

I.D. #	
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Subject	Phys. 560
Course	Section
Instructor	
Date	

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Receiving or giving aid in a final examination is a cause for dismissal from the University.

Lecture 17:

1.) Review: Single defect

Last class, we showed that for a single impurity

$$|R|^2 = \frac{(W/t)^2}{(W/t)^2 + 4\sin^2 k}$$

Today we want to study how the spectrum rearranges as a result of the impurity. From there we will study the general problem of impurity scattering and the advent of Anderson localization.

2.) Method of Defects:

In the band in the TBM, all the states indexed by $E = -2t \cos k$ are extended. The question we

$$EC_n = -t(C_{n+1} + C_{n-1}) + W\delta_{n,m}C_n$$

Step 1: multiply by e^{ikn} and the \sum_n

$$\sum_n e^{ikn} C_n = -t \sum_n e^{ikn} (C_{n+1} + C_{n-1}) + W \sum_n e^{ikn} \delta_{n,m} C_n$$

$$\Rightarrow (E - \epsilon(k)) C_k = W e^{ikm} C_m.$$

$$\Rightarrow C_k = \frac{W e^{ikm} C_m}{(E - \epsilon(k))}$$

Step 2: Multiply by e^{-ikm} and then integrate

$$\int_{-\pi}^{\pi} e^{-ikm} C_k dk = W \int \frac{C_m}{E - \epsilon(k)} dk$$

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-ikm} \sum_l e^{ikl} C_l = W C_m \int_{-\pi}^{\pi} \frac{dk}{E - \epsilon(k)}$$

$$\frac{1}{2\pi} \int dk \cdot e^{i k(l-m)} = \delta_{l,m}.$$

$$\Rightarrow C_m = W C_m \int_{-\pi}^{\pi} \frac{1}{2\pi} \frac{dk}{E - \epsilon(k)}.$$

The Green function is $G(E) = [E - \epsilon(k)]^{-1}$.

$$\langle G \rangle = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{dk}{E - \epsilon(k)}.$$

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$\Rightarrow 1 = W \langle G \rangle$ is the equation we must solve

Step 3: Perform integral in $\langle G \rangle$: This is our old friend.

$$\langle G \rangle = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{dk}{E - E(k)} = \int_{-\pi}^{\pi} \frac{dk}{2\pi} \frac{1}{E + 2t \cos k}.$$

Recall we did this integral before. But now we want $\langle G \rangle = \frac{1}{W}$ ← a real number. Let $z = e^{ik}$.

$$dz = iz dk.$$

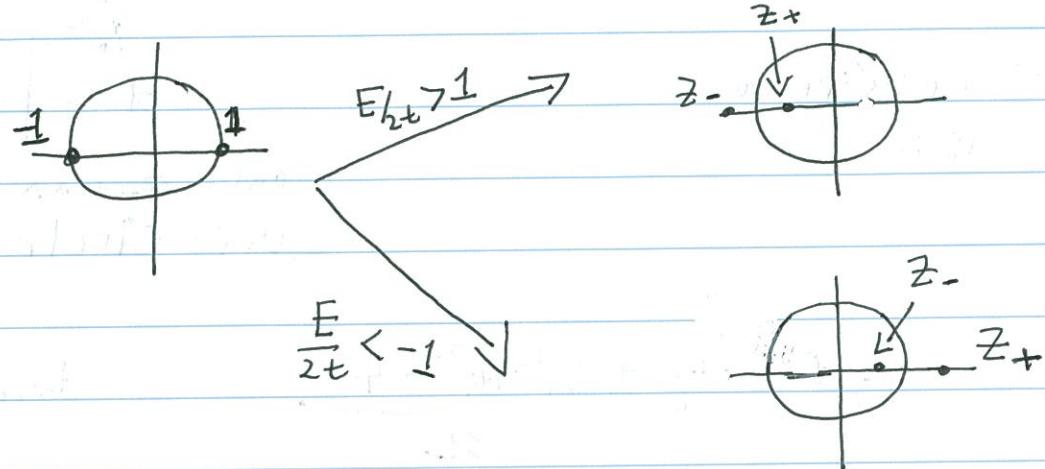
$$\Rightarrow \langle G \rangle = \frac{1}{t} \oint \frac{dz}{2\pi i} \frac{1}{(z - z_+)(z - z_-)}$$

$$z_{\pm} = \frac{-E/t \pm \sqrt{(E/t)^2 - 4}}{2}.$$

when $|E/t| < 2$, z_{\pm} is complex which means that we cannot satisfy the condition $\langle G \rangle = \frac{1}{W}$ ← a real number. \Rightarrow

We either have no solution or we have to look outside the range of the bands. For $|E/t| > 2$, z_{\pm} is surely real and a solution exists. For the impurity, new electronic states occur outside the band. There are two cases.

First, consider the position of the poles. $z \pm \left(\frac{E}{2t} \mp 1\right) = \mp 1$



This means that the integral can take on one of two values

Case 1: $E/2t > 1$

$$\langle G \rangle = \frac{1}{t} \frac{1}{2\pi i} \frac{2\pi i}{z_+ - z_-} = \frac{1}{2t} \frac{1}{\sqrt{(E/2t)^2 - 1}} \\ = \frac{1}{\sqrt{E^2 - 4t^2}}$$

$$\langle G \rangle = \frac{1}{W} \Rightarrow W > 0. \text{ (repulsive defect)}$$

Solve for E .

$$W^2 = E_b^2 - 4t^2$$

$$\Rightarrow E_b = \sqrt{W^2 + 4t^2}$$

\Rightarrow there is a new state that lies outside the band when $W > 0$. It, in fact, lies above the band.

Case II: $E_b/t < -1$.

Now enclose only Z_- .

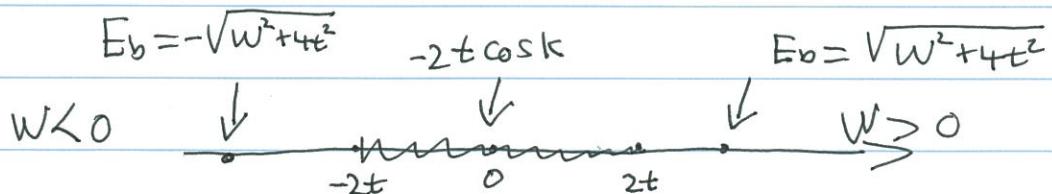
$$\Rightarrow \langle G \rangle = \frac{1}{t} \int_{-2\pi}^{2\pi} 2\pi i \cdot \frac{1}{z - z_+} = \frac{-1}{\sqrt{E_b^2 - 4t^2}}$$

$$\Rightarrow \frac{1}{W} = -\frac{1}{\sqrt{E_b^2 - 4t^2}} \Rightarrow W < 0 \quad (\text{attractive defect})$$

$$\Rightarrow E_b = -\sqrt{W^2 + 4t^2}.$$

Now the new state lies below the band.

So here's the picture.



What are these new states?

If the defect is at site m , the w.f. to the left of the defect is

$$T e^{ikm}$$

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Except for a single state, the dispersion is still $-2t \cosh k$.

Let's apply this dispersion even if $E > \pm 2t$. That is,

$$E_b = \pm 2t \cosh k_b \Rightarrow \sinh k_b = \sqrt{\left(\frac{E_b}{2t}\right)^2 - 1}$$

$$\text{but } \sqrt{E_b^2 - 4t^2} = W$$

$$\Rightarrow \sinh k_b = W/2t$$

k_b is purely imaginary in the sense that $k \rightarrow i k_b$.

VI Compute T.

$$T = 1 + R = 1 - \frac{W/t}{W/t - 2i \sinh k}$$

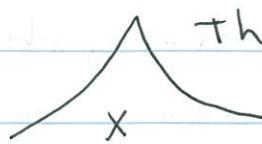
$$T = \frac{-2i \sinh k}{\frac{W}{t} - 2i \sinh k}$$

evaluate at $k \rightarrow i k_b$.

$$\Rightarrow T = \frac{2t \sinh k_b}{W + 2t \sinh k_b} = \frac{W}{W + W} = \frac{1}{2}$$

\Rightarrow the wave function is

$$\frac{1}{\sqrt{2}} k_b |X|$$



This is a localized state or a bound state.

General rule:

$d=1$, 1 impurity always leads to a bound state.
regardless of the size of W .

$d=2$, ditto.

$d=3$, a bound state only for $W > W_c$.

\Rightarrow dimensionality plays a crucial role.

Now let's consider a finite number of impurities.

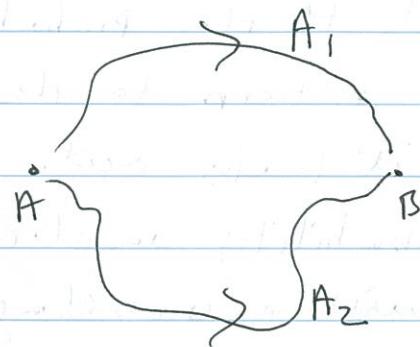
$\left\{ \begin{array}{l} d=1 \Rightarrow \text{all states are exponentially localized} \\ \quad \& W \neq 0. (\text{We will look at one exception}). \end{array} \right.$

$d=2$, ditto.

$d=3$ metallic states for $W < W_c$.

This is observed experimentally.

The physical mechanism for localization is coherent backscattering. This leads to the return probability of a particle being non-zero. A particle is localized only if it has a finite probability of remaining where it started. Let's look at backscattering.



$$P_{A \rightarrow B} = \left| \sum_i A_i \right|^2 = \sum_i |A_i|^2 + \sum_{i \neq j} \underbrace{A_i^* A_j}_{\text{interference}}$$

i = sum over all paths

When A and B are spatially separated, there is no special phase relationship between A_i and A_j and only the first term survives.

Now consider closed paths.



forward and backward paths contribute equally.

$$P_{cl} = |A_f|^2 + |A_c|^2$$

$$P_{QM} = |A_f + A_c|^2$$

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Since A_f and A_r are time reverses of one another $A_f = A_r$ if $T \cdot R$ is present.

$\Rightarrow P_{QM} = 2 P_{cl} \Rightarrow$ return processes $^{in M}$ are more effective in localizing a particle than are classical paths. The conductivity should be proportional to the probability that the particle does not return to the origin, that is to the probability it leabs out. We focus on calculating the probability an electron travels a closed tube of width a wavelength $\lambda_F = \lambda/V_F m.$

$$\lambda_F^{d-1} \quad \text{---} \quad \Delta t = V_F dt \quad \Rightarrow dV = \lambda_F^{d-1} V_F dt.$$

$$V_{max} = (\langle r^2 \rangle)^{1/2}$$

if D_0 is the diffusion coefficient
 $\Rightarrow \langle r^2 \rangle = D_0 t \Rightarrow$

$$V_{max} = (D_0 t)^{1/2}.$$

The electron will remain the tube as long as phase coherence is maintained. \Rightarrow we must integrate from $[\tau, \tau\phi]$ where $\tau\phi$ is the time at which phase coherence is lost.

$$P_{loc} \sim \int_{\tau}^{\tau\phi} \frac{dv}{V_{max}} = v_F \lambda_F^{d-1} \int_{\tau}^{\tau\phi} \frac{dt}{(D_0 t)^{d/2}}$$

$$= v_F \lambda_F^{d-1} \begin{cases} d=1 \left[\left(\frac{\tau\phi}{\tau} \right) - 1 \right] \frac{1}{2} \ln \frac{\tau\phi}{\tau} \\ d=2 \left(\ln \frac{\tau\phi}{\tau} \right) \frac{1}{D_0} \\ d=3 \left(\frac{\tau\phi}{\tau} \right)^{\frac{1}{2}} \end{cases}$$

We have calculated the T-dependence of various scattering mechanisms. We obtained that

$$\gamma_{\tau\phi} \sim T^\eta \quad \eta > 0$$

\Rightarrow in $d=2$ the correction is logarithmic in T .

This has been seen experimentally. It confirms the absence of a metallic state in $d=2$. Note in $d=1$ P_{loc} is not proportional to t . \Rightarrow the $d>1$ result is not Q.M. in nature. even in com. all paths are closed even in $d=1$.