Lecture 19:

1.) RDM Redux

Furstenberg's theorem tells us that if the transfer matrices don't commute, then the electronic states are all localized. Such is the case in random systems. The RDM shows how this fails. In the RDM, each pair has a vanishing reflection coefficient at some energy. All it takes is a plane of symmetry. Rule: If the defects possess a plane of symmetry, then any random assortment of such defects will lead to delocalization.

2.) Semiconductors:

Semiconductors are a special kind of insulator with a hard gap. The more basic quantity to start this section with is a
Mobility edge. This is the energy separating localized from extended states.

This idea will play out in semiconductor physics.

There are two kinds of semiconductors classified by their gaps. Si

Direct

Indirect

The largest gap occurs at the band center. The excitation gap is from $P \rightarrow X$. 
Recall, for holes and electrons, the effective masses have differing signs:

\[ \Sigma(k) = -2 + \cos k a \]

\[ \frac{1}{m} = \frac{a^2 + \cos k a}{\frac{2}{a^2} \Sigma k} = \frac{2^2 \Sigma k}{a^2} - \frac{2^2}{2} k^2 \]

- \( m < 0 \) (holed), \( (\text{valence band in semic}) \)
- \( m > 0 \) \( (e^2/\hbar^2) \) (cond. band in SC)

\[ \epsilon_k = \epsilon_c + \frac{a^2}{2} k^* M^{-1} k \]

\[ \epsilon_V = \epsilon_V - \frac{a^2}{2} k^* M^{-1} k \]

Let's estimate some carrier densities:

\[ n_e = \int_{\epsilon_c}^{\infty} D(\epsilon) \, f(\epsilon) \, d\epsilon \]

\[ p_h = \int_{-\infty}^{\epsilon_V} D(\epsilon) \left( 1 - f(\epsilon) \right) d\epsilon \]

\[ = \int_{-\infty}^{\epsilon_V} D(\epsilon) \left( 1 - \frac{1}{e^{(\epsilon-\epsilon_F)/kT} + 1} \right) = \int_{-\infty}^{\epsilon_V} D(\epsilon) \, f(-(-\epsilon - \mu)) \, d\epsilon \]
Non-degenerate limit: The probability of occupying states far from the band center is small.

\[ \Rightarrow \quad \varepsilon_c - m \gg k_B T \]
\[ \Rightarrow \quad m - E_v \gg k_B T. \]

\[ \Rightarrow \quad N_e = \int_{\varepsilon_c}^{\infty} D(\varepsilon) \frac{1}{e^{\beta(\varepsilon - m + \varepsilon_c - E_v)} + 1} \]
\[ \text{using } \varepsilon_c - m \gg k_B T \]

\[ = \quad \bar{e} e^{\beta(\varepsilon_c - m)} \int_{\varepsilon_c}^{\infty} D(\varepsilon) e^{\beta(\varepsilon - \varepsilon_c)} \]
\[ \quad - \rho(\varepsilon_c - m) \]
\[ = \quad e^{-\rho(\varepsilon_c - m)} N_e \]

\[ P_n = \int_{-\infty}^{E_v} D(\varepsilon) \frac{1}{e^{\beta(\varepsilon - m + E_v - E_v)} + 1} \]
\[ = \quad e^{-\beta(m-E_v)} \int_{-\infty}^{E_v} D(\varepsilon) e^{\beta(\varepsilon - E_v)} d\varepsilon \]
\[ = \quad e^{-\beta(m-E_v)} N_v. \]

Let's take the product

\[ N_e P_n = N_c N_v e^{-\beta (\varepsilon_c - E_v)} \]
\[ = \quad N_c N_v \bar{e} e^{-\beta E_g} \]
In an intrinsic semiconductor, \( n = p \)

\[
n = \frac{\sqrt{N_c N_p}}{e^{E_g/2}} \cdot \frac{1}{e}
\]

In intrinsic semiconductors, every hole left behind in the valence results in an electron in the conduction band. This need not be the case if dopants are present. The dopants can either be donors or acceptors. Donor states lie just below the conduction band while acceptor states at the top of the valence band.

Consider the case of Si:P. The donor band can be partially filled but yet no conduction transpires. This is the question Matt asked. He reasoned that this must arise from a competition between the kinetic energy and the Coulomb...
repulsion. A metal obtains once

\[
\frac{E_{\text{kin}}}{E_{\text{corr}}} > \#.
\]

Recall \( E_{\text{kin}} = n_e^{2/3} \) while \( E_{\text{corr}} = n_e^{1/3} \)

\[
\frac{E_{\text{kin}} \cdot \frac{k^2}{2m} n_e^{2/3}}{E_{\text{corr}} \cdot \frac{e^2}{e^2 n_e^{1/3}}} = a_{\text{H}} n_e^{1/3}
\]

\[\Rightarrow n_e^{1/3} a_{\text{H}} > 1.\]

Can we establish this more rigorously. Si is a dielectric material. Electrons in Si experience a charge \( e \rightarrow e/e^{1/2} \). The binding energy on a atom is

\[
E_b = \frac{e^4}{2 \alpha_0^2} = \frac{e^2}{2} \frac{k^2}{m^* e^{*2}}
\]

\[= \frac{e^2}{\varepsilon^{1/2} k^2} \frac{1}{m^* m e^{*2}}
\]

\[= \frac{1}{\varepsilon^2} \frac{m^*}{m} \left( \frac{e^2}{2 \alpha_0} \right)
\]

\[= \frac{1}{1000} 13.6 \text{eV}.
\]
The binding energy is greatly suppressed. The physical picture is that the atoms must have overlapping wave functions for the donor band to mediate electric transport.

\[ \frac{E-1}{E+2} = \frac{4\pi N_p \alpha}{2} \]

Insulator

To derive a criterion consider the Clausius-Mossotti equation:

\[ \frac{E-1}{E+2} = \frac{4\pi N_p \alpha}{2} \]

\[ \Rightarrow 3E - 3 = 4\pi N_p \alpha (E+2) \]

\[ \Rightarrow E = \frac{3 + 8\pi N_p \alpha}{3 - 4\pi N_p \alpha} \]

The denominator diverges when

\[ 3 = 4\pi N_p \alpha \]

\[ \Rightarrow 3 = 4\pi N_p \alpha \frac{1}{2} \alpha^*^2 \]

\[ \Rightarrow \frac{3}{4\pi N_p \alpha} = \frac{1}{2} \alpha^*^2 \]

\[ \Rightarrow n_p \alpha^*^2 = 0.38 \]
Should $n_p a^* > 0.38$ a metallic state forms. For Si $a^* = 4.89\text{Å}$ and $N_{cn'} = 2.33 \times 10^8 \text{cm}^{-3}$. The experimental number is $3.74 \times 10^8 \text{cm}^{-3}$. So this phenomenological argument is damn good!