Physics 419: Lecture 13: Curvature and Spacetime
9 March, 2021

1 Themes

- Gravitational Red Shift
- GR Oddities
- Space and Time

2 Gravitational Red Shift

Consider a falling object. Its speed increases as it is falling. Hence, if we were to associate a frequency with that object the frequency should increase accordingly as it falls to earth. Because of the equivalence between gravitational and inertial mass, we should observe the same effect for light. So lets shine a light beam from the top of a very tall building. If we can measure the frequency shift as the light beam descends the building, we should be able to discern how gravity affects a falling light beam. This was done by Pound and Rebka in 1960 (here is the link to their paper $http://prl.aps.org/pdf/PRL/v4/i7/p337_1$). They shone a light from the top of the Jefferson tower at Harvard and measured the frequency shift. The frequency shift was tiny but in agreement with the theoretical prediction. Consider a light beam that is travelling away from a gravitational field. Its frequency should shift to lower values. This is known as the gravitational red shift of light. This effect should be distinguished from the Doppler red shift of a receding star. We will talk about the latter next class. Any way, the gravitational red shift is a key prediction of the general theory of relativity.

In fact, the gravitational red shift implies that spacetime is curved. Consider two observers at different heights in a graviational field. Lets say the length of the pulse is given by $2\pi N$ where N is the number of cycles of the pulse. Let $\delta t_{\rm bot}$ be the time it takes the sender to send the signal and $\delta t_{\rm top}$ the time it takes the observer at the top to receive the signal. These times are given by $\delta t_{\rm top} = 2\pi N/\omega_{\rm top}$ and $\delta t_{\rm bot} = 2\pi N/\omega_{\rm bot}$. From the gravitational red shift, it must be that $\omega_{\rm bot} > \omega_{\rm top}$. Consequently, $\delta t_{\rm top} > \delta t_{\rm bot}$. Now consider a space-time diagram in which the height in the gravitational field is the y axis and time is the horizontal axis as shown. Light pulses sent from bottom to top must be sent at 45^o angles. Consider two such pulses.

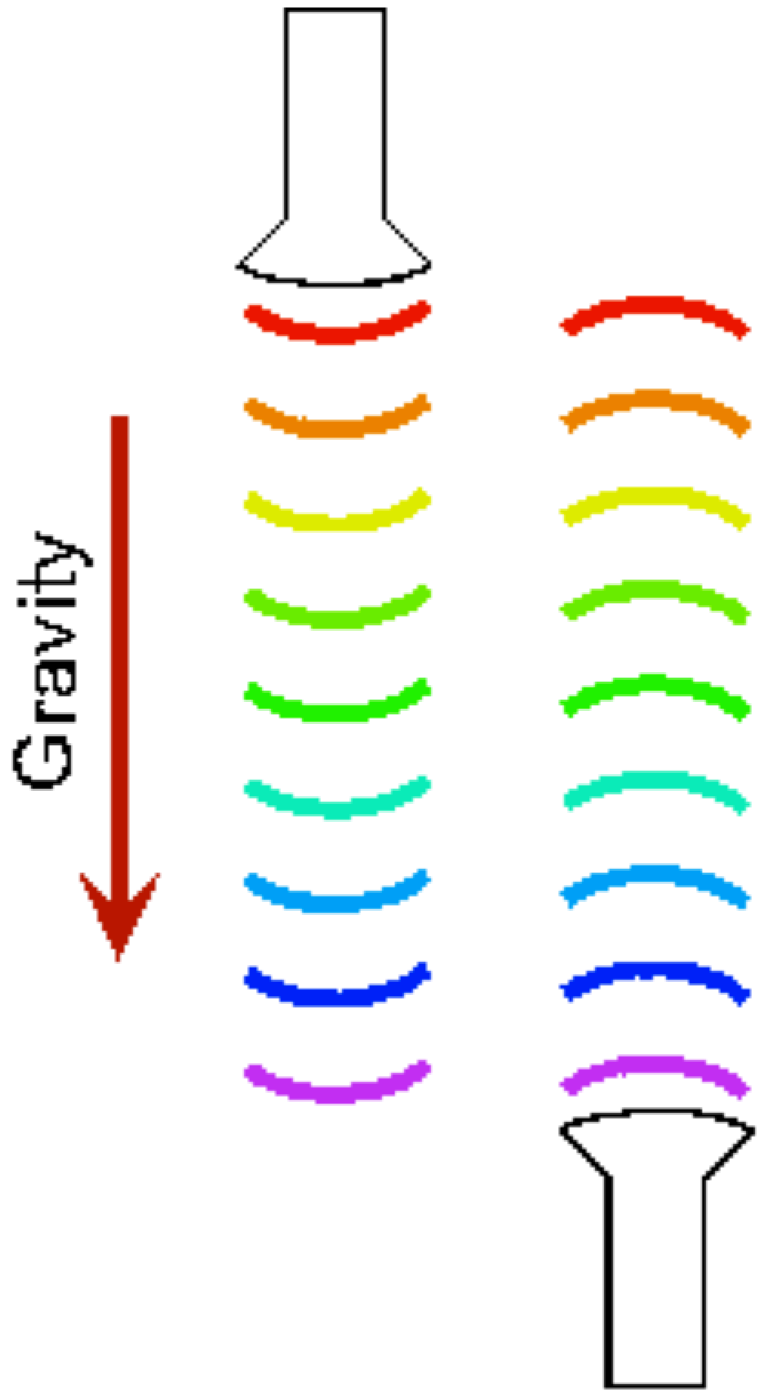

Figure 1: Gravitational red shift of light.

The time delay between the two pulses is $t_{\rm top}$ and $t_{\rm bot}$ as indicated. The two light paths must be parallel to one another. However, $t_{\rm top} > t_{\rm bot}$. This is impossible in flat Euclidean space. Hence, space-time must be

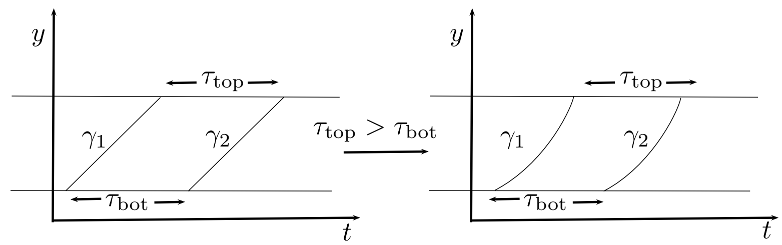

Figure 2: Warping of spacetime. Successive pulses of light, γ_1 and γ_2 are sent and received by observers separated by a vertical distance in a gravitational field. The light pulses must be at 45^o relative to one another. However, the time lapses $\tau_{\rm top}$ and $\tau_{\rm bot}$ are not equal. The only solution is that the space is not flat.

curved to accommodate a parallelogram in which the two legs are of different lengths but the sides are of equal length. It is not that space is curved separately from time. The only sensible conception is that the union of the two is curved: space-time is curved.

Important for devices that rely on high-altitude clocks:

GPS devices positioning would drift off by about 7 miles a day, without GR corrections!

Back to the twins: What if Alice doesn't accelerate back toward Earth using rockets, but instead just slings around using gravity from some big planet. She feels no accelerations. So we can't argue anymore that the one who's younger is the one who "feels no accelerations"- neither feels any. Nonetheless, Alice is younger, because she spent time DOWN in the gravitational field of that big planet, so her clocks were all running slow.

You can see G.R. outgrow some of the arguments which led into it.

Gravitational space-warp:

On the merry-go-round, there was missing space inside the circle. Near a star, there should be the opposite effect. A wave passing by the star has to traverse more distance on the part near the star than on the part farther away. The wavefronts farther away gets ahead, which turns the wave as if it were being pulled toward the star. That's the extra curvature of light observed in 1919.

It looks like the wavefronts are closer together near the star, but that's only because I've forced the picture onto this nearly Euclidean page. Really there's extra space there, and all the fronts are equally spaced, as measured locally.

Note the necessity for an operational definition of distance (and time).

The new equations to describe the gravitational interactions are not the same as Newton's. (Obviously not, since we've now found experimental predictions which differ.) The new equations used to describe how

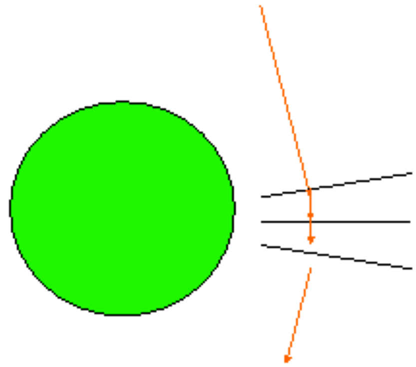

Figure 3: Bending of light rays by the sun.

things look from accelerated frames and ones with gravity (i.e. all the frames which exist in the actual world) are called General Relativity.

GR also solved a long-standing problem. Remember way back when we were describing Newton's triumphs?

The orbit of Mercury had been known since about 1800 to deviate from its Newtonian prediction. (The axis of the ellipse rotated, i.e., precessed.) Many unsuccessful searches had been made for objects, such as other planets, that would explain the motion. GR predicts that the orbit of a planet should follow an ellipse but the closest point on the orbit (perihelion) should precess slowly about the sun. Such an orbit is known as the precession of the perihelion. This is a small effect but nonetheless can be measured. In 1845, the precession of the perihelion of Mercury was observed. This effect is largest for Mercury because it is closest to the sun. Einstein's GR predicts that the perihelion of Mercury should precess about the sun 43.0 seconds of arc per century. Values of the precession for Venus and Earth are 8.63 and 3.8 seconds of arc per century. The predictions from GR agree with these experimental results within 1%. Hence, this is a key triumph of general relativity.

Some other confirmations of GR : see transparency

Modern versions of the light-deflection experiment: repeatedly confirmed, now to better than 1% accuracy.

Gravitational slowing of clocks: repeatedly confirmed, in one case to 0.03% accuracy.

Time-delays are affected by travel near sun: repeatedly confirmed to within 5% accuracy.

Existence of gravitational lenses

Probable existence of black holes

Slowing of pulsar rotation due to gravitational radiation (quantitative)

etc

Modern versions of the light-deflection experiment: repeatedly confirmed, now to better than 1% accuracy.

Gravitational slowing of clocks: repeatedly confirmed, in one case to 0.03% accuracy.

Time-delays are affected by travel near sun: repeatedly confirmed to within 5% accuracy.

Existence of gravitational lenses.

Existence of gravitational waves confirmed in 2016.

Probable existence of black holes

Slowing of pulsar rotation due to gravitational radiation (quantitative)

etc

Do we need curved spacetime?

Just to be sure it is clear, the 2-d examples of curved surfaces allow us to look at things from ”outside.” However, a 2-d person confined to the surface and with no knowledge of the 3rd dimension could still infer the curvature from his own geometrical measurements (e.g., the angles of triangles). That's the situation we 4-d people find ourselves in. The curvature can be completely defined and all experimental predictions, etc made with no reference to some other dimensions.

The immediate question is do we need curved spacetime, or can we get away with ”flat” geometry? (Sklar, 55-67) We have already seen that if we want to put accelerated observers on a par with inertial ones, we need curvature. If we don't do this, then we need to say that the forces accelerated observers feel are pseudoforces akin to the centrifugal force. Are these two views equivalent?

They are equivalent locally (if no singularities), but not globally. Only special configurations of gravity (uniform gravity, for example) can be completely eliminated by going to an accelerated reference frame. In particular, nonuniform gravity gives tidal forces which can't be transformed away. One can maintain flat geometry if one attributes effects such as the slowing of clocks to dynamics (similar to the pre-SR Lorentz contraction, etc.), but that gains one nothing and is vulnerable to the same criticism (it's ad hoc) that Einstein made of the pre-SR theories.

If the universe really has the topology of a sphere, that has global consequences (e.g., one might be able to go around) which can't happen in flat space. (Sklar does not discuss this.) The large scale geometry of the universe is still not known. More on this later. However it is known that the geometry must have at least one singularity- an infinite deviation from the simple flat-space picture, which would be extremely hard to mimic in some fancy flat-space model.

Furthermore, GR makes many detailed predictions which have been beautifully confirmed, and which beat the predictions of a whole slew of rivals- most of which are curved space, anyway. There is no competing theory with the simplicity and predictive power, except theories whose predictions are already known to be wrong.

3 Space, Time and Being

After GR and SR, we are finally able to tackle the problem of what is space and what is time, or so it seems. GR seems to indicate that we should not think of space and time separately but as a unit. Spacetime as a unit has tangible stuff associated with it. It is a fabric which has dents, wrinkles and warps. So Leibniz is wrong. It is always dangerous to use a scientific argument to settle a metaphysical problem. Here's why.

Let's first look at time and being. For spacetime to represent a coherent notion, space and time would have to be on the same footing. Everything about space seems to be captured independent of perspective. We can either specify coordinates or use here or there to specify everything about the location of an object. Nothing seems to be left out. Is the same thing true for time? Sklar points out that one runs into an immediate problem. Here and there do not carry tense. Not true for the corresponding words for time. Sklar gives the example of Caesar's death. Is the pastness of Caesar's death captured in its entirety by saying that Caesar died earlier than now. Relationists say yes. Now and then are just indexicals like here and there. Indexicals are either words or phrases that are context sensitive. This does not mean that they are ambiguous. Its just that the referrent is not fixed. Today, I, you, tomorrow, and there are all indexicals and are certainly not ambiguous. Words such as bank or chair are ambiguous, however as it is uncertain what is being picked out is agreed upon socially. Now just refers to the time an utterance is made just like here and there specify locations of objects. Antirelationists argue that there is more to the utterance "now" because it carries tense with it. Hence, they argue that words such as now are not true indexicals. Now picks out that which exists. Here and there do not function like this. Neither does elsewhere. Only that which exists now exists, regardless of our memory of the past or view of the future. Hence, the antirelationist would counter that the pastness of Caesar's death is not equivalent to the statement that Caesar died earlier than now. The latter statement is timelessly true. However, that Caesar died is true only after he died. So we have equated a timelessly true statement with one that is not timeless. So something must be left over. Equivalently, the phrase elsewhere is perfectly valid; however, elsewhen is not. The antirelationist would argue that elsewhen does not exist because talk of time is inherently linked to tense and now picks out that which exists. As Sklar points out, there is a great parallel here with the Augustinian notion that only that which exists now exists at all.

But it turns out that even the expression only that which is now exists at all is problematic. Let's substitute the meaning "is real" for exists. As you well know, what an observer views as now depends on his velocity. Let's review the example in Sklar. Consider two observers, A and B who are in motion relative to one another. Both observers are at the same location when event e obtains. As a result of their relative motion, an event **a** could be after e for A but simultaneous with e for B. Here's the question: Does it make sense to say that event **a** is unreal to A at e when it is perfectly real to B given that both are coincident at e. The problem is that observer B and A do not share the same notion of 'is real' even though they are coincident. Shouldn't 'is real' mean what is real regardless of the observers as long as they are coincident? Consider a more complicated case. We say that an event **b** is absolutely after **e** regardless of frame all observers agree that it is after **e**. Consider an observer whose life event **e'** is observed to be simultaneous with **b**. Let **e'** be simultaneous with **e** for A. However, b is unreal to A at e. Hence, we have that **e'** is real to B, **b** is real to B, **e'** is real to A but transitivity seems to break down. One could conclude that all this means is that 'is real' is not transitive. Hence, not only is "is simultaneous" relativised but so is "is real" as well. In relativity, all past, present, future are all equally real, just not at the same place and time.

A ploy is to deny the existence of the elsewhen and the elsewhere. This is a radical solution and not one that I am sure we should take seriously.

Additionally, the debate between the substantivilist and the relationist rages. The substantivilist claims that spacetime solves the problem of which frames experience non-inertial effects. It is those frames that are accelerated relative to spacetime. Inertial frames are those that do not accelerate relative to spacetime. Hence, the very structure of spacetime isn't directly accessible only its causal effects. The relationsist says this is not acceptable and posits that those frames that are inertial are brute facts of nature and the substantivilist is engaged in self deception. Inertial frames are a brute fact of nature that never get explained.