(A) The original potential is:

\[ \hat{V}(x) = \begin{cases} 0 & 0 < x < a \\ \infty & \text{elsewhere} \end{cases} \]

The original energy eigenstates are:

\[ \psi_n(x) = \begin{cases} \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right) & 0 < x < a \\ 0 & \text{elsewhere} \end{cases} \]

The original energy eigenvalues are:

\[ E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}. \quad (16) \]

In this system, we take \( n = 1, 2, 3, \ldots \). The system starts out in the ground state, \( n = 1 \).

The potential suddenly changes:

\[ \hat{V}'(x) = \begin{cases} 0 & 0 < x < 2a \\ \infty & \text{elsewhere} \end{cases} \]

The new energy eigenstates are:

\[ \psi'_n(x) = \begin{cases} \sqrt{\frac{2}{2a}} \sin\left(\frac{n\pi}{2a}x\right) & 0 < x < 2a \\ 0 & \text{elsewhere} \end{cases} \]
The new energy eigenvalues are:
\[ E'_n = \frac{n^2 \pi^2 \hbar^2}{2m(2a)^2}. \] (17)

We need to construct the old ground state, \( \psi_1(x) \), as a linear combination of the new energy eigenstates:
\[ \psi_1(x) = \sum_{n=1}^{\infty} c'_n \psi'_n(x). \] (18)

The coefficients are
\[ c'_n = \int_{-\infty}^{\infty} \psi'_n(x) \psi_1(x) dx \] (19)

\[ = \int_{-\infty}^{0} \psi'_n(x) \psi_1(x) dx + \int_{0}^{a} \psi'_n(x) \psi_1(x) dx + \int_{a}^{\infty} \psi'_n(x) \psi_1(x) dx \]
\[ = \frac{\sqrt{2}}{a} \int_{0}^{a} \sin(\frac{n\pi}{2a} x) \sin(\frac{\pi}{a} x) dx \]
\[ = \frac{\sqrt{2}}{a} \int_{0}^{a} \frac{(e^{i\frac{n\pi}{2a} x} - e^{-i\frac{n\pi}{2a} x}) (e^{i\frac{\pi}{a} x} - e^{-i\frac{\pi}{a} x})}{2i} \frac{dx}{2i} \]
\[ = -\frac{1}{2\sqrt{2}a} \int_{0}^{a} (e^{i\frac{3n\pi}{2a} x} - e^{i\frac{n\pi}{2a} x} - e^{i\frac{n\pi}{2a} x} + e^{i\frac{3n\pi}{2a} x}) dx \] (20)

These terms can be integrated and then turned back into trig functions for easy evaluation. The end result is
\[ P'_n = |c'_n|^2 = \begin{cases} 
\frac{1}{2} & n = 2 \\
\pm \frac{4\sqrt{2}}{\pi(n^2-4)} & n \text{ odd} \\
0 & \text{otherwise}
\end{cases} \]

The most probable result of an energy measurement is \( E'_2 = \frac{\pi^2 \hbar^2}{8ma^2} \), with a probability of \( P'_2 = 50\% \).

(B) The next most probable result is \( E'_1 = \frac{\pi^2 \hbar^2}{8ma^2} \), with a probability of \( P'_1 \approx 36\% \).

(C) The expectation value of the Hamiltonian for this state is
\[ < H >_{\psi_1} = \int_{-\infty}^{\infty} \psi_1^*(x) \hat{H} \psi_1(x) dx \] (21)
\[ = \frac{2}{a} \int_{0}^{a} \sin(\frac{\pi}{a} x) (-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \sin(\frac{\pi}{a} x)) dx \]
\[ = \frac{\pi^2\hbar^2}{2ma^2}, \] (22)

which is the same as before the potential was altered. The calculation makes this obvious.