Hund's rules state that, to determine the (a) \((S,L,J)\) ground state of an atom, we

① First, maximize \(S\).

② Then, maximize \(L\).

③ Finally, for more than half-filled shells, maximize \(J\); otherwise minimize \(J\).

When applying Hund's rules, we need to take care to only consider valid states.

Applying Hund's rules to \(\text{Li, B, N, Na, and K}\), we get

<table>
<thead>
<tr>
<th>Element</th>
<th>(S)</th>
<th>(L)</th>
<th>(J)</th>
<th>Term Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{Li (Z=3)})</td>
<td>1/2</td>
<td>0</td>
<td>1/2</td>
<td>(^2S_{1/2})</td>
</tr>
<tr>
<td>(\text{B (Z=5)})</td>
<td>1/2</td>
<td>1</td>
<td>1/2</td>
<td>(^2P_{1/2})</td>
</tr>
<tr>
<td>(\text{N (Z=7)})</td>
<td>3/2</td>
<td>0</td>
<td>3/2</td>
<td>(^4S_{3/2})</td>
</tr>
<tr>
<td>(\text{Na (Z=11)})</td>
<td>1/2</td>
<td>0</td>
<td>1/2</td>
<td>(^2S_{1/2})</td>
</tr>
<tr>
<td>(\text{K (Z=19)})</td>
<td>1/2</td>
<td>0</td>
<td>1/2</td>
<td>(^2S_{1/2})</td>
</tr>
</tbody>
</table>

Nitrogen is a little tricky. It has three valence electrons in the 2p shell. They have quantum numbers \(l_1, m_e, m_s, l_2, m_e, m_s, l_3, m_e, m_s\) with \(l_1 = l_2 = l_3 = 1\).
To maximize $S$ to $3/2$, we need that
\[ m_{s_1} = m_{s_2} = m_{s_3} = \pm 1/2, \]

in other words that all three electrons' spins are aligned.

So, in the $|l_1 m_1, m_{s_1}; l_2 m_2, m_{s_2}; l_3 m_3, m_{s_3}\rangle$ basis, we know that the ground state must satisfy
\[ |l_1 m_1, 1/2; l_2 m_2, 1/2; l_3 m_3, 1/2\rangle. \]

Since these are electrons, they are fermions and cannot occupy the same state.

Therefore, $m_{e_1}, m_{e_2}, m_{e_3}$ must all be different.

Since $m_{e_i} \in \{-1, 0, 1\}$ for $l_i = 1$, the only possible state satisfying $S = 3/2$ and $l_1 = l_2 = l_3 = 1$ is the $m_{e_1} = -1$, $m_{e_2} = 0$, $m_{e_3} = 1$ state:
\[ |1, -1 1/2; 0 1/2; 1 1/2\rangle. \]

This state has $M_L = 0$, which implies that $L = 0$.

This means that the only possible $J$ is $J = 3/2$. 

(b) The lowest frequency line in the absorption spectrum of Na corresponds to an electron in the 3s level absorbing a photon and being excited to a 3p state. There are two such states, split by the spin-orbit effect. The splitting is given by

$$E_{so} \propto -\vec{\mu}_s \cdot \vec{B}$$

where $\vec{\mu}_s = -g_s u_\beta \vec{S}/\hbar$ is the magnetic moment of the electron and $\vec{B}$ is an effective magnetic field.

Semiclassically, we can think of the $\vec{B}$ field as arising from a charged nucleus rotating around the electron:

$$\vec{B} \propto q \vec{E} \times \vec{r}.$$  (related to Biot-Savart law)

The $\vec{E}$ field is that of a point charge

$$\vec{E} \propto \frac{e}{r^2} \hat{r}.$$  

So

$$\vec{B} \propto \frac{q e}{c r^2} \vec{r} \times \hat{r} = \frac{q e}{c m r^2} (m \vec{v} \times \hat{r}) \quad (\vec{r} = r \hat{r})$$

$$= \frac{q e}{c m r^3} (m \vec{\omega} \times \hat{r}) = \frac{q e}{c m r^3} \vec{L}, \quad (\vec{L} = m \vec{\omega} \times \hat{r})$$
This then tells us that the (b-an) spin-orbit splitting can also be written as

\[ E_{\text{so}} = -\vec{p}_S \cdot \vec{B} \]

\[ = - \left[ \frac{g_S MB}{h} \right] \cdot \left[ \frac{g_e}{cm^3} \vec{L} \right] \]

\[ = - \frac{g_S MB g_e}{mc^2} \cdot \frac{1}{r^3} \vec{L} \cdot \vec{S} \]

\[ \propto \vec{L} \cdot \vec{S} \cdot \frac{1}{r^3} \]

For the particular states considered, \( L \) and \( S \) are fixed and non-zero, so the splitting is proportional to \( \langle \frac{1}{r^3} \rangle \), \( n = -3 \).

(c) For potassium \( K (Z=19) \), there is one 4s valence electron in the ground state. The term symbol corresponding to this \( (L=0, S=\frac{1}{2}, J=\frac{1}{2}) \) state is \( ^2S_{1/2} \).

The lowest single valence electron states are

\[
\begin{align*}
\text{[Ar]}4s & & ^2S_{1/2} & \quad (S, L, J) = (\frac{1}{2}, 0, \frac{1}{2}) \\
\text{[Ar]}4p & & ^2P_{1/2} & \quad (\frac{1}{2}, 1, \frac{1}{2}) \\
\text{[Ar]}4p & & ^2P_{3/2} & \quad (\frac{1}{2}, 1, \frac{3}{2})
\end{align*}
\]
The energy levels look like:

\[(\text{can}) \quad (4p) \quad ^2P_{3/2} \quad (M_J = -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}) \]
\[(M_J = -\frac{1}{2}, \frac{1}{2}) \]
\[(4s) \quad ^2S_{1/2} \quad (M_J = -\frac{1}{2}, \frac{1}{2}) \]

The energy of the electron's magnetic dipole in a magnetic field \( \vec{B} = B \hat{z} \) is:

\[ \hat{\mathcal{H}}' = -\mu_J \cdot \vec{B} = \frac{e}{2m} \mathbf{g} \cdot \vec{S} \cdot \vec{B} = \frac{e}{2m} g J_z B \propto g M_J, \]

where \( \vec{\mu}_J = -\frac{e}{2m} \mathbf{g} \) is the magnetic dipole of the electron in an \((S,L,J)\) atomic state (\( g \) is the Landé-\( g \)-factor, which depends on \( S,L,J \)).

So, larger \( g \) and \( M_J \) correspond to larger changes in energies due to magnetic fields.

The perturbed energy levels look like:

\[(4p) \quad ^2P_{3/2} \quad (g = \frac{1}{2}) \]
\[ ^2P_{1/2} \quad (g = \frac{1}{2}) \]
\[(4s) \quad ^2S_{1/2} \quad (g = 2) \]

Energy \( \uparrow \) \( B \) = magnetic field (exaggerated slopes)
(d) All of the allowed El (dipole) transitions to the \( ^2S_{1/2} \) states are

which were determined by following the selection rules.