RELATIVITY

Transformation rules from static frame S to frame S′ moving with speed βc

	INDEX NOTATION	MATRIX NOT$^{\underline{n}}$
scalars:	$\phi' = \phi$	$\phi' = \phi$
4-vectors:	$A^{\mu\prime} = \Lambda^{\mu}{}_{\nu} A^{\nu}$	$A' = \Lambda A$
4-tensors:	$F^{\mu\nu\prime} = \Lambda^{\mu}{}_{\alpha} \Lambda^{\nu}{}_{\beta} F^{\alpha\beta}$	$F' = \Lambda F \Lambda^{T}$

where $\Lambda = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

when $\vec{\beta}$ along $\hat{x}$ direction

... and $\gamma \equiv 1/\sqrt{1-\beta^2}$

Covariant vs Contravariant

UPPER index = contravariant $\longrightarrow$ transform with Λ

LOWER index = covariant $\longrightarrow$ transform with $\bar{\Lambda} = (\Lambda^{-1})^{T} = \Lambda$ with sign of β reversed

Changing covariant $\Leftrightarrow$ contrav.: $A_{\mu} = g_{\mu\nu} A^{\nu}$, $A^{\mu} = g^{\mu\nu} A_{\nu}$ where metric tensor $g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$ (i.e. change sign of spatial components 1,2,3)

★ SCALAR PRODUCT $A^{\mu} \cdot B_{\mu}$ is always Lorentz-invariant for any 4-vectors A, B

Physical Quantities in "covariant form"

- proper time $\tau = \frac{t}{\gamma_u}$ (scalar) for object w speed u
- space/time $x^{\mu} = [ct, \vec{x}]$
- derivatives $\partial_{\mu} = [\frac{1}{c}\frac{\partial}{\partial t}, \vec{\nabla}]$
- sources $J^{\mu} = [c\rho, \vec{J}]$
- potentials $A^{\mu} = [\frac{V}{c}, \vec{A}]$
- velocity $\eta^{\mu} = \frac{dx^{\mu}}{d\tau} = \gamma_u [c, \vec{u}]$
- force $K^{\mu} = \frac{dp^{\mu}}{d\tau} = \gamma_u [\frac{\vec{F}\cdot\vec{u}}{c}, \vec{F}]$
- energy / momentum $p^{\mu} = [\frac{E}{c}, \vec{p}]$
- fields (anti-sym. tensors) $F^{\mu\nu} = \begin{pmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{pmatrix}$ $G^{\mu\nu} = \begin{pmatrix} 0 & -B_x & -B_y & -B_z \\ B_x & 0 & E_z/c & -E_y/c \\ B_y & -E_z/c & 0 & E_x/c \\ B_z & E_y/c & -E_x/c & 0 \end{pmatrix}$

Electrodynamic Equations

- continuity $\partial_{\mu} J^{\mu} = \emptyset$
- Maxwell $\partial_{\mu} F^{\mu\nu} = \mu_0 J^{\nu}$ (inhomog) & $\partial_{\mu} G^{\mu\nu} = \emptyset$ (homog.)
- Lorentz gauge $\partial_{\mu} A^{\mu} = \emptyset$
- Maxwell i.t.o. potentials $\Box^2 A^{\mu} = \partial_{\nu}\partial^{\nu} A^{\mu} = -\mu_0 J^{\mu}$
- Lorentz force $K^{\mu} = q F^{\mu\nu} \eta_{\nu}$
- Fields from potentials $F^{\mu\nu} = \partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu}$
- Momentum i.t.o. velocity $p^{\mu} = m \eta^{\mu}$ where $p^{\mu} = [\frac{E}{c}, \vec{p}]$