

Note: A square-root sign is to be understood over every coefficient, e.g., for $-8/15$ read $-\sqrt{8/15}$.

Notation:

J	J	...
M	M	...
m_1	m_2	Coefficients
m_1	m_2	
\vdots	\vdots	
\vdots	\vdots	

$$1/2 \times 1/2 \begin{array}{|c|c|c|} \hline 1 & & \\ \hline +1/2+1/2 & 1 & 0 & 0 \\ \hline +1/2 & -1/2 & 1/2 & 1/2 & 1 \\ \hline -1/2 & +1/2 & 1/2 & -1/2 & -1 \\ \hline & & -1/2 & -1/2 & 1 \\ \hline \end{array}$$

$$Y_1^0 = \sqrt{\frac{3}{4\pi}} \cos \theta$$

$$2 \times 1/2 \begin{array}{|c|c|c|} \hline 5/2 & & \\ \hline +5/2 & 1 & 5/2 & 3/2 \\ \hline +2 & +1/2 & 1 & +3/2 & +3/2 \\ \hline +2 & -1/2 & 1/5 & 4/5 & 5/2 & 3/2 \\ \hline +1 & +1/2 & 4/5 & -1/5 & +1/2 & +1/2 \\ \hline \end{array}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}} \left(\frac{3}{2} \cos^2 \theta - \frac{1}{2} \right)$$

$$\begin{array}{|c|c|c|} \hline +1 & -1/2 & 2/5 & 3/5 & 5/2 & 3/2 \\ \hline 0 & +1/2 & 3/5 & -2/5 & -1/2 & -1/2 \\ \hline \end{array}$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}} \sin \theta \cos \theta e^{i\phi}$$

$$\begin{array}{|c|c|c|} \hline 0 & -1/2 & 3/5 & 2/5 & 5/2 & 3/2 \\ \hline -1 & +1/2 & -3/5 & -3/5 & -3/2 & -3/2 \\ \hline \end{array}$$

$$Y_2^2 = \frac{1}{4} \sqrt{\frac{15}{2\pi}} \sin^2 \theta e^{2i\phi}$$

$$3/2 \times 1/2 \begin{array}{|c|c|c|} \hline 2 & & \\ \hline +3/2 & +1/2 & 1 & +1 & +1 \\ \hline +3/2 & -1/2 & 1/4 & 3/4 & 2 & 1 \\ \hline +1/2 & +1/2 & 3/4 & -1/4 & 0 & 0 \\ \hline \end{array}$$

$$1 \times 1/2 \begin{array}{|c|c|c|} \hline 3/2 & & \\ \hline +3/2 & 3/2 & 1/2 \\ \hline +1 & +1/2 & 1 & +1/2 & +1/2 \\ \hline +1 & -1/2 & 1/3 & 2/3 & 3/2 & 1/2 \\ \hline 0 & +1/2 & 2/3 & -1/3 & -1/2 & -1/2 \\ \hline \end{array}$$

$$2 \times 1 \begin{array}{|c|c|c|} \hline 3 & & \\ \hline +3 & 3 & 2 \\ \hline +2 & +1 & 1 & +2 & +2 \\ \hline +2 & 0 & 1/3 & 2/3 & 3 & 2 & 1 \\ \hline +1 & +1 & 2/3 & -1/3 & +1 & +1 & +1 \\ \hline \end{array}$$

$$3/2 \times 1 \begin{array}{|c|c|c|} \hline 5/2 & & \\ \hline +5/2 & 5/2 & 3/2 \\ \hline +3/2 & +1 & 1 & +3/2 & +3/2 \\ \hline +3/2 & 0 & 2/5 & 3/5 & 5/2 & 3/2 & 1/2 \\ \hline +1/2 & +1 & 3/5 & -2/5 & +1/2 & +1/2 & +1/2 \\ \hline \end{array}$$

$$1 \times 1 \begin{array}{|c|c|c|} \hline 2 & & \\ \hline +2 & 2 & 1 \\ \hline +1 & +1 & 1 & +1 & +1 \\ \hline +1 & 0 & 1/2 & 1/2 & 2 & 1 & 0 \\ \hline 0 & +1 & 1/2 & -1/2 & 0 & 0 & 0 \\ \hline \end{array}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

0	-1	1/2	1/2	2
-1	0	1/2	-1/2	-2
-1	-1	1	1	2

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M \rangle = (-1)^{J-j_1-j_2} \langle j_2 j_1 m_2 m_1 | j_2 j_1 J M \rangle$$

3/2 x 3/2

$$\begin{array}{|c|c|c|} \hline 3 & & \\ \hline +3 & 3 & 2 \\ \hline +3/2 & +3/2 & 1 & +2 & +2 \\ \hline +3/2 & +1/2 & 1/2 & 1/2 & 3 & 2 & 1 \\ \hline +1/2 & +3/2 & 1/2 & -1/2 & +1 & +1 & +1 \\ \hline +3/2 & -1/2 & 1/5 & 1/2 & 3/10 & 3 & 2 & 1 & 0 \\ \hline +1/2 & +1/2 & 3/5 & 0 & -2/5 & 0 & 0 & 0 & 0 \\ \hline -1/2 & +3/2 & 1/5 & -1/2 & 3/10 & 0 & 0 & 0 & 0 \\ \hline \end{array}$$

2 x 3/2

$$\begin{array}{|c|c|c|} \hline 7/2 & & \\ \hline +7/2 & 7/2 & 5/2 \\ \hline +2 & +3/2 & 1 & +5/2 & +5/2 \\ \hline +2 & +1/2 & 3/7 & 4/7 & 7/2 & 5/2 & 3/2 \\ \hline +1 & +3/2 & 4/7 & -3/7 & +3/2 & +3/2 & +3/2 \\ \hline +2 & -1/2 & 1/7 & 16/35 & 2/5 & 7/2 & 5/2 & 3/2 & 1/2 \\ \hline +1 & +1/2 & 4/7 & 1/35 & -2/5 & +1/2 & +1/2 & +1/2 & +1/2 \\ \hline 0 & +3/2 & 2/7 & -18/35 & 1/5 & +3/2 & -3/2 & 1/20 & 1/4 & 9/20 & 1/4 \\ \hline +1/2 & -1/2 & 9/20 & 1/4 & -1/20 & -1/4 & -1/2 & +1/2 & 9/20 & 1/4 & -1/20 & -1/4 \\ \hline -3/2 & +3/2 & 1/20 & -1/4 & 9/20 & -1/4 & 3 & 2 & 1 & 0 \\ \hline & & -1 & -1 & -1 & 1/5 & 1/2 & 3/10 & 1/5 & 1/2 & 3/10 \\ \hline & & & & & -1/2 & -1/2 & 3/5 & 0 & -2/5 & 3 & 2 \\ \hline & & & & & -3/2 & +1/2 & 1/5 & -1/2 & 3/10 & -2 & -2 \\ \hline \end{array}$$

2 x 2

$$\begin{array}{|c|c|c|} \hline 4 & & \\ \hline +4 & 4 & 3 \\ \hline +2 & +2 & 1 & +3 & +3 \\ \hline +2 & +1 & 1/2 & 1/2 & 4 & 3 & 2 \\ \hline +1 & +2 & 1/2 & -1/2 & +2 & +2 & +2 \\ \hline +2 & 0 & 3/14 & 1/2 & 2/7 & 4 & 3 & 2 & 1 \\ \hline +1 & +1 & 4/7 & 0 & -3/7 & +1 & +1 & +1 & +1 \\ \hline 0 & +2 & 3/14 & -1/2 & 2/7 & +1 & +1 & +1 & +1 \\ \hline +2 & -1 & 1/14 & 3/10 & 3/7 & 1/5 & +1 & -3/2 & 4/35 & 27/70 & 2/5 & 1/10 \\ \hline +1 & 0 & 3/7 & 1/5 & -1/14 & -3/10 & 0 & -1/2 & 18/35 & 3/35 & -1/5 & -1/5 \\ \hline 0 & +1 & 3/7 & -1/5 & -1/14 & 3/10 & -1 & +1/2 & 12/35 & 5/14 & 0 & -3/10 \\ \hline -1 & +2 & 0 & +1/2 & 18/35 & -3/35 & -1/5 & 1/5 & 0 & +1/2 & 7/2 & 5/2 & 3/2 & 1/2 \\ \hline -1 & +3/2 & -1 & +3/2 & 4/35 & -27/70 & 2/5 & -1/10 & -1/2 & -1/2 & -1/2 & -1/2 \\ \hline +1 & -3/2 & 4/35 & 27/70 & 2/5 & 1/10 & -1 & -3/2 & 2/7 & 18/35 & 1/5 \\ \hline 0 & -1/2 & 18/35 & 3/35 & -1/5 & -1/5 & -1 & -1/2 & 4/7 & -1/35 & -2/5 \\ \hline -1 & +1/2 & 12/35 & -5/14 & 0 & 3/10 & -2 & +3/2 & 7/2 & 5/2 & 3/2 \\ \hline -2 & +3/2 & 1/35 & -6/35 & 2/5 & -2/5 & -3/2 & -3/2 & -3/2 \\ \hline 0 & -3/2 & 2/7 & 18/35 & 1/5 & -1 & -1/2 & 4/7 & -1/35 & -2/5 & 7/2 & 5/2 \\ \hline -2 & +1/2 & -2 & +1/2 & 1/7 & -16/35 & 2/5 & -5/2 & -5/2 & -1 & -3/2 & 4/7 & 3/7 & 7/2 \\ \hline & & & & & & & & & & -2 & -1/2 & 3/7 & -4/7 & -7/2 \\ \hline +2 & -2 & 1/70 & 1/10 & 2/7 & 2/5 & 1/5 & -1 & -3/2 & 4/7 & 3/7 & 7/2 \\ \hline +1 & -1 & 8/35 & 2/5 & 1/14 & -1/10 & -1/5 & 0 & 0 & 18/35 & 0 & -2/7 & 0 & 1/5 \\ \hline 0 & 0 & 18/35 & 0 & -2/7 & 0 & 1/5 & -1 & +1 & 8/35 & -2/5 & 1/14 & 1/10 & -1/5 \\ \hline -1 & +1 & 8/35 & -2/5 & 1/14 & 1/10 & -1/5 & 4 & 3 & 2 & 1 \\ \hline -2 & +2 & 1/70 & -1/10 & 2/7 & -2/5 & 1/5 & -1 & -1 & -1 & -1 \\ \hline +1 & -2 & 1/14 & 3/10 & 3/7 & 1/5 & 0 & -1 & 3/7 & 1/5 & -1/14 & -3/10 \\ \hline 0 & -1 & 3/7 & -1/5 & -1/14 & -3/10 & -1 & 0 & 3/7 & -1/5 & -1/14 & 3/10 \\ \hline -2 & +1 & 1/14 & -3/10 & 3/7 & -1/5 & 4 & 3 & 2 & -2 & -2 & -2 \\ \hline 0 & -2 & 3/14 & 1/2 & 2/7 & 0 & -1 & -1 & 4/7 & 0 & -3/7 & 4 & 3 \\ \hline -1 & -1 & 4/7 & 0 & -3/7 & -3 & -3 \\ \hline -2 & 0 & 3/14 & -1/2 & 2/7 & -1 & -2 & -1 & -2 & -2 & -2 \\ \hline -1 & -2 & 1/2 & 1/2 & 4 \\ \hline -2 & -1 & 1/2 & -1/2 & -4 \\ \hline -2 & -2 & 1 & 1 \\ \hline \end{array}$$