Ang. Mom. Addition → see review video

- rule: $j = |j_1 - j_2|, \ldots (j_1 + j_2)$ steps of 1
- reading CG tables: given $j_1 \times j_2$: $|m_1, m_2\rangle \leftrightarrow |J, M\rangle$ individual ↔ composite
- q.nos vs e-values:

j	of $\hat{j}^2 = \hbar^2 j(j+1)$	& $m = -j, \ldots +j$ steps of 1
m	of $\hat{j}_z = \hbar m$	

Identical Particles

- Axiom 6: $\hat{P}_{ij}\,\psi(1,2,\ldots i,\ldots j,\ldots) = \begin{cases} -\psi_A(1,2,\ldots) & \text{if fermions} \\ +\psi_S(1,2\ldots) & \text{if bosons} \end{cases}$

  $\hat{P}_{ij}$: exchange $i \leftrightarrow j$; ij = identical pcle

  A = anti-symmetric, S = symmetric under exchange of any pair i, j of identical pcles

- $|\text{FULL QM STATE}\rangle$ = space part × spin part

  ⇑ this is what must obey Axiom 6

e.g. $|\text{STATE}\rangle_A$ (2 electrons →) $= \begin{cases} \psi_S(\vec{r}_1, \vec{r}_2) \cdot \chi_{A\,1,2} \\ \psi_A(\vec{r}_1, \vec{r}_2) \cdot \chi_{S\,1,2} \end{cases}$

e.g. Helium: G 5.2.1

COROLLARY

- Pauli exclusion for identical fermions
- Exchange force:

spatial w.f.

$$\langle |\vec{r}_1 - \vec{r}_2| \rangle = \begin{matrix} \text{larger for } \psi_A(\vec{r}_1, \vec{r}_2) = \text{"repulsive"} \\ \text{smaller for } \psi_S(\vec{r}_1, \vec{r}_2) \quad \text{"attractive"} \end{matrix}$$

- 2-spin 1/2 combo's: (e.g. 2 electrons)

$S=1$ TRIPLET, symmetric:

$$\begin{cases} |1\ +1\rangle_{SM} = |\uparrow\uparrow\rangle_{m_1 m_2} \\ |1\ 0\rangle_{SM} = \frac{1}{\sqrt{2}}\left[|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle\right]_{m_1 m_2} \\ |1\ -1\rangle_{SM} = |\downarrow\downarrow\rangle_{m_1 m_2} \end{cases}$$

$S=0$ SINGLET, antisymmetric:

$$|0\ 0\rangle_{SM} = \frac{1}{\sqrt{2}}\left[|\downarrow\uparrow\rangle - |\uparrow\downarrow\rangle\right]_{m_1 m_2}$$

TECHNIQUE

- (T) Building ψ_A for >2 particles: Slater determinant
- (T) 2-particle operators: use particle subscripts

e.g. $\hat{H}_{\text{2 atomic electrons}} = -\frac{\hbar^2}{2m}\nabla_1^2 - \frac{\hbar^2}{2m}\nabla_2^2 + A\,\hat{S}_{1x}\hat{S}_{2x}$

- $-\frac{\hbar^2}{2m}\nabla_1^2$: acts on r_1, θ_1, ϕ_1 only
- $-\frac{\hbar^2}{2m}\nabla_2^2$: acts on x_2, y_2, z_2 only
- $A\,\hat{S}_{1x}\hat{S}_{2x}$: $\left(\frac{\hbar}{2}\right)^2 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_1 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_2$

e.g. Disc 6 Q3

One-e^- Atoms: Bohr + spin-orbit

- Mag. Dipole Moment $\vec{\mu} = \gamma \vec{J}$

with $\gamma = g \cdot \left(\frac{e}{2m}\right)$

CLASSICAL

- 1: $\vec{J}$ = OAM
- 2: $\vec{J}$ = spin of a Dirac p'cle (pointlike spin ½ pcle)
- other values: $\vec{J}$ = spin of particles w structure e.g. nucleons

- Effect 1: $V_{zeeman} = -\vec{\mu} \cdot \vec{B}_{EXTERNAL}$
- Effect 2: $V_{spin\text{-}orbit} = -\vec{\mu} \cdot \vec{B}_{INTERNAL} \sim \hat{\vec{s}} \cdot \hat{\vec{l}}$

($\vec{\mu}$ due to $\vec{s}$ of atomic electron) ($\vec{B}$ due to $\vec{l}$ of electron around nucleus)

- good q. nos: when spin-orbit present,

NOT (m_s, m_ℓ) .... YES (j, m_j)

- atomic notation → ℓ: s, p, d, f, ...
  → electronic config: [He] $2s^2 2p^3$
  → term symbols: $^{2S+1}L_J$

- Hydrogenic energy corrections: (in order)
  Bohr
  Fine Structure = spin-orbit ($\vec{\ell}\cdot\vec{s}$) + relativistic KE
  Lamb shift [QED]
  Hyperfine structure = $\vec{s}\cdot\vec{s}_{NUC}$

Many-e Atoms : Bohr + Vee + $V_{spin-orbit}$ + ...

- screening & central-force / mean field } approx to approximately deal with Vee
- filling order : $E_{n\ell} \sim (n+\ell)$
- closed shells : $S=0, L=0$
- good q. nos : L-S coupling scheme → S, L, J
  ($V_{ee} \gg V_{s\text{-}o}$)
- Hund rules #(1, 2) → minimize Vee ... #3 → min. $V_{s\text{-}o}$

e.g. energy-level diagram for Carbon : end of Lec 5B