**1D SHO** $\hat{H}(x)=\frac{1}{2m}\left(\hat{p}^2+m^2\omega^2x^2\right)$ Define $x_0\equiv\sqrt{\frac{\hbar}{m\omega}}$, $\xi\equiv\frac{x}{x_0}$ $\rightarrow$ $\hat{H}(\xi)=\frac{\hbar\omega}{2}\left(\xi^2-\frac{d^2}{d\xi^2}\right)$

$E_n=\left(n+\tfrac{1}{2}\right)\hbar\omega$, $\psi_n(x)=\left(\frac{1}{\pi x_0^2}\right)^{1/4}\frac{1}{\sqrt{2^n n!}}H_n\left(\frac{x}{x_0}\right)e^{-\frac{x^2}{2x_0^2}}$ where $H_0(\xi)=1$, $H_2(\xi)=4\xi^2-2$, $H_1(\xi)=2\xi$, $H_3(\xi)=8\xi^3-12\xi$,

$\hat{a}_\pm=\frac{1}{\sqrt{2}}\left(\xi\mp\frac{d}{d\xi}\right)\rightarrow$ $\hat{a}_+\psi_n=\sqrt{n+1}\,\psi_{n+1}$, $\hat{a}_-\psi_n=\sqrt{n}\,\psi_{n-1}$, $\hat{H}=\hbar\omega\left(\hat{a}_+\hat{a}_-+\tfrac{1}{2}\right)$, $\left[\hat{a}_-,\hat{a}_+\right]=1$, $H_n(\xi)=(-1)^n e^{\xi^2}\frac{d^n}{d\xi^n}e^{-\xi^2}$

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for $-8/15$ read $-\sqrt{8/15}$.

Notation:

| | | J J … |
|---|---|---|
| | | M M … |
| $m_1$ | $m_2$ | |
| $m_1$ | $m_2$ | Coefficients |
| . | . | |

$Y_1^0=\sqrt{\frac{3}{4\pi}}\cos\theta$

$Y_1^1=-\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$

$Y_2^0=\sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta-\frac{1}{2}\right)$

$Y_2^1=-\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$

$Y_2^2=\frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$

$Y_\ell^{-m}=(-1)^m Y_\ell^{m*}$

$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$

### 1/2×1/2

| $m_1$ $m_2$ | J | | |
|---|---|---|---|
| | 1, +1 | | |
| +1/2 +1/2 | 1 | | |
| | 1, 0 | 0, 0 | |
| +1/2 −1/2 | 1/2 | 1/2 | |
| −1/2 +1/2 | 1/2 | −1/2 | |
| | 1, −1 | | |
| −1/2 −1/2 | 1 | | |

### 1×1/2

| $m_1$ $m_2$ | | |
|---|---|---|
| | 3/2, +3/2 | |
| +1 +1/2 | 1 | |
| | 3/2, +1/2 | 1/2, +1/2 |
| +1 −1/2 | 1/3 | 2/3 |
| 0 +1/2 | 2/3 | −1/3 |
| | 3/2, −1/2 | 1/2, −1/2 |
| 0 −1/2 | 2/3 | 1/3 |
| −1 +1/2 | 1/3 | −2/3 |
| | 3/2, −3/2 | |
| −1 −1/2 | 1 | |

### 2×1/2

| $m_1$ $m_2$ | | |
|---|---|---|
| | 5/2, +5/2 | |
| +2 +1/2 | 1 | |
| | 5/2, +3/2 | 3/2, +3/2 |
| +2 −1/2 | 1/5 | 4/5 |
| +1 +1/2 | 4/5 | −1/5 |
| | 5/2, +1/2 | 3/2, +1/2 |
| +1 −1/2 | 2/5 | 3/5 |
| 0 +1/2 | 3/5 | −2/5 |
| | 5/2, −1/2 | 3/2, −1/2 |
| 0 −1/2 | 3/5 | 2/5 |
| −1 +1/2 | 2/5 | −3/5 |
| | 5/2, −3/2 | 3/2, −3/2 |
| −1 −1/2 | 4/5 | 1/5 |
| −2 +1/2 | 1/5 | −4/5 |
| | 5/2, −5/2 | |
| −2 −1/2 | 1 | |

### 3/2×1/2

| $m_1$ $m_2$ | | |
|---|---|---|
| | 2, +2 | |
| +3/2 +1/2 | 1 | |
| | 2, +1 | 1, +1 |
| +3/2 −1/2 | 1/4 | 3/4 |
| +1/2 +1/2 | 3/4 | −1/4 |
| | 2, 0 | 1, 0 |
| +1/2 −1/2 | 1/2 | 1/2 |
| −1/2 +1/2 | 1/2 | −1/2 |
| | 2, −1 | 1, −1 |
| −1/2 −1/2 | 3/4 | 1/4 |
| −3/2 +1/2 | 1/4 | −3/4 |
| | 2, −2 | |
| −3/2 −1/2 | 1 | |

### 2×1

| $m_1$ $m_2$ | | | |
|---|---|---|---|
| | 3, +3 | | |
| +2 +1 | 1 | | |
| | 3, +2 | 2, +2 | |
| +2 0 | 1/3 | 2/3 | |
| +1 +1 | 2/3 | −1/3 | |
| | 3, +1 | 2, +1 | 1, +1 |
| +2 −1 | 1/15 | 1/3 | 3/5 |
| +1 0 | 8/15 | 1/6 | −3/10 |
| 0 +1 | 2/5 | −1/2 | 1/10 |
| | 3, 0 | 2, 0 | 1, 0 |
| +1 −1 | 1/5 | 1/2 | 3/10 |
| 0 0 | 3/5 | 0 | −2/5 |
| −1 +1 | 1/5 | −1/2 | 3/10 |
| | 3, −1 | 2, −1 | 1, −1 |
| 0 −1 | 2/5 | 1/2 | 1/10 |
| −1 0 | 8/15 | −1/6 | −3/10 |
| −2 +1 | 1/15 | −1/3 | 3/5 |
| | 3, −2 | 2, −2 | |
| −1 −1 | 2/3 | 1/3 | |
| −2 0 | 1/3 | −2/3 | |
| | 3, −3 | | |
| −2 −1 | 1 | | |

### 1×1

| $m_1$ $m_2$ | | | |
|---|---|---|---|
| | 2, +2 | | |
| +1 +1 | 1 | | |
| | 2, +1 | 1, +1 | |
| +1 0 | 1/2 | 1/2 | |
| 0 +1 | 1/2 | −1/2 | |
| | 2, 0 | 1, 0 | 0, 0 |
| +1 −1 | 1/6 | 1/2 | 1/3 |
| 0 0 | 2/3 | 0 | −1/3 |
| −1 +1 | 1/6 | −1/2 | 1/3 |
| | 2, −1 | 1, −1 | |
| 0 −1 | 1/2 | 1/2 | |
| −1 0 | 1/2 | −1/2 | |
| | 2, −2 | | |
| −1 −1 | 1 | | |

### 3/2×1

| $m_1$ $m_2$ | | | |
|---|---|---|---|
| | 5/2, +5/2 | | |
| +3/2 +1 | 1 | | |
| | 5/2, +3/2 | 3/2, +3/2 | |
| +3/2 0 | 2/5 | 3/5 | |
| +1/2 +1 | 3/5 | −2/5 | |
| | 5/2, +1/2 | 3/2, +1/2 | 1/2, +1/2 |
| +3/2 −1 | 1/10 | 2/5 | 1/2 |
| +1/2 0 | 3/5 | 1/15 | −1/3 |
| −1/2 +1 | 3/10 | −8/15 | 1/6 |
| | 5/2, −1/2 | 3/2, −1/2 | 1/2, −1/2 |
| +1/2 −1 | 3/10 | 8/15 | 1/6 |
| −1/2 0 | 3/5 | −1/15 | −1/3 |
| −3/2 +1 | 1/10 | −2/5 | 1/2 |
| | 5/2, −3/2 | 3/2, −3/2 | |
| −1/2 −1 | 3/5 | 2/5 | |
| −3/2 0 | 2/5 | −3/5 | |
| | 5/2, −5/2 | | |
| −3/2 −1 | 1 | | |

### 3/2×3/2

| $m_1$ $m_2$ | | | | |
|---|---|---|---|---|
| | 3, +3 | | | |
| +3/2 +3/2 | 1 | | | |
| | 3, +2 | 2, +2 | | |
| +3/2 +1/2 | 1/2 | 1/2 | | |
| +1/2 +3/2 | 1/2 | −1/2 | | |
| | 3, +1 | 2, +1 | 1, +1 | |
| +3/2 −1/2 | 1/5 | 1/2 | 3/10 | |
| +1/2 +1/2 | 3/5 | 0 | −2/5 | |
| −1/2 +3/2 | 1/5 | −1/2 | 3/10 | |
| | 3, 0 | 2, 0 | 1, 0 | 0, 0 |
| +3/2 −3/2 | 1/20 | 1/4 | 9/20 | 1/4 |
| +1/2 −1/2 | 9/20 | 1/4 | −1/20 | −1/4 |
| −1/2 +1/2 | 9/20 | −1/4 | −1/20 | 1/4 |
| −3/2 +3/2 | 1/20 | −1/4 | 9/20 | −1/4 |
| | 3, −1 | 2, −1 | 1, −1 | |
| +1/2 −3/2 | 1/5 | 1/2 | 3/10 | |
| −1/2 −1/2 | 3/5 | 0 | −2/5 | |
| −3/2 +1/2 | 1/5 | −1/2 | 3/10 | |
| | 3, −2 | 2, −2 | | |
| −1/2 −3/2 | 1/2 | 1/2 | | |
| −3/2 −1/2 | 1/2 | −1/2 | | |
| | 3, −3 | | | |
| −3/2 −3/2 | 1 | | | |

### 2×3/2

| $m_1$ $m_2$ | | | | |
|---|---|---|---|---|
| | 7/2, +7/2 | | | |
| +2 +3/2 | 1 | | | |
| | 7/2, +5/2 | 5/2, +5/2 | | |
| +2 +1/2 | 3/7 | 4/7 | | |
| +1 +3/2 | 4/7 | −3/7 | | |
| | 7/2, +3/2 | 5/2, +3/2 | 3/2, +3/2 | |
| +2 −1/2 | 1/7 | 16/35 | 2/5 | |
| +1 +1/2 | 4/7 | 1/35 | −2/5 | |
| 0 +3/2 | 2/7 | −18/35 | 1/5 | |
| | 7/2, +1/2 | 5/2, +1/2 | 3/2, +1/2 | 1/2, +1/2 |
| +2 −3/2 | 1/35 | 6/35 | 2/5 | 2/5 |
| +1 −1/2 | 12/35 | 5/14 | 0 | −3/10 |
| 0 +1/2 | 18/35 | −3/35 | −1/5 | 1/5 |
| −1 +3/2 | 4/35 | −27/70 | 2/5 | −1/10 |
| | 7/2, −1/2 | 5/2, −1/2 | 3/2, −1/2 | 1/2, −1/2 |
| +1 −3/2 | 4/35 | 27/70 | 2/5 | 1/10 |
| 0 −1/2 | 18/35 | 3/35 | −1/5 | −1/5 |
| −1 +1/2 | 12/35 | −5/14 | 0 | 3/10 |
| −2 +3/2 | 1/35 | −6/35 | 2/5 | −2/5 |
| | 7/2, −3/2 | 5/2, −3/2 | 3/2, −3/2 | |
| 0 −3/2 | 2/7 | 18/35 | 1/5 | |
| −1 −1/2 | 4/7 | −1/35 | −2/5 | |
| −2 +1/2 | 1/7 | −16/35 | 2/5 | |
| | 7/2, −5/2 | 5/2, −5/2 | | |
| −1 −3/2 | 4/7 | 3/7 | | |
| −2 −1/2 | 3/7 | −4/7 | | |
| | 7/2, −7/2 | | | |
| −2 −3/2 | 1 | | | |

### 2×2

| $m_1$ $m_2$ | | | | | |
|---|---|---|---|---|---|
| | 4, +4 | | | | |
| +2 +2 | 1 | | | | |
| | 4, +3 | 3, +3 | | | |
| +2 +1 | 1/2 | 1/2 | | | |
| +1 +2 | 1/2 | −1/2 | | | |
| | 4, +2 | 3, +2 | 2, +2 | | |
| +2 0 | 3/14 | 1/2 | 2/7 | | |
| +1 +1 | 4/7 | 0 | −3/7 | | |
| 0 +2 | 3/14 | −1/2 | 2/7 | | |
| | 4, +1 | 3, +1 | 2, +1 | 1, +1 | |
| +2 −1 | 1/14 | 3/10 | 3/7 | 1/5 | |
| +1 0 | 3/7 | 1/5 | −1/14 | −3/10 | |
| 0 +1 | 3/7 | −1/5 | −1/14 | 3/10 | |
| −1 +2 | 1/14 | −3/10 | 3/7 | −1/5 | |
| | 4, 0 | 3, 0 | 2, 0 | 1, 0 | 0, 0 |
| +2 −2 | 1/70 | 1/10 | 2/7 | 2/5 | 1/5 |
| +1 −1 | 8/35 | 2/5 | 1/14 | −1/10 | −1/5 |
| 0 0 | 18/35 | 0 | −2/7 | 0 | 1/5 |
| −1 +1 | 8/35 | −2/5 | 1/14 | 1/10 | −1/5 |
| −2 +2 | 1/70 | −1/10 | 2/7 | −2/5 | 1/5 |
| | 4, −1 | 3, −1 | 2, −1 | 1, −1 | |
| +1 −2 | 1/14 | 3/10 | 3/7 | 1/5 | |
| 0 −1 | 3/7 | 1/5 | −1/14 | −3/10 | |
| −1 0 | 3/7 | −1/5 | −1/14 | 3/10 | |
| −2 +1 | 1/14 | −3/10 | 3/7 | −1/5 | |
| | 4, −2 | 3, −2 | 2, −2 | | |
| 0 −2 | 3/14 | 1/2 | 2/7 | | |
| −1 −1 | 4/7 | 0 | −3/7 | | |
| −2 0 | 3/14 | −1/2 | 2/7 | | |
| | 4, −3 | 3, −3 | | | |
| −1 −2 | 1/2 | 1/2 | | | |
| −2 −1 | 1/2 | −1/2 | | | |
| | 4, −4 | | | | |
| −2 −2 | 1 | | | | |

## Atomic Structure

Bohr magneton: $\mu_B=\frac{e\hbar}{2m_e}$

gyromag. ratio $\gamma$: $\vec{\mu}_J=\gamma\vec{J}$, $\gamma_{\text{classical}}=\frac{e}{2m}$

g factor: $\vec{\mu}_L=\frac{e}{2m}\vec{L}$, $\vec{\mu}_S=g\frac{e}{2m}\vec{S}$, $g_{\text{spin-1/2 point particle}}=2$

Hund rules: 1. Max $S$ 2. Max $L$ 3. Min $J$ for $\leq$ 1/2-filled shells

$l=$ 0 1 2 3 4… ; *s p d f g*…

term symbol: ${}^{2S+1}L_J$