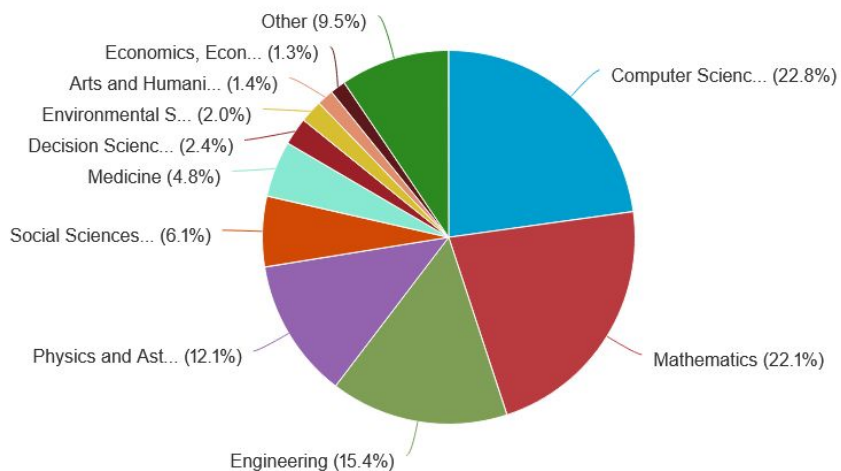


# What defines an area of research?

Documents with the Keyword Opinion Dynamics



Documents by subject area





# Citations

1. SCOPUS for being the homie
2. Siegenfeld, A. F. & Bar-Yam, Y. Negative representation and instability in democratic elections. *Nature Physics* **16**, 186–190 (2020).
3. Siegenfeld, A. F., Taleb, N. N. & Bar-Yam, Y. What models can and cannot tell us about COVID-19. *Proceedings of the National Academy of Sciences of the United States of America* **117**, 16092–16095 (2020).
4. Siegenfeld, A. F. & Bar-Yam, Y. The impact of travel and timing in eliminating COVID-19. *Communications Physics* **3**, 1–8 (2020).
5. Siegenfeld, A. F. & Bar-Yam, Y. An Introduction to Complex Systems Science and Its Applications. *Complexity* **2020**, (2020).
6. Sznajd-Weron, K. & Sznajd, J. Opinion evolution in closed community. *International Journal of Modern Physics C* **11**, 1157–1165 (2000).
7. Galam, S. Sociophysics: A review of galam models. *International Journal of Modern Physics C* **19**, 409–440 (2008).
8. McPherson, M., Smith-Lovin, L. & Cook, J. M. Birds of a feather: Homophily in social networks. *Annual Review of Sociology* **27**, 415–444 (2001).