M. Stone, 2024

Syllabus for Physics 509: Mathematical Physics II

Course Title: CRN 49905 ; Physics 509: Mathematical Physics II

Instructor S2024, Professor Michael Stone

Course description: Continuation of PHYS 508. Further core techniques of mathematical physics widely used in the physical sciences. Complex variables; group theory in classical and quantum systems; tensors in physics; differential forms and their applications in mechanics; electromagnetism.

Prerequisites and Co-requisites: Course explorer lists PHYS 508 as a requirement, but I do not enforce this. .

Credit Hours: 4 hrs

Current course website: https://courses.physics.illinois.edu/phys509/sp2024/ This site includes Includes all homeworks, information about grading, and links to other sites of interest.

Grade assignment: Grades will be determined from total scores weighted as follows: Homework 60%, Term Paper and Presentation 40%.

Texts and Supplies: Recommended but not required: Michael Stone and Paul Goldbart *Mathematics for Physics: A guided tour for graduate students*, (Cambridge University Press 2009). (A draft version of the book is linked to on the course website.)

Also recommended: *Differential Forms with applications to the Physical Sciences* by Harley Flanders, *Group Theory* by Morton Hammermesh, and *Mathematics for Physicists* by Phillipe Dennery and Andre Krzywicki. All three are published by Dover Publications, and should cost no more than $25 or so each.

Course outline: This is the second semester of a graduate-level course on the mathematics used in physics research. The course will have three related components:

1) Vectors, Tensors, Differential Geometry and Topology: Covariant and contravariant vectors and tensors. Vector fields: Lie bracket and integrability. Differential forms: exterior differentiation, Poincare's theorem, integration of p-forms, Stokes' theorem. Introduction to topology: homology and cohomology.
2) Groups and Group Representations: Elementary group theory: subgroups, cosets and conjugacy classes. Representations of finite groups:

characters and orthogonality. Physical applications: molecular vibrations, quantum mechanics. Lie groups and Lie algebras: semisimiple and compact groups, representations, roots and weights.

3) Complex Analysis: Complex differentiability. Conformal mapping and its physical applications. Cauchy, Taylor, and Laurent theorems, smooth *vs*. analytic functions. Applications to contour integration, solution of differential equations and asymptotics.

Weekly Schedule: This guide is *approximate* because the actual progress through the course material will depend on questions and discussions that arise during the class-time.

Week1: Tensor algebra, covariant and contravariant tensors and their transformation laws

Week 2: Manifolds, coordinate systems. Vector and Tensor fields on manifolds.

Week 3: Differential Calculus on manifolds, Lie brackets and Lie derivatives.

Week 4: Differential forms and the exterior calculus. Applications to Electromagnetism and Hamiltonian mechanics.

Week 5: Riemann geometry, connections and curvature .

Week 6: Integration on manifolds, differential forms and Stokes' theorem.

Week 7: Homotopy and Hopf map. Application to the spin-statistics theorem

Week 8: Algebraic topology. Simplicial homology, cohomology and de Rahm's theorem.

Week 9: Exact sequences and cohomological calculations

Week 10: A review/introductions to the theory of finite groups. The idea of representations.

Week 11: Representations of finite groups, character tables, applications.

Week 12: The geometry of Lie groups. Invariant vector fields and Maurer-Cartan forms. Applications in physics.

Week 13: Introduction to complex analysis. Complex differentiability. Stokes's theorem and Cauchy's theorem

Week 14: Applications of complex variables. Contour integral solutions of differential equations. Hypergeometric functions.

Learning Objectives and Outcomes: For each topic in the list above you will master the basic concepts to the level that they can use them to solve physics problems. You will acquire the necessary skills though working illustrative examples in homework assignments. You will demonstrate competence by your performance in the homework and by an end-of-term paper and oral presentation on a topic relevant to the course .

Disability Access: The Department of Physics is committed to being an open and welcoming environment for all of our students. We are committed to helping all of our students succeed in our courses. To obtain disability-related academic adjustments and/or auxiliary aids, students with disabilities must contact the course instructor and the Disability Resources and Educational Services (DRES) as soon as possible. To contact DRES, you may visit 1207 S. Oak St., Champaign, call 333-4603, e-mail disability@illinois.edu, or go to the DRES website. If you are concerned you have a disability-related condition that is impacting your academic progress, there are academic screening appointments available on campus that can help diagnosis a previously undiagnosed disability by visiting the DRES website and selecting "Sign-Up for an Academic Screening" at the bottom of the page.

Academic Integrity: All activities in this course are subject to the Academic Integrity rules as described in*Article 1, Part 4, Academic Integrity*, of the Student Code.

Infractions include, but are not limited to:

- Cheating, plagiarism, fabrication
- facilitating infractions of academic integrity.
- academic interference
- computer-related infractions
- unauthorized use of university resources
- sale of class materials or notes

Violations of any of these rules will be prosecuted and reported to the student's home college in compliance with the Student Code: *Article 1, Part 4, Academic Integrity*, of the Student Code.