(a) A two-body system, such as an electron-proton system, can be expressed as an effective one-body system with a reduced mass

$$\mu = \frac{m_1 m_2}{m_1 + m_2}.$$

This means that we can use the results derived for the hydrogen atom, but replacing $m \to \mu$.

For example, the ground state (binding) energy of hydrogen, accounting for the proton mass, is

$$E_1' = -\left[\frac{\mu}{2\hbar^2}\left(\frac{e^2}{4\pi\epsilon_0}\right)^2\right] = \frac{\mu}{m}\left[-\frac{m}{2\hbar^2}\left(\frac{e^2}{4\pi\epsilon_0}\right)^2\right]$$

$$= \frac{\mu}{m} E_1$$

where $E_1 = -13.6\,\text{eV}$ is the usual ground state energy of hydrogen, m is the mass of the electron, $\mu = \frac{m m_p}{m + m_p}$ is the reduced mass, and m_p is the mass of the proton.

(a con) The percent error for ignoring the proton mass is

$$\frac{|E_1 - E_1'|}{E_1'} \times 100\% = 100\% \times \frac{E_1\left(1 - \frac{\mu}{m}\right)}{E \frac{\mu}{m}}$$

$$= 100\% \times \left[\frac{m}{\mu} - 1\right] = 100\% \times \left[\frac{m + m_p}{m_p} - 1\right]$$

$$= 100\% \times \frac{m}{m_p} = 100\% \times \left(\frac{\text{mass of electron}}{\text{mass of proton}}\right)$$

$$= 5.4 \times 10^{-2}\% = 0.054\%.$$

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(b) The energy levels for hydrogen and deuterium are

$$E_n' = E_1'/n^2 = \frac{\mu}{m} E_1 \frac{1}{n^2} \quad (n=1,2,3,\dots)$$

The energy difference between the $n=2$ and $n=3$ levels are

$$\Delta E' = E_3' - E_2' = E_1'\left(\frac{1}{9}-\frac{1}{4}\right) = -\frac{5}{36} E_1'$$

This correspond to the following wavelength of light

$$\Delta E' = \frac{hc}{\lambda'} = -\frac{5}{36} E_1' \Rightarrow \lambda' = -\frac{36}{5}\frac{hc}{E_1'} = -\frac{36}{5}\frac{hc}{E_1}\frac{m}{\mu}.$$

For hydrogen, accounting for the proton mass $m_p \gg m$,

$$\mu_h = \frac{m_p m}{m_p + m} = \frac{m}{1+\frac{m}{m_p}}.$$

For deuterium, the nucleus mass is approximately $2m_p$, so the reduced mass is

$$\mu_d = \frac{2m_p m}{2m_p + m} = \frac{m}{1+\frac{m}{2m_p}}.$$

The difference in wavelength of the $n=2 \to 3$ lines for hydrogen and deuterium are

$$\lambda_h - \lambda_d = -\frac{36}{5}\frac{hc}{E_1}\left(\frac{m}{\mu_h} - \frac{m}{\mu_d}\right) = -\frac{36}{5}\frac{hc}{E_1}\left[\left(1+\frac{m}{m_p}\right) - \left(1+\frac{m}{2m_p}\right)\right]$$

$$= -\frac{36}{10}\frac{hc}{E_1}\frac{m}{m_p} = 1.79\times10^{-9}\,\text{m}.$$

A muon has mass $m_\mu = 207m$, so the muon atom's

(c) reduced mass is $\mu = \frac{m_\mu m_p}{m_\mu + m_p}$.

The $n=2 \to n=1$ line has energy difference

$$\Delta E' = E_2' - E_1' = E_1' \left(\frac{1}{4} - \frac{1}{1} \right) = -\frac{3}{4} E_1'$$

which corresponds to the wavelength

$$\lambda' = \frac{hc}{\Delta E'} = -\frac{4}{3} \frac{hc}{E_1'} = -\frac{4}{3} \frac{hc}{E_1} \frac{m}{\mu}$$

$$= 6.54 \times 10^{-10} \, m.$$