(A) The original potential is:

$$\hat{V}(x) = \begin{cases} 0 & 0 < x < a \\ \infty & elsewhere \end{cases}$$

The orignal energy eigenstates are:

$$\psi_n(x) = \begin{cases} \sqrt{\frac{2}{a}} \sin(\frac{n\pi}{a}x) & 0 < x < a \\ 0 & elsewhere \end{cases}$$

The original energy eigenvalues are:

$$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}. \tag{16}$$

In this system, we take $n = 1, 2, 3, \ldots$. The system starts out in the ground state, $n = 1$. The potential suddenly changes:

$$\hat{V}'(x) = \begin{cases} 0 & 0 < x < 2a \\ \infty & elsewhere \end{cases}$$

The new energy eigenstates are:

$$\psi'_n(x) = \begin{cases} \sqrt{\frac{2}{2a}} \sin(\frac{n\pi}{2a}x) & 0 < x < 2a \\ 0 & elsewhere \end{cases}$$

The new energy eigenvalues are:

$$E'_n = \frac{n^2\pi^2\hbar^2}{2m(2a)^2}. \tag{17}$$

We need to construct the old ground state, $\psi_1(x)$, as a linear combination of the new energy eigenstates:

$$\psi_1(x) = \sum_{n=1}^{\infty} c'_n \psi'_n(x). \tag{18}$$

The coeffecients are

$$c'_n = \int_{-\infty}^{\infty} \psi'_n(x)\psi_1(x)dx \tag{19}$$

$$= \int_{-\infty}^{0} \psi'_n(x)\psi_1(x)dx + \int_{0}^{a} \psi'_n(x)\psi_1(x)dx + \int_{a}^{\infty} \psi'_n(x)\psi_1(x)dx$$

$$= \frac{\sqrt{2}}{a}\int_0^a \sin(\frac{n\pi}{2a}x)\sin(\frac{\pi}{a}x)dx$$

$$= \frac{\sqrt{2}}{a}\int_0^a (\frac{e^{i\frac{n\pi}{2a}x} - e^{-i\frac{n\pi}{2a}x}}{2i})(\frac{e^{i\frac{n\pi}{a}x} - e^{-i\frac{n\pi}{a}x}}{2i})dx$$

$$= -\frac{1}{2\sqrt{2}a}\int_0^a (e^{i\frac{3n\pi}{2a}x} - e^{i\frac{n\pi}{2a}x} - e^{i\frac{n\pi}{2a}x} + e^{i\frac{3n\pi}{2a}x})dx \tag{20}$$

These terms can be integrated and then turned back into trig fuctions for easy evaluation. The end result is

$$P'_n = |c'_n|^2 = \begin{cases} \frac{1}{2} & n = 2 \\ \pm\frac{4\sqrt{2}}{\pi(n^2-4)} & n \text{ odd} \\ 0 & \text{otherwise} \end{cases}$$

The most probable result of an energy measurement is $E'_2 = \frac{\pi^2\hbar^2}{2ma^2}$, with a probability of $P'_2 = 50\%$.

(B) The *next* most probable result is $E'_1 = \frac{\pi^2\hbar^2}{8ma^2}$, with a probability of $P'_1 \approx 36\%$.

(C) The expectation value of the Hamiltonian for this state is

$$< H >_{\psi_1} = \int_{-\infty}^{\infty} \psi_1^*(x)\hat{H}\psi_1(x)dx \tag{21}$$

$$= \frac{2}{a}\int_0^a \sin(\frac{\pi}{a}x)(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2})\sin(\frac{\pi}{a}x)dx$$

$$= \frac{\pi^2\hbar^2}{2ma^2}, \tag{22}$$

which is the same as before the potential was altered. The calculation makes this obvious.