Physics 419: Lecture 4: Newton's Laws
Feb. 4, 2021

We will establish today the key underlying physical principle behind Kepler's laws. The themes today are :

- The equivalence between Kepler's laws and Newton's inverse square law of gravity
- The relation between measurement and reality.
- The relation between methodology and metaphysics.

1 Setting the Stage

To do so, we need to define two concepts: velocity and acceleration. Velocity is defined as the rate of change of the position and hence has units of length/time. Acceleration is the rate at which the velocity changes. Consequently its units are length/time x time). Velocity and acceleration have both a direction and a magnitude. Quantities which have both magnitude and direction are called vectors. The velocity and acceleration vectors point in the direction of the motion. The acceleration is nonzero anytime the magnitude (speed) or direction of the velocity vector changes. Hence, a ball being whirled around a horizontal circle on a taught string has a non-zero acceleration as a result of the directional change its velocity vector experiences. If it were not for the string, the ball would not move in a circle. Hence, its acceleration vector points toward the center. Likewise planets moving in an orbit around the sun have acceleration vectors that point toward the sun. The center-pointing acceleration is called the centripetal acceleration. It is this acceleration that is the key to understanding planetary motion.

2 Kepler and Newton

Planets about sun, $T^2 \propto R^3$.

But for circular motion, let us cheat and use the formula for the centripetal acceleration: $a = mv^2/r$. Let us assume that the velocity is just the perimeter of the circle divided by the time. Then, $a = m(2\pi R/T)^2/R = m4\pi^2 R/T^2$. Now lets use Kepler's third law and substitute for T^2 that is proportional to R^3. What we find is that $a \propto 1/R^2$. So Kepler's law implies that there is an acceleration pointing inwards that falls off as $1/R^2$. This is Newton's gravitational law.

What do you get if you reason in the other direction?

I.e. assume the $a \propto 1/R^2$ toward the Sun.

Here's what follows (math by Isaac Newton):

- elliptical orbits with the Sun at one focus
- $T^2 \propto R^3$ with R the long axis
- equal areas swept per equal times!

All of Kepler's laws boil down to one statement about accelerations.

Question:

What is special about the Sun that makes things accelerate toward it?

Answer: Nothing.

The Earth and Jupiter have moons which accelerate toward them, and all sorts of stuff falls to Earth.

Are all these accelerations for the same reason? Probably yes, if a similar description holds. So is the acceleration toward the Earth inversely proportional to the square of the distance from the Earth? Yes- comparison of the moon and an apple follows the $1/R^2$ law if R is taken to be the distance to the center of the Earth.

Why should R be taken to the center of the Earth?

Assume that each clod of dirt has this attractive power, causing things to accelerate toward it. You have to add up the little acceleration vectors from all the parts of the Earth- some near, some far. How can you add all those little things?

Invent integral calculus.

The result: the net attraction of anything not actually inside the Earth is exactly the same as it would be if all the Earth's stuff were at the center.

3 Newton's Laws

A modest generalization: Every object in the universe attracts every other object, with the strength of the resulting acceleration is inversely proportional to the square of the distance, and proportional to the "mass" of the attracting object.

This generalization is due to Isaac Newton. He applied the same law seen on Earth to all celestial objects. The successful theory of gravity (see below) is powerful evidence for the validity of this approach. Newton's faith in the approach allowed him to spend much of the period between 1665 and 1687 inventing the Calculus in order to solve a problem. (Can a spherical object be considered to be a point in so far as its gravitational effects are concerned? Yes.)

Objects have properties and relationships other than the usual geometrical ones (position and motion) which can be described mathematically. For example, mass and force.

- Law of inertia. If no forces act, a body's velocity is constant.
- $F = ma$. A body's acceleration is proportional to the force acting on it. (Actually, Newton used a slightly more general statement which did not presume that m is constant. F= time rate of change of momentum, which is the product of m and v.)
- $F_{12} = -F_{21}$. When object 1 exerts a force on object 2, object 2 exerts an opposite force on object 1.

Note the similarity to the Laws of motion written down by Descartes some 30 years earlier:

Law 1. Each thing, in so far as it is simple and undivided, always remains in the same state, as far as it can, and never changes except as a result of external causes... Hence we must conclude that what is in motion always, so far as it can, continues to move. (Principles Part II, art. 37)

Law 2. Every piece of matter, considered in itself, always tends to continue moving, not in any oblique path but only in a straight line. (Principles Part II, art. 39)

Law 3. When a moving body collides with another, if its power of continuing in a straight line is less than the resistance of the other body, it is deflected so that, while the quantity of motion is retained, the direction is altered; but if its power of continuing is greater than the resistance of the other body, it carries that body along with it, and loses a quantity of motion equal to that which it imparts to the other body. (Principles Part II, art. 40)

Newton's laws are identical to these. What do you make of this? Note the power of continuing that Descartes talks about is the concept of momentum that is implied by Newton.

Commentary on the Laws of Newton/Descartes

The origin of the first law is a complete mystery. That is, the law of inertia has no known origin. We know things coast forever if there are no forces acting on them, but we do not know why. The third law is an example of a new concept: momentum is conserved. Newton also stated the law of conservation of mass. Conservation laws turn out to have a deep and subtle connection to the underlying properties of the universe, which we will discuss more later.

You can reformulate Newton's laws as follow:

1.) Total momentum is conserved. (law 3) 2.) When two objects trade momentum, they do so following some rules which we can discover. We call momentum- trading the exertion of forces. (law 2) 3.) There should be some conditions in which "nothing happens"- no momentum is traded. (law 1)

Assuming that we know how to assign masses, measure accelerations, and know all the force laws, in order to make predictions, one must also specify initial conditions, namely the initial positions and velocities. Given this information, one can then calculate the accelerations and predict the future motions. We'll see next time how this is related to the concepts of causality and determinism.

What Newton's Laws do not do:

1.) They do not tell us how to determine masses of objects.

2.) They do not tell us how to calculate forces.

3.) Even if the first two problems can be solved, they do not tell us how to determine positions and velocities, only accelerations once we know the positions.

What then is the meaning of Newton's 3 laws? If they together form a testable proposition, we need some way of assigning values to F, m, and a.

1. We need some way of measuring "a", say by comparison with the average motion of all the observed stars. 2. We need some way to measure m's- say by introducing some test force which we think is known, and measuring accelerations. 3. Now we need to find F- but the only general rule about F is the third law.

So the general laws now make only one prediction: the sum of the momenta of all the parts of a closed system (no external forces) doesn't change. Until we know something about the forces, we don't know if closed systems exist! Here's the sole prediction of Newton's grand laws of motion:

Sum total of Newton's laws: The acceleration of the center-of-mass of everything is zero

That's good, but now let's remember what we were measuring motion with respect to.

Our reference frame was the average motion of everything.

So now we have the sole prediction of Newton's three laws:

More accurate reduction: The acceleration of the center-of-mass of everything is zero relative to the center-of-mass of everything!

Already in Kant and Descartes. So what is new after all the math?

Why do we then consider Newton's laws to be a major achievement?

How can we invest Newton's laws with some meaning, i.e. make them more than a tautology?

Ways to make Newton's general laws of motion a fully testable theory:

1. Specify all the forces. (Not yet done!)

2. Make some implicit assumptions about the forces

It might sound as if our rules for constraining the force laws are hopelessly fuzzy, i.e. that we'll always be able to invent forces to make Newton's laws work.

That can't be quite true, however, because these days we agree that Newton's laws are NOT TRUE, in particular that it is not true that F=ma. (Newton didn't exactly say F=ma, but that's the way his force law usually appears in text-books.)

How would we have concluded if their connection to reality was completely flexible?

Note the philosophical lesson: "Survival of the fittest" is often criticized as a tautological principle. It is- but becomes fleshed-out and meaningful in the same way Newton's laws of motion do, by the added imprecise assumption that "fitness" is comprehensible, predictable, etc. Even the most precise, mathematical laws acquire their meaning through less precise connections with the observable world.

4 Gravity

Newton described one general force law. He made the huge generalization from knowing that the Earth pulls on all sorts of nearby things and on the moon, the Sun pulls on the planets, and Jupiter pulls on its moons, to the following:

Everything pulls on everything else.

The law of gravitational attraction:

$$F = \frac{GMm}{R^2}. \tag{1}$$

By combining this general law with the math (integral calculus, invented for the occasion!) needed to add up the effects of all the little bits of stuff in a ball (the Earth), he concluded that the Earth acts as if its mass were all at the center. That justifies the comparison of moon-apple accelerations.

It is often alleged that a scientific theory should not make claims about phenomena other than those specifically shown to obey it. If that were the grading criterion, what grade would Newton have received on the law of universal gravitation?

A modern scientific theory is expected to have two features:

1.) An economical description of phenomena

2.) The ability to predict new phenomena or explain (in the Galilean sense) previously inexplicable ones.

All three of Kepler's laws can be derived from Newton's gravitational law. (which might count as two laws, since the toward-the-Sun part and the $1/R^2$ part are logically independent.) Kepler had tried and failed to do the same thing (he tried a force that went as $1/r$).

Furthermore Newton's gravity described erasers, apples, etc by the same law that described Jupiter, Venus, etc!

Newton's law of gravity explains the multiple centers of motion. There is nothing special about the Sun, except that it is very massive. Jupiter and its moons interact in the same way as do the Sun and the planets.

It predicts small deviations from Kepler's laws.

It explains the tides. A phenomenon which was previously thought to be irrelevant turns out to be pertinent.

How, in general, do we know which are the relevant phenomena?

Newton's gravity forms a complete theory for astronomy if you add the auxiliary assumption that no other forces are significant for that scale of phenomena. It predicts that Kepler's laws should be about right (assuming only that the Sun has a much bigger mass than the planets), with small deviations due to the mutual attractions of the planets. So far so good.

What happens when predicitions are not supported experimentally?

The motion of Jupiter's moons lag or lead the predictions of Newton's theory by about eight minutes. This is a big error.

The motion of Uranus didn't quite fit the theory.

The motion of Mercury didn't quite fit the theory.

Does this mean that Newton is wrong?

Jupiter's moons: Interpreted by Romer in 1672 as due to a finite speed of light, and the variable distance from Earth to Jupiter. This is a good example of how knowing one law allows us to construct other laws. The law of universal gravitation allows us to understand that something else must be going on with Jupiter. Our interpretation of the experimental observations requires a new concept. Namely, that light is not an instantaneously propagating medium. It has a finite velocity. Once this was realised, it was easy to estimate the speed of light. The current value has not changed significantly from the early estimate. The orbit of Uranus (predicted by Kant in 1755 and found by Herschel in 1845): An ad-hoc patch was proposed- another planet, whose orbit could be determined from the deviations of its observed neighbour. So the discrepancy turned into a prediction. The discovery of Neptune in 1845 was the crowning triumph of Newton's theory.

The orbit of Mercury: An ad-hoc patch was proposed- another planet, whose orbit could be determined from the deviations of its observed neighbour. The other planet wasn't found. What gives? Stay tuned.