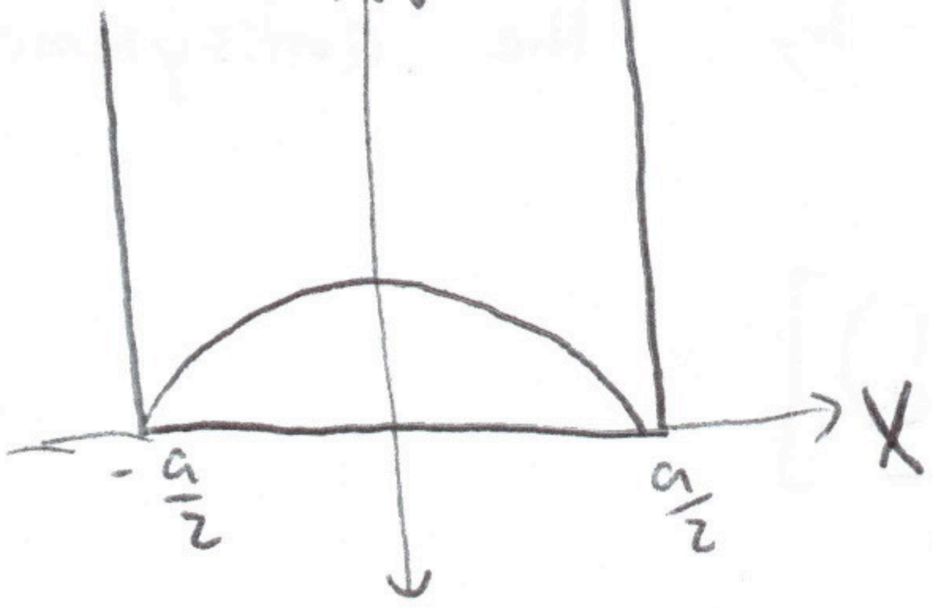
1a) 

$$\psi(x) = \sqrt{\frac{2}{a}}\cos\left(\frac{\pi x}{a}\right)$$

$$E\psi = H\psi = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = \frac{\hbar^2\pi^2}{2ma^2}\psi, \quad \text{so } E = \frac{\hbar^2\pi^2}{2ma^2}$$

b) We know $E^{(0)} = \frac{\hbar^2\pi^2}{2ma^2}$. We need $E^{(1)}$.

$$E^{(1)} = \langle\psi|V(x)|\psi\rangle = \int_{-a/2}^{a/2}\frac{2}{a}\cos^2\left(\frac{\pi x}{a}\right)\frac{2\varepsilon|x|}{a}\,dx$$

$$= 2\int_0^{a/2}\frac{2}{a}\cos^2\left(\frac{\pi x}{a}\right)\frac{2\varepsilon x}{a}\,dx$$

$$= \frac{8\varepsilon}{a^2}\int_0^{a/2}x\cos^2\left(\frac{\pi x}{a}\right)dx$$

$$= \frac{8\varepsilon}{a^2}\int_0^{a/2}x\left[\frac{1+\cos\left(\frac{2\pi x}{a}\right)}{2}\right]dx$$

$$= \frac{8\varepsilon}{a^2}\left[\int_0^{a/2}\frac{x}{2}\,dx + \frac{1}{2}\int_0^{a/2}x\cos\left(\frac{2\pi x}{a}\right)dx\right]$$

Let $u = x$, $dv = \cos\left(\frac{2\pi x}{a}\right)dx$
$v = \frac{a}{2\pi}\sin\left(\frac{2\pi x}{a}\right)$

$$= \frac{8\varepsilon}{a^2}\left[\frac{x^2}{4}\Big|_0^{a/2} + \frac{1}{2}\frac{xa}{2\pi}\sin\left(\frac{2\pi x}{a}\right)\Big|_0^{a/2} - \frac{1}{2}\int_0^{a/2}\frac{a}{2\pi}\sin\left(\frac{2\pi x}{a}\right)dx\right]$$

$$= \frac{8\varepsilon}{a^2}\left[\frac{a^2}{16} + 0 + \frac{a}{4\pi}\frac{a}{2\pi}\cos\left(\frac{2\pi x}{a}\right)\Big|_0^{a/2}\right]$$

$$= \frac{8\varepsilon}{a^2}\left[\frac{a^2}{16} - \frac{a^2}{4\cancel{8}\pi^2}\right]$$

$$= \left(\frac{\varepsilon}{2} - \frac{2\varepsilon}{\pi^2}\right)$$