Midterm Exam 1 Formula Sheet

Feb 29, 2024

Reference formulae

Time-dependent Schrödinger equation: $i\hbar\frac{\partial}{\partial t}\psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t) + V(x,t)\psi(x,t)$

Normalization: $\int_{-\infty}^{\infty} dx\, |\psi(x,t)|^2 = 1$

Operators: momentum $p \leftrightarrow -i\hbar\frac{\partial}{\partial x}$; position $x \leftrightarrow x$; Hamiltonian $H \leftrightarrow p^2/2m + V(x,t)$

Expectation values: $\langle \mathcal{O} \rangle = \int_{-\infty}^{\infty} dx\, \psi^*(x,t)\mathcal{O}\psi(x,t)$

standard deviation σ: $\sigma_{\mathcal{O}}^2 = \langle \mathcal{O}^2 \rangle - \langle \mathcal{O} \rangle^2$

Time-independent Schrödinger equation: $H\psi_n(x) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi_n(x) + V(x)\psi_n(x) = E_n\psi_n(x)$

$\psi(x,t) = \sum_n c_n \psi_n(x) e^{-iE_n t/\hbar}$

Orthonormality: $\int_{-\infty}^{\infty} dx\, \psi_m^*(x)\psi_n(x) = \delta_{mn}$

Infinite square well, $V(x) = 0$ for $0 < x < L$, $V(x) = \infty$ elsewhere:

$\psi_n(x) = \sqrt{\frac{2}{L}}\sin(n\pi x/L)$, $E_n = \frac{\hbar^2}{2m}\left(\frac{\pi n}{L}\right)^2$

Free particle, $V = 0$. Momentum eigenstates $\psi_k(x) = \frac{1}{\sqrt{2\pi}}e^{ikx}$

Continuum orthonormality, $\int_{-\infty}^{\infty} dx\, \psi_{k_1}^*(x)\psi_{k_2}(x) = \delta(k_1 - k_2)$

Integrals: Gaussian, $\int_{-\infty}^{\infty} dx\, e^{-(\alpha x^2 + \beta x)} = \sqrt{\pi/\alpha}\, e^{\beta^2/4\alpha}$ for $\mathrm{Re}\,\alpha > 0$

$\int_{-\infty}^{\infty} dx\, x^2 e^{-\alpha x^2} = \sqrt{\pi/4\alpha^3}$

Delta function, $\delta(x) = 0$ for $x \neq 0$, ∞ for $x = 0$, $\int_{-\infty}^{\infty} dx\, \delta(x-a)f(x) = f(a)$

Fourier transform: $f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dk \tilde{f}(k)e^{ikx}$, $\tilde{f}(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dx f(x)e^{-ikx}$