$$\hat{\vec{j}}_1 + \hat{\vec{j}}_2 = \hat{\vec{J}}$$

$[\hat{j}_x, \hat{j}_y] = i\hbar \hat{j}_z$ & cyclic permut^{n}s

$[\hat{j}^2, \hat{j}_i] = \emptyset$ for $i = x, y, z$

$\Downarrow$

$\underbrace{|j_1, m_1\rangle}_{\text{2 quantum numbers}}$ where j_1 specifies e-value of $\hat{j}_1^2$: $\hbar^2 j_1(j_1+1)$

m_1 " " of $\hat{j}_{1z}$: $\hbar m_1$

to specify an angular momentum state completely

Now for the COMBINED angular momentum $\hat{\vec{J}} = \hat{\vec{j}}_1 + \hat{\vec{j}}_2$

2 representations = bases:

$\boxed{1}$ INDIVIDUAL BASIS

$|j_1, j_2; m_1, m_2\rangle$

$= |j_1, m_1\rangle |j_2, m_2\rangle$

$\boxed{2}$ COMPOSITE BASIS

$|j_1, j_2; J\, M\rangle$

connection between these bases:

CLEBSCH-GORDAN Coefficients / tables

Angular Momentum Addition Rule for $\hat{\vec{j}}_1 + \hat{\vec{j}}_2 = \hat{\vec{J}}$

(a) quantum no. $\boxed{J = |j_1 - j_2|, \ldots\ldots, j_1 + j_2 \quad \text{steps of } 1}$

(b) quantum no. $\boxed{M = m_1 + m_2}$ from $\hat{j}_{1z} + \hat{j}_{2z} = \hat{J}_z$

Using CG tables

e.g. add $j_1 = 1$ and $j_2 = 2$

$|j_1, m_1\rangle |j_2, m_2\rangle$
$= |j_1, j_2; m_1, m_2\rangle$
$= |m_1, m_2\rangle$

$|j_1, j_2; JM\rangle$
$= |JM\rangle$ given that we're adding "1×2"

total # states: $3 \times 5 =$ (m_1 values, m_2 values) $= 15$

m_1		m_2
-1		-2
0	$\times$	-1
		0
$+1$		$+1$
		$+2$

J	M	total # states:
1	$-1, 0, +1$	3
2	$-2, \ldots\ldots, +2$	5
3	$-3, \ldots\ldots, +3$	7
		15

e.g. $|m_1\, m_2\rangle = |0\ 0\rangle = \underline{\text{WHAT?}}$ in the $|J M\rangle$ basis

oops, we only have a 2×1 table $\Rightarrow$ our states will be $|m_2\, m_1\rangle$ & $|J\ M\rangle$

Find the state $|m_1\, m_2\rangle = |0\ 0\rangle_{m_1 m_2}$ in the 2×1 table ... $\Leftarrow$ ... we chose $|m_2\, m_1\rangle = |0\ 0\rangle$ $\therefore$ no change!

$$|0\ 0\rangle_{m_1 m_2} = \sqrt{\frac{3}{5}}\,|3\ 0\rangle_{JM} - \sqrt{\frac{2}{5}}\,|1\ 0\rangle_{JM}$$

$\therefore$ If we measure J, we have probability $3/5$ to get $J=3$

" 0 " $J=2$

" $2/5$ " $J=1$