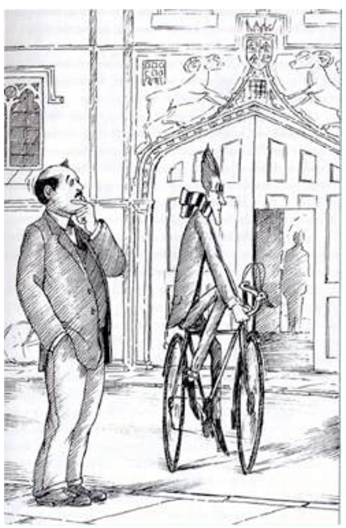
Principle of Relativity: The laws of physics are the same in all inertial frames.

The Speed of Light c in vacuum is the same for all inertial observers, independent of the motion of the source.

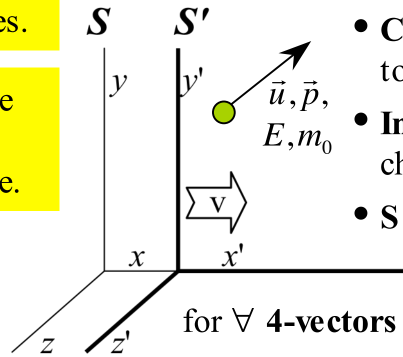
$$\beta \equiv \frac{v}{c} \qquad \gamma \equiv \frac{1}{\sqrt{1-\beta^2}}$$

- **Time Dilation**: moving clocks tick slower by factor γ
- **Length Contraction**: moving objects are shorter by factor γ along direction of motion
- **Loss of Simultaneity**
- **Lattice of Rods & Clocks**: synchronize clocks on grid of rigid rulers → how to think about time @ distant location



Week 7 = Final

Lorentz Boosts and 4-Vectors



- **Convention**: S' speed rel. to S is $v=\beta c$ in $+x$ direction.
- **Inverse**: swap S ↔ S′ by changing the sign of β.
- **Synch origins** to drop Δ's

for ∀ **4-vectors** a^μ, b^μ: • $a'^\mu = \Lambda\ a^\mu$

- $a^\mu \cdot b^\mu \equiv a^0b^0 - a^1b^1 - a^2b^2 - a^3b^3$ is frame invariant

$$\Lambda \equiv \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\Delta x^\mu \equiv (c\Delta t, \Delta x, \Delta y, \Delta z)$$

$$\eta^\mu \equiv \frac{dx^\mu}{d\tau} = \gamma_u\left(c, u_x, u_y, u_z\right)$$

$$p^\mu \equiv m_0\eta^\mu = \left(\frac{E}{c}, p_x, p_y, p_z\right)$$

where $d\tau = \dfrac{dt}{\gamma_u} \qquad \gamma_u = \dfrac{1}{\sqrt{1-u^2/c^2}}$

- Boost **velocity**:

$$u_x = \frac{u'_x + v}{1 + u'_x v/c^2}$$

$$u_{y,z} = \frac{u'_{y,z}}{\gamma(1 + u'_x v/c^2)}$$

- Boost **frequency** of light ray (parallel case) $\qquad \dfrac{f'}{f} = \sqrt{\dfrac{1-\beta}{1+\beta}}$

Invariant Interval

$$I = (c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2$$

is invariant under boosts

Timelike $I_{A\text{-}B} > 0$
- object can travel from A-B
- boost can change Δx_{AB} sign

Spacelike $I_{A\text{-}B} < 0$
- cannot travel from A-B
- boost can change Δt_{AB} sign

The **proper time** interval $\Delta\tau_{AB} \equiv \sqrt{I_{AB}}/c$ is the "**watch-time**" that elapses on the wristwatch of an inertial observer who travels from A to B.

Dynamics

$$\vec{F} = \frac{d\vec{p}}{dt} \qquad W = \int \vec{F} \bullet d\vec{l} = \Delta E$$

$$m_{\text{inert}} = \gamma m_0 \qquad \vec{p} = m_{\text{inert}}\vec{v} \qquad E = m_{\text{inert}}c^2$$

$$E = \sqrt{(pc)^2 + (m_0c^2)^2}$$

$$\vec{p} = \gamma m_0 \vec{v} \qquad E = \gamma m_0 c^2 \qquad \beta = \frac{pc}{E}$$

$KE \equiv E - m_0c^2$ **E,p conserved**

- **(Rest) mass not conserved**, can be converted ↔ energy
- **Photon**: $m_0 = 0$, $E = hf$, equ's with γ factors useless

Causality: Causal relationships exist

Nothing – even information – can travel faster than c.

Minkowski Diagrams

- **worldline**: path of an object in (ct,x) diagram
- **boost hyperbola**: locus of coordinates $(t,x)_B$ in all possible inertial frames relative to $(t,x)_A$ at (0,0); defined by $I = (c\Delta t)^2 - (\Delta x)^2$
- **tilted axes**: ct' and x' axes plotted in S-frame are tilted and stretched rel to ct, x axes

1 Basic SR Effects from LT

1. Time dilation of moving clock:
   $\Delta x' = 0 \rightarrow \Delta t = \gamma \Delta t'$

2. Length meas of moving object:
   $\Delta t = 0 \rightarrow \Delta x = \Delta x' / \gamma$

3. Simultaneous events in S′:
   $\Delta t' = 0 \rightarrow \Delta t = \gamma\beta\Delta x'$

2 Derivation of LT

1. LT must be **linear**, as straight lines (constant v) map onto straight lines to preserve relativity

2. **Inverse** S↔S′ equivalent to $t \leftrightarrow -t$
   ∴ (1) $x'=ax+bt$ and (2) $x=ax'-bt'$

3. **Relative speed** of S,S′ is v
   ∴ $x'=0$ maps onto $x=vt$
   ∴ (1) $x'=0=a(vt)+bt \rightarrow b=-av$

4. **Light ray** $x=ct$ maps onto $x'=ct'$:
   (1) $x'=ct'=a(x-vt)=a(c-v)t$ and
   (2) $x=ct=a(x'+vt')=a(c+v)t'$
   ∴ t'/t = <algebra> $\rightarrow a = \gamma$

3 Argument for Speed Limit *c*

- Hypothesis: X travels FTL from A to B
  ∴ Emission at A *causes* detec^n at B

- FTL → $I_{A\text{-}B}$ is spacelike (negative)
  ∴ Can change frames so that $t_B < t_A$
  ∴ A *cannot* have caused B

Week 7 = Final

4 Derive velocity addit^n

Boost space-time interval
$\Delta x'^{\mu}=(cT', u_x'T', u_y'T', u_z'T')$
betw two points on trajectory of particle moving with speed u'
→ get Δx^{μ} and so (u_x,u_y,u_z)

Alternate: boost η^{μ}; get rid of unknown γ_u in result using η'^0

5 Derive Doppler shift

Calculate intersection of two wave crests with path of moving observer S′; boost to S′ frame to get $\Delta t'=1/f'$

Easier: boost p^{μ} of photon, use $E=hf$
→ get general case $f'=f\gamma(1-\beta\cos\theta)$

6 Derive relativ. mech.

Motivation:
- Incorporate $v \le c$ speed limit
- Incorporate photon, with $E=pc$

Photon-in-a-box thought expt: preserve principle of inertia by assigning photon mass $m=E/c^2$

Hypotheses for normal particles:
- inertial mass in $p=mv$ grows w v
- total energy $E=mc^2$ as for photon
- keep $F=dp/dt$ and $W=\int F\cdot dl$

→ derive new energy-momentum relation $E^2 = (pc)^2 + (m_0c^2)^2$ where m_0 is **rest mass** of particle

→ find $m_0=0$ for photon and inertial mass $m=\gamma m_0$ for massive particles