Lecture 18: March 25, 2021

PHYSICS 419 - Spring 2021

I. Introduction

In this lecture, we present 1) the Copenhagen interpretation of quantum mechanics (the standard model as it were), 2) measurement in quantum mechanics and 3) the Schrödinger cat paradox.

1 Size in Quantum Mechanics

We saw that marbles do not give rise to an interference pattern but electrons do. The question is why. We can answer this by looking at the difference between a bullet and an electron. A bullet is much more massive than an electron. Its velocity (typically) is also much lower than that of an electron's. So let's first make the velocities equal. Do we now get interference with the bullet? We need to determine the wavelength of a bullet. The wavelength is about 10^{-28} smaller than that of an electron even if both are moving at the same speed. This arises because of the mass difference between a bullet and an electron. As the wavelength of a particle gets smaller and smaller, can we resolve its wave-like characteristics? The answer is no. Wavelike nature goes away as the wavelength gets smaller and smaller. Objects with vanishingly small wavelengths appear continuous. Hence, there is no interference. This is the key. Quantum mechanics becomes important when objects have wavelengths that are on the atomic scale. Should the wavelength be less than atomic length scales, then we need not worry about quantum mechanics. Such objects appear continuous and we can think about them in our usual classical way. Another way of thinking about this is to consider what happens when the spacing between the slits goes off to infinity. There is no overlap of the waves and hence no interference. This tells us that if we were to devise an experiment to see the interference of marbles, we would have to have the slits so close together that it would be impossible to determine which slit the marbles went through. The separation between the slits would have to be on the order of half the wavelength of a bullet. Such length scales are not possible. This further shows you that marbles and electrons only differ quantitatively. In quantum mechanics, matter has wave-like properties entirely because size matters. One should be careful here because this implies that matter always has wavelike properties–its just a matter of being clever enough to detect it.

2 Copenhagen Interpretation

We will start with the most widely espoused interpretation of what QM “means.” The (Copenhagen Interpretation) CI was developed by Bohr, Heisenberg, etc, and never accepted by Einstein, Schrodinger, nor by de Broglie.

Three principles are central to the CI view:

1.) The (macroscopic) measuring apparatus must be described in non-QM terms. When one looks at a volt meter, one always obtains a definite value. There are no non-classical probabilities or uncertainty principles at work.

2.) Microscopic objects (electrons) do not have any properties in the abence of measurements by a macroscopic apparatus, but only a description in terms of a wavefunction. The wavefunction describes all possible mutually compatible states of the system. The uncertainty principle is not a result of the disbturbance of the electron by the apparatus, but is inherent in the fact that the wave function tells us all there is to know about the electron (that is all mutually compatible states). This is the completeness assumption of quantum mechanics.

3.) Complementarity: It is impossible to gain all knowledge of a quantum mechanical object at the same time. For example, we cannot think of a system as being a particle and wave at a given instant of time if we wanted this description to fully account for all observable properties of the system. Likewise, we cannot specify the position and the momentum of a particle infinitely accurately simultaneously. We are chained by the Heisenberg principle to uncertainty.

The dichotomy between QM and classical mechanics is fundamental to the Copenhagen Interpretation. Some other QM interpretations and modifications try to avoid it, and some try to make it a result of a unified theory, not a dualistic assumption.

There are several issues to be studied:

- Is QM (the theory) correct? I.e., does it make correct predictions?
- Is the probabilistic wave function necessary? Are there some "hidden variables" which really determine the outcome?
- Is the CI distinction between macroscopic and microscopic necessary? Does it have to be so qualitative and extreme?
- What other interpretations are there? What do they signify?

What I will do right now is analyze quantum mechanics in terms of determinism, objectivity and probability and parallel it with classical mechanics.

Determinism in classical physics:

Remember that, in principle, with adequate information predictions can be made with certainty. Some difficulties:

1.) Integrable systems are well behaved, but chaotic systems are not. Our predictive power is severely constrained.

2.) Systems are never completely isolated, so there will always be unforeseen disturbances.

3.)Newtonian determinism does not always admit a causal interpretation.

Objectivity: When does a phenomenon objectively exist?

Weak form: Independent observers can verify it (intersubjectivity). Berkeley claimed that was the only objectivity there is.

There are three points:

1.) Objectivity $\neq$ determinism.

2.) Classical physics is compatible with objectivity. That is, objects have positions and velocities even when they aren't observed.

3.) We saw with relativity that the "objective", i.e. invariant, quantities don't have to be the same ones that you might guess. We have to be careful in QM interpretation not to describe it as non-objective just because the properties we would like to have be independent of "observation" turn out not to be. The question is, are there some good set of properties which are independent of observation? And what is observation anyway? Is it any interaction with large-scale stuff, or is it something more subjective?

3 Determinism, objectivity, and probability in QM

Measurement

Quantum mechanics represents the system of interest by something called a state vector. This represents all mutually compatible states of the system. Such a collection is called a linear superposition. Formally, QM describes an ensemble of identically prepared systems. "Identical" means that they are all described by the same wave function. Then, we can use our machinery of probability to describe what will happen. But in practice, we all assume that QM really provides some statement about ANY system, since Heraclitus is still right that the world does not contain ensembles of absolutely identically prepared systems. In the absence of a “measurement”, the state vector evolves in a smooth, deterministic way with time, as described by Schrödinger’s equation.

Measurements are made on individual systems taken from the ensemble. The results of measurements are always one of the eigenvalues (this is a fancy word for mutually compatible outcomes) of the measured property of the system. In fact, that’s just a definition:

A measurement of some property of a micro-system is an interaction with a macro-system, such that the observed outcomes correspond to eigenvalues of the operator representing that property of the microsystem.

This is called the collapse of the wave function. All subsequent measurements will not see any sign of the superposed state but rather the only collapsed state that resulted from the first measurement.

Determinism

Despite QM’s probabilistic nature, there remains a lot of determinism:

The evolution of the wave function is deterministic as long as no “measurement” occurs.

The statistical distributions are determined by the wave function and are therefore deterministic.

There is statistical determinism, similar to what we have in thermodynamics. However, unlike classical physics, standard QM says that the probabilities are not the result of a pseudo-indeterminism arising from the lack of knowledge of the system. There is no more knowledge to be had, no hidden information.

Probability

What is measureable in quantum mechanics is the square of the wave function.

Objectivity

There is none. The wavefunction collapses upon measurement. So what is real here? Perhaps quantities such as mass and electrical charge, because they don’t change when we look. Maybe the probability distribution itself. We'll investigate this later.

Two important questions:

1.) Is the state vector an objective property of the object?

2.) If the state vector was not an eigenstate before the measurement, it becomes one afterwards. Thus, after the measurement, there seems to be some objective existence. The question arises, did it exist before the measurement?

Either answer to 1 gives us problems. If yes, how can it change discontinuously when a measurement is made? This is not only a time discontinuity, but also a spatial one. If we can avoid the collapse of the wave function, then we might avoid this problem. If no, then how does it give rise to physical effects such as interference? We'll deal with question 2 later.

The central problem, which we will keep coming back to, is that the idea of "measurement" as a break in the behavior keeps coming up- but we haven't said why some interactions between things constitute measurements and others don't. If the only problems with QM were that it was not fully deterministic or allowed objective reality, or that it said that our old ideas about waves and particles didn't describe the world, or that quantities like "momentum" do not generally have precise values, my response would be "Live with it." Relativity has already taught us not to put too much stake in our prior prejudices. We'll see, however, that QM asks us to give up much more, so that it is not quite clear what it is saying about the world, even though we know exactly how to use it to make predictions about almost all experiments. We'll also see that just plain experiment forces us to give up nearly as much of our basic worldview as QM theory does.

Here is the classic story of Schrödinger's cat.

Say that the micro-variable is a quantum spin, and the measurement apparatus is set up to kill a cat if the spin is up, and give it some food and water if the spin is down. This is not a science-fiction idea, but a relatively trivial thing to set up in an ordinary lab.

The result of the solution of the linear wave equation is that the cat is both alive and dead, in a superposition.

$$|\psi\rangle = |\text{livecat}\rangle + |\text{deadcat}\rangle \tag{1}$$

This does not mean “in a coma” or ”almost dead” but BOTH fully alive and purring or thoroughly dead and decomposing.

Furthermore, once you look, your wave function becomes entangled with those of the cat, etc. The solution of the linear wave equation now describes a superposition of a you who has seen the dead cat and a you who has seen the live cat!

Which of youse guys is for real?

The linear wave equation by itself does not describe the world of our experience!

What does describe our experience?

The most extreme version of the orthodox Copenhagen view is that the wave function was never real, just an algorithm for predicting experiences, which are the only reality. Note, this is instrumentalism.

The common folk view among physicists is that the wave function somehow “collapses” to one of the possibilities, following probability rules.

Some physicists hold to a sort of hybrid of these rules.

Many physicists will give this resolution, or something similar.

There are two steps to the argument.

In a macroscopic apparatus, it is impossible in practice to observe interference between the macroscopically distinct states. This contrasts with the microscopic situation. As a consequence, it is valid to conclude that a particular outcome has been realized. Remember the two slit experiment. The reason we get in trouble when we try to say that the electron really went through one slit or the other is that it would destroy the interference between the two paths. However, when we are in a situation where there is no interference pattern to begin with, then what prevents us from making that kind of either/or statement?

The first step of the argument is not very controversial. A macroscopic apparatus is very complex and much larger than any relevant wavelengths, so interference vanishes to truly immeasurable levels. This phenomenon is called decoherence. It appears in the classical physics of waves and is not a quantum mystery.

The second step is more problematic. We must face the question, “Exactly when is a particular result realized?” The problem with the orthodox resolution is that it forces us to attach different words (meanings) to the same mathematical quantities in different situations. It confuses truth with evidence.