I. COURSE DESCRIPTION

Systematic introduction to Einstein’s theory, with emphasis on modern coordinate-free methods of computation. Review of special relativity, differential geometry, foundations of general relativity, laws of physics in the presence of a gravitational field, linearized theory, and experimental tests of gravitation theories.

**What is this class about?** General Relativity is an advanced graduate course that teaches the foundations of Einstein’s theory of General Relativity, with emphasis on modern coordinate-free methods of computation. This class is a mathematically intensive and lays the foundations for black hole theory, post-Newtonian theory and numerical relativity. In fact, General Relativity was initially taught in the mathematics department of universities! It is impossible to teach this subject without doing a deep-dive into the mathematics that are important in General Relativity, so the first half of this class is quite mathematical. Topics covered in the first half of the course include modern differential geometry, tensor analysis, and the foundations of General Relativity. The second half of the course presents the physical consequences of Einstein’s theory, with a brief tour of its greatest hits: non-spinning (Schwarzschild) black holes and neutron stars, Solar System tests of gravitation, gravitational waves and linearized theory, and an introduction to cosmology. Students interested in these physical applications are encouraged to take subsequent courses on General Relativity, cosmology and astrophysics.

**Who should take this class?** This course is intended for advanced graduate students, although (highly-motivated) advanced undergraduate students are also welcome provided they have fulfilled the pre-requisites for the course. All students are assumed to have prior knowledge of Einstein’s theory of *special* relativity, Newtonian gravitation and classical mechanics, Maxwell’s theory of electrodynamics and advanced mathematics, including differential equations, advanced Calculus and advanced linear algebra. Other advanced mathematical machinery of General Relativity (e.g. differential geometry) will be covered in the course, and no computational knowledge is required. The primary target of the class is students who wish to specialize in General Relativity and gravitation (analytical or numerical), relativistic astrophysics and cosmology. This class will lay the foundations required to take more advanced classes and do research on the subject matter. The secondary target is students with broad interests in high-energy physics and phenomenology, nuclear theory, particle physics, condensed matter theory, field theory, string theory, and mathematical and computational physics; this class will provide a firm foundation in relativity and the ability to calculate in relativistic field theories. Other students with only a mild or minor interest in relativity are also welcome to take this class, but they should be advised that there may be other (perhaps less intensive) courses they can take to fulfill their elective requirements.

**What is expected of students who take this class?** Students are expected to attend class, complete all homework assignments and complete a midterm exam and a final exam. In addition, students are expected to be mature enough to independently do some amount of self-learning outside of class, including reading the assigned textbook(s) and papers mentioned in class. Since this is a graduate course, readings will not be assigned weekly, but students are expected to find the topics that are being covered in class in the course’s textbook and read about them in the textbook; in addition to the required class textbook, there are also other additional (recommended) textbooks that students can and should refer to if and when needed. Questions are always welcomed, either in class, or outside of class during office hours.
II. TEXTBOOKS

Required textbooks:


Recommended additional textbooks:

- P. T. Chrusciel, “Elements of general relativity”, Birkhäuser, 2019

Further reading: If you wish to read about the latest developments physics, you can find open-access preprints of scientific articles on the [https://arxiv.org/](https://arxiv.org/)

III. TOPICS

- 27 lectures of 80 minutes each
- Spring break: 11-19 March 2023
- No class on 18 April 2022 due to APS April meeting
- Detailed weekly course schedule online at [https://courses.physics.illinois.edu/phys515/sp2023/schedule.html](https://courses.physics.illinois.edu/phys515/sp2023/schedule.html)

1. Flat Spacetime and Manifolds [Chapters 1 and 2]
   - Special Relativity
   - Tensor Calculus
   - Classical Field Theory

2. Curvature [Chapters 3]
   - Covariant Derivatives
   - Geodesics
   - Curvature Tensors
   - Killing Vectors

3. Midterm [in-class, 2-hour block]

4. Gravity [Chapters 4]
   - Einstein’s Equations
   - Lagrangian Formulation
   - Equivalence Principle
5. Black Holes and Neutron Stars [Chapters 5]
   - Properties
   - Geodesics
   - Experimental Tests

6. Gravitational Waves [Chapters 7]
   - Linearized Theory and the Properties of Waves
   - Generation of Waves
   - Energy Loss and Detection

7. Cosmology [Chapters 8]
   - Robertson–Walker Spacetimes
   - Friedman Equations
   - Redshift and Lensing

8. Final Exam (Take Home)

IV. CREDIT AND GRADE SCALE

Credit Points: Everyone starts with 0 credit points (CPs). You gain CPs by doing homework, doing the final exam and participating in class or in piazza (asking or answering questions). There will be 12 homework sets and in total they will be worth \( \sim 70\% \) of your grade. There will be one midterm and one final exam, worth \( \sim 12.5\% \) of your grade each. You can earn up to 27 bonus points by answering gradescope questions in class.

Grading: The grader for homework assignments is Rohit Subbarayan Chandramouli, and exams will be graded by Prof. Witek and Rohit Subbarayan Chandramouli.

Homework: handed-out (online) on Wednesdays before class and due one week later on Wednesdays online. Unless a valid, verifiable excuse is given, homework sets which are submitted late will receive a 10% penalty per day (between Friday and Monday a 20% penalty applies). Homework sets which are turned in more than a week late will receive no credit.

Exams: The midterm exam will be closed-book and take 1.5 hours during class. The final exam will be both open-book and take-home. The only book your are allowed to use during the exam is Carroll’s “Spacetime and Geometry” listed above.

Grade Scale: Your overall grade is correlated with your CPs, as given by the following ranking table:
   - Wizard: \( \geq 890 \) CPs, corresponds to an A.
   - Sorcerer: \( \geq 770 \) and \( < 890 \) CPs, corresponds to a B.
   - Mage: \( \geq 650 \) and \( < 770 \) CPs, corresponds to a C.
   - Enchanter: \( \geq 500 \) and \( < 650 \) CPs, corresponds to a D.
   - N/A: \( < 500 \) CPs, corresponds to an F.

V. ADDITIONAL COURSE POLICY

Academic integrity: All activities in this course are subject to the Academic Integrity rules as described in Article 1, Part 4, Academic Integrity, of the Student Code; see [https://studentcode.illinois.edu/article1/part4/1-401/].

Academic dishonesty may result in a failing grade. Every student is expected to review and abide by the Academic Integrity Policy. Ignorance is not an excuse for any academic dishonesty. It is your responsibility
to read this policy to avoid any misunderstanding. Do not hesitate to ask the instructor(s) if you are ever in doubt about what constitutes plagiarism, cheating, or any other breach of academic integrity.

**COVID-19** Classes are expected to take place in person. Please follow updates of the University’s COVID-19 policy.

Following University policy, all students are required to engage in appropriate behavior to protect the health and safety of the community. Wearing a facial covering properly is strongly recommended (recommended N95, KN95, FFP2). Students who are ill should not come to class. These students are judged to have excused absences for the class period and should contact the instructor via email about making up the work.

**Anti-Racism and Inclusivity Statement** The Grainger College of Engineering is committed to the creation of an anti-racist, inclusive community that welcomes diversity along with a number of dimensions, including, but not limited to, race, ethnicity and national origins, gender and gender identity, sexuality, disability status, class, age, or religious beliefs. The College recognizes that we are learning together in the midst of the Black Lives Matter movement, that Black, Hispanic, and Indigenous voices and contributions have largely either been excluded from, or not recognized in, science and engineering, and that both overt racism and micro-aggressions threaten the well-being of our students and our university community.

The effectiveness of this course is dependent upon each of us to create a safe and encouraging learning environment that allows for the open exchange of ideas while also ensuring equitable opportunities and respect for all of us. Everyone is expected to help establish and maintain an environment where students, staff, and faculty can contribute without fear of personal ridicule, or intolerant or offensive language. If you witness or experience racism, discrimination, micro-aggressions, or other offensive behavior, you are encouraged to bring this to the attention of the course director if you feel comfortable. You can also report these behaviors to the Bias Assessment and Response Team (BART); see [https://bart.illinois.edu/](https://bart.illinois.edu/). Based on your report, BART members will follow up and reach out to students to make sure they have the support they need to be healthy and safe. If the reported behavior also violates university policy, staff in the Office for Student Conflict Resolution may respond as well and will take appropriate action.

**Sexual Misconduct Reporting Obligation** The University of Illinois is committed to combating sexual misconduct. Faculty and staff members are required to report any instances of sexual misconduct to the University’s Title IX Office. In turn, an individual with the Title IX Office will provide information about rights and options, including accommodations, support services, the campus disciplinary process, and law enforcement options.

A list of the designated University employees who, as counselors, confidential advisors, and medical professionals, do not have this reporting responsibility and can maintain confidentiality, can be found here: [wecare.illinois.edu/resources/students/#confidential](http://wecare.illinois.edu/resources/students/#confidential).

Other information about resources and reporting is available here: [wecare.illinois.edu](http://wecare.illinois.edu).

**Religious Observances** Illinois law requires the University to reasonably accommodate its students’ religious beliefs, observances, and practices in regard to admissions, class attendance, and the scheduling of examinations and work requirements. You should examine this syllabus at the beginning of the semester for potential conflicts between course deadlines and any of your religious observances. If a conflict exists, you should notify your instructor of the conflict and follow the procedure [https://odos.illinois.edu/community-of-care/resources/students/religious-observances/](https://odos.illinois.edu/community-of-care/resources/students/religious-observances/) to request appropriate accommodations. This should be done in the first two weeks of classes.

**Disability-Related Accommodations** To obtain disability-related academic adjustments and/or auxiliary aids, students with disabilities must contact the course instructor and the Disability Resources and Educational Services (DRES) as soon as possible. To contact DRES, you may visit 1207 S. Oak St., Champaign, call 333-4603, e-mail [disability@illinois.edu](mailto:disability@illinois.edu) or go to [https://www.disability.illinois.edu](https://www.disability.illinois.edu). If you are concerned you have a disability-related condition that is impacting your academic progress, there are academic screening appointments available that can help diagnose a previously undiagnosed disability. You may access these by visiting the DRES website and selecting ”Request an Academic Screening“ at the bottom of the page.

**Family Educational Rights and Privacy Act (FERPA)** Any student who has suppressed their directory information pursuant to the Family Educational Rights and Privacy Act (FERPA) should self-identify to the instructor to ensure the protection of the privacy of their attendance in this course. See [https://registrar.illinois.edu/academic-records/ferpa/](https://registrar.illinois.edu/academic-records/ferpa/) for more information.