2 Problem 2

(a) The phrase "a particle of spin s" ***does not mean*** that if you were to measure the particle's spin in an experiment, you'd find that it's equal to s! This is simply not true. Why? **In experiments, we can only measure observables - specifically, eigenvalues of Hermitian operators.** So to measure the spin of a particle in an experiment means finding the magnitude of its spin vector.

$\hat{S}^2$ is a Hermitian operator which has eigenvalue given by $\hbar^2 s(s+1)$. The magnitude of the particle's spin (which is what you can actually measure in an experiment) is $\boxed{\hbar\sqrt{s(s+1)}}$. For $s=1$, this is $\boxed{\sqrt{2}\hbar}$.

So the lesson here is that s is not an eigenvalue of a Hermitian operator - it's just a *quantum number*, which you can use to label eigenstates (if the Hamiltonian commutes with the operator corresponding to that quantum number). Don't confuse a quantum number with a measurable eigenvalue of a Hermitian operator!

(b) We are told that a particle is in a central potential with orbital angular momentum *quantum number* $l=2$ and spin *quantum number* $s=1$. Recall that from angular momentum addition, the possible values of the total angular momentum *quantum number* j are

$$j = |l-s|, |l-s|+1, \ldots, l+s \tag{6}$$

subject to the rule that $m_j = m_l + m_s$ where $m_j = -j, -j+1, \ldots, j-1, j$ for each possible value of j.

For the values of l and s in this problem, we find that the possible values of j are $j = 1, 2, 3$. From equation (5), the eigenstates of $\hat{H}_{\text{s-o}}$ are given by $|j\, l\, s\, m_j\rangle$. The eigenstates are therefore,

$$\begin{aligned} &j=1 \text{ eigenstates} \quad \{|1\,2\,1\,-1\rangle, |1\,2\,1\,0\rangle, |1\,2\,1\,1\rangle\} \\ &j=2 \text{ eigenstates} \quad \{|2\,2\,1\,-2\rangle, |2\,2\,1\,-1\rangle, |2\,2\,1\,0\rangle, |2\,2\,1\,1\rangle, |2\,2\,1\,2\rangle\} \\ &j=3 \text{ eigenstates} \quad \{|3\,2\,1\,-3\rangle, |3\,2\,1\,-2\rangle, |3\,2\,1\,-1\rangle, |3\,2\,1\,0\rangle, |3\,2\,1\,1\rangle, |3\,2\,1\,2\rangle, |3\,2\,1\,3\rangle\} \end{aligned} \tag{7}$$

Notice that for each value of j, there are $2j+1$ eigenstates (because there are $2j+1$ values of m_j).

The energies E_{jsl} in equation (5) **are independent of** m_j. This implies the $\boxed{j=1 \text{ eigenstates are 3-fold degnerate}}$, $\boxed{j=2 \text{ eigenstates are 5-fold degnerate}}$, and $\boxed{j=3 \text{ eigenstates are 7-fold degnerate}}$ in energy. The energy eigenvalues are

$$\begin{aligned} &\boxed{j=1 \text{ eigenstates} \quad E_{121} = \frac{\beta\hbar^2}{2}(1(1+1) - 2(2+1) - 1(1+1)) = -3\beta\hbar^2} \\ &\boxed{j=2 \text{ eigenstates} \quad E_{221} = \frac{\beta\hbar^2}{2}(2(2+1) - 2(2+1) - 1(1+1)) = -\beta\hbar^2} \\ &\boxed{j=3 \text{ eigenstates} \quad E_{321} = \frac{\beta\hbar^2}{2}(3(3+1) - 2(2+1) - 1(1+1)) = 2\beta\hbar^2} \end{aligned} \tag{8}$$