First, normalization. We have

$$1 = \langle\Psi|\Psi\rangle = A^2 \int_{-\infty}^{\infty} x^2 e^{-2bx^2}\,dx = \sqrt{\frac{\pi}{32b^3}}\,A^2, \quad \text{or} \quad A = \left(\frac{32b^3}{\pi}\right)^{1/4}$$

Now, we compute $\langle\Psi|H|\Psi\rangle$.

$$\langle\Psi|H|\Psi\rangle = \sqrt{\frac{32b^3}{\pi}} \int_{-\infty}^{\infty}\left[\frac{-\hbar^2}{2m}\, x e^{-bx^2}\frac{\partial^2}{\partial x^2} x e^{-bx^2} + \frac{1}{2} m\omega^2 x^4 e^{-2bx^2}\right]dx$$

$$= \underbrace{\sqrt{\frac{32b^3}{\pi}}\,\frac{\hbar^2}{2m}\int_{-\infty}^{\infty}\left|\frac{\partial}{\partial x} x e^{-bx^2}\right|^2 dx}_{①} + \underbrace{\sqrt{\frac{32b^3}{\pi}}\,\frac{m\omega^2}{2}\int_{-\infty}^{\infty} x^4 e^{-2bx^2}\,dx}_{②}$$

$$① = \sqrt{\frac{8b^3\hbar^4}{\pi m^2}}\int_{-\infty}^{\infty}\left(\cancel{x}e^{-bx^2} - 2bx^2 e^{-bx^2}\right)^2 dx$$

$$= \sqrt{\frac{8b^3\hbar^4}{\pi m^2}}\int_{-\infty}^{\infty}\left(e^{-2bx^2} - 4bx^2 e^{-2bx^2} + 4b^2x^4 e^{-2bx^2}\right)dx$$

$$= \sqrt{\frac{8b^3\hbar^4}{\pi m^2}}\left(\sqrt{\frac{\pi}{2b}} - \sqrt{\frac{\pi}{2b}} + \frac{3}{4}\sqrt{\frac{\pi}{2b}}\right)$$

$$= \frac{3}{4}\sqrt{\frac{4b^2\hbar^4}{m^2}} = \frac{3b\hbar^2}{2m}$$

$$② = \sqrt{\frac{8b^3m^2\omega^4}{\pi}}\int_{-\infty}^{\infty} x^4 e^{-2bx^2}\,dx$$

$$= \sqrt{\frac{8b^3m^2\omega^4}{\pi}}\left(\frac{3}{16}\sqrt{\frac{\pi}{2b^5}}\right)$$

$$= \frac{3}{8}\sqrt{\frac{m^2\omega^4}{b^2}} = \frac{3m\omega^2}{8b}$$

Thus, $\langle\Psi|H|\Psi\rangle = \dfrac{3b\hbar^2}{2m} + \dfrac{3m\omega^2}{8b}$. To minimize, we set the derivative to zero:

$$0 = \partial_b\langle\Psi|H|\Psi\rangle = \frac{3\hbar^2}{2m} - \frac{3m\omega^2}{8b^2}, \quad \text{or} \quad b^2 = \frac{m^2\omega^2}{4\hbar^2}, \quad \text{or} \quad b = \frac{m\omega}{2\hbar}$$

Then $\langle\Psi|H|\Psi\rangle_{min} = \dfrac{3\hbar^2}{2m}\left(\dfrac{m\omega}{2\hbar}\right) + \dfrac{3m\omega^2}{8}\left(\dfrac{2\hbar}{m\omega}\right) = \dfrac{3}{4}\hbar\omega + \dfrac{3}{4}\hbar\omega = \dfrac{3}{2}\hbar\omega$, or $E_1 \leq \dfrac{3}{2}\hbar\omega$

Note that $\langle\Psi|H|\Psi\rangle$ gives an upper bound for the energy of $|\Psi_1\rangle$, the first excited state, not just an upper bound for the energy of $|\Psi_0\rangle$. This is because we know $|\Psi_0\rangle$ is even, and our $|\Psi\rangle$ is always odd, thus we know $\langle\Psi|\Psi_0\rangle = 0$ for any b.