EM RADIATION as Perturbation

jargon: E1 = elec. dipole
E2 = elec. quadr.
...
M1 = mag. dipole
...

Electric Dipole (E1) Radiation

E1 radiation is dominant when

① $F_B \ll F_E$ on charges in your system

② $\vec{k} \cdot \vec{r} \ll 1$

} very common in ~~atomic~~ physics

① $F_E = qE$ ... compare with $F_B = |q\vec{v} \times \vec{B}| \leq qvB$

- compare E & B: for EM waves in vacuum, $B = \frac{E}{c}$
- how big is v for atomic electrons?

Virial Theorem for p'cles bound in $-\frac{1}{r}$ potential:

$$\underset{\text{NEG}}{\langle E \rangle} = \underset{\text{NEG}}{\langle V \rangle} + \underset{\text{POS}}{\langle T \rangle}$$

... Virial Thm for 1/r:

$$\boxed{|\langle V \rangle| = 2\langle T \rangle}$$

$$= -\langle T \rangle = -\frac{mv^2}{2}$$

$$E_n = -\frac{(Z\alpha)^2}{\cancel{2}n^2}(\cancel{m_e c^2}) = -\frac{\cancel{m_e c^2}}{\cancel{2}} \cdot \frac{v^2}{c^2}$$

Bohr

$$\therefore \frac{v}{c} = \frac{Z\alpha}{n} \underset{n=1}{\leq} Z\alpha = \frac{Z}{137} \approx 1\% \text{ for light atoms}$$

-10%

$\Longrightarrow \frac{F_B}{F_E}$ on e's in ~light atoms $\le \frac{qvB}{qE} = \frac{v}{c} = \mathcal{O}(1\%) \ll 1$ //

-10%

in presence of EM waves

② $\vec{k}\cdot\vec{r} \le \frac{2\pi r}{\lambda}$ $\rightarrow$ $r \simeq 1$ Å $= 0.1$ nm

$\lambda \rightarrow$ e.g. visual range $\lambda = 400-700$ nm

of EM wave with ($\vec{E} = \vec{E}_0 e^{i\vec{k}\cdot\vec{r}-\omega t}$) PLANE WAVE

cf. RF waves have MUCH larger λ

$\Longrightarrow$ in huge range of cases, $\vec{k}\cdot\vec{r} \ll 1$ //

With approx ① $F_B \ll F_E$ ② $\vec{k}\cdot\vec{r} \ll 1$

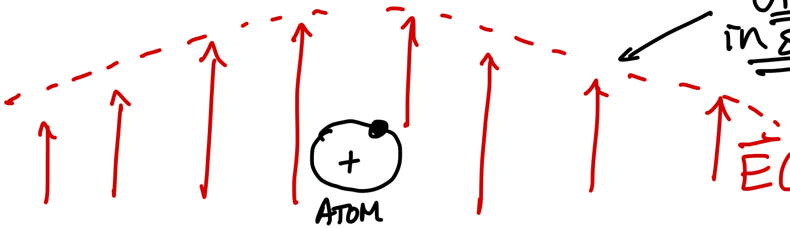
part of EM wave to consider:

$$\vec{E}(\vec{r},t) = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)} = \vec{E}_0 e^{i\vec{k}\cdot\vec{r}} e^{-i\omega t}$$

where $e^{i\vec{k}\cdot\vec{r}} = 1 + i\vec{k}\cdot\vec{r} + \frac{(i\vec{k}\cdot\vec{r})^2}{2!} + \dots$

≈ 1 for $\vec{k}\cdot\vec{r} \ll 1$ ie. $\vec{E}(\vec{r},t) \approx \vec{E}_0 e^{-i\omega t}$

UNIFORM in space, oscillating in time



$\vec{E}(\vec{r}) \simeq \vec{E}_0$ = uniform over size of system

Goal: Evaluate FGR $W_{i\to f}$ for E1 radiation

need ① average V_{fi} over polariz^{n} states of $\vec{E}_0 = \hat{\varepsilon} E_0$
(assuming INCOHERENT source of radiation)

② $|\overline{V}_{fi}|$ transition matrix elem → selection rules

③ $n(E_f)$ density of states

Potential of a charge q in an E1 rad^{n} field:

$$\boxed{V(\vec{r}) = -q\vec{E}_0\cdot\vec{r}}$$

just electrostatic potential energy $-\int_{\vec{r}_{0,REF}}^{\vec{r}} \vec{F}_E\cdot\vec{dl}$

= WORK integral

⊕ spatial part ↑ is what goes into F.G.R. (as part of V_{fi})

① Assuming <u>INCOHERENT source</u> of radiation
(e.g. flashlight, blackbody source)

$$V_{fi} = \langle f|-q\vec{E}_0\cdot\vec{r}|i\rangle$$

... define polariz^{n} vector $\hat{\varepsilon}$: $\vec{E}_0 = E_0\hat{\varepsilon}$

$$= -qE_0\langle f|\hat{\varepsilon}\cdot\vec{r}|i\rangle$$

(classical)

↳ is ⊥ to $\vec{k}$ of photon but

randomly oriented: $\vec{r}\cdot\hat{\varepsilon} = r\cos\theta$

⇒ average over random θ [Gr p.353]

$$\overline{|\langle \vec{r}\cdot\hat{\varepsilon}\rangle_{if}|^2} = \boxed{\frac{1}{3}}\, |\langle \vec{r}\rangle_{if}|^2 \sim \overline{|V_{fi}|^2}$$

① NOT an expectation value!

$$\langle \hat{Q}\rangle_{if} \equiv \langle i|\hat{Q}|f\rangle$$

= TRANSITION matx. elem

② When is V_{fi} between atomic* states non-zero?

⇒ **SELECTION RULES** for E1 transitions:

Take $|i\rangle, |f\rangle = |n\ell m\rangle_{i,f}$ single-particle states

*(or any central potential)

Transition Matx elem is

$$V_{fi} = -E_0 \underbrace{\langle n_f \ell_f m_f | q\vec{r} | n_i \ell_i m_i\rangle}_{\equiv\, \vec{p}_{fi}}$$

$\equiv \vec{p}_{fi}$: $q\vec{r}$ is electric dipole moment of a single charge q relative to the origin

... BUT $\vec{p}_{fi}$ is "TRANSITION electric dipole moment",

When is V_{fi} non-zero?

Gr. p.360-361: when

$$\boxed{\begin{array}{l} \bullet\ \ell_i - \ell_f = \pm 1 \\ \bullet\ m_i - m_f = 0, \pm 1 \end{array}}$$

E1 selection rules

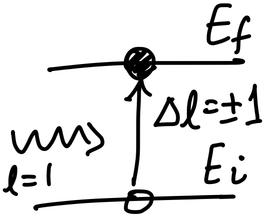
What does this tell us about an E1 photon?

NOT actual EDM of any single charge distribution

⇒ E1 photon must carry angular momentum of 1

QFT: E1 photon is a "SPIN* 1 BOSON"

*i.e. orbital ang. momentum of 1, but intrinsic to an E1 photon



② E1 transitions are so dominant that transitions mediated by

E2 (electric quadrupole)
M1 (magnetic dipole)
⋮
terms in expansion of EM wave

are called FORBIDDEN transition

③ Finally, need density of states $n(E_f)$

...