Physics 419
Lecture 11: Causality within special relativity
March 2, 2021

1 Themes

- $E = mc^2$
- Minkowski Space
- Causality

2 Mass and energy

There is no simple way to derive $E = mc^2$. I attach with this lecture a note which gets at the heart of the problem. According to Newton, mass is an invariant. With Einstein, we find that this is not so. Mass is energy and hence can just be thought of as another form of energy. There is a new invariant:

$$E^2 = m_0^2 c^4 + p^2 c^2. \tag{1}$$

Photons have no rest mass (that is no inertial mass). Such objects in classical mechanics would have no kinetic energy. However, this is not so. We find that $E = pc$. What is p? p is the momentum. We need quantum mechanics to interpret this correctly. Quantum mechanics does allow massless particles to propagate but they have to do so at the speed of light. More on this when we get to QM.

What does "mass" mean?

For an object at rest, $E = m_0 c^2$. Newton's concept of mass as "quantity of matter" is gone. It has been replaced by a Lorentz invariant relationship between energy and momentum.

The reason why N's concept works so well is, as usual, that unless we are dealing with objects moving at or near the speed of light, the effects are tiny. For example, in a macroscopic object at room temperature, the extra mass due to the thermal energy (in SR, all energy contributes to mass) is about 10^{-10} of the mass of the atoms. That's why Newton was able to assume conservation of mass. In the Newtonian limit, mass does tell us "the quantity of matter" (i.e., the total number of protons and neutrons), but this is only an approximation.

3 Minkowski space: 4-dimensional spacetime

In 1908, Minkowski realized that there is a beautiful geometrical interpretation of SR. It is based on the mathematical similarity between rotations and Lorentz transformations. However, you need not stick to this presentation. By far the simplest presentation is that of Rohrlich's on pages 76-86. In fact, this is what I will present in class.

World lines In barest terms, the history of an object consists of a graph of position versus time, called a world line:

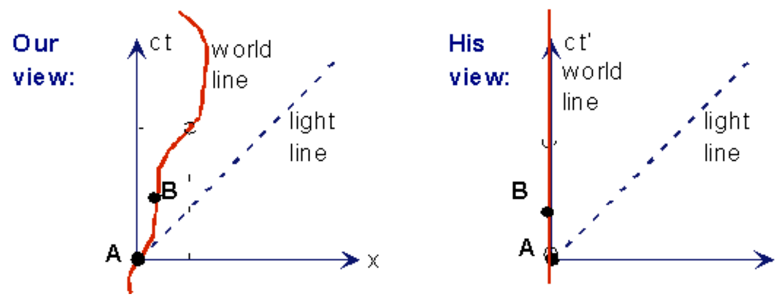

Figure 1: World lines in Minkowski space.

If an object is at rest in any inertial reference frame, its speed is less than c in every reference frame.

Cuasality in SR

The speed limit divides the spacetime diagram into causally distinct regions.

Consider events A, B, C, and D. A might be a cause of B, because effects produced by A can propagate to B. They cannot get to D without travelling faster than light, nor to C because it occurs before A. Because of special relativity, we say that two causation must occur forward within a light cone. That is, there is no such thing as simultaneous cause and effect.

Similarly, C might be a cause of A, B, and/or D. D could be a cause of B, because light can get from D to B (they lie on a 45 line).

We can determine possible causal connections by evaluating the interval, $(ct)^2 - x^2$, separating pairs of events. If the interval is positive, then a causal connection is possible. This is known as a "timelike" interval. If it is negative ("spacelike") then no causal connection is possible.

Strong form: No event can be affected in any way by events outside its past light cone. This applies to conversation laws as well.

Weak form: No information may be transmitted except forward within a light cone.

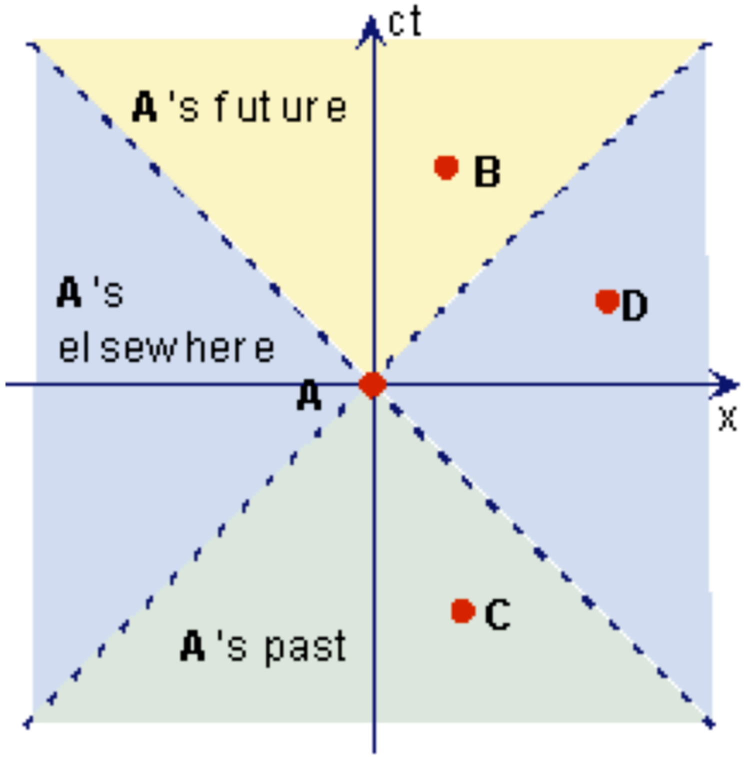

Figure 2: Causally connected regions in Minkowski spacetime.

Weaker form: No information can be transmitted except within a light cone.

You may wonder why we make such pointless distinctions. Can't any "effect" be used to transmit information? Stay tuned.

In a deterministic world, The Strong form would mean that an event would be completely predictable on the basis of knowledge of its past cone alone. Observations OUTSIDE the past light cone might provide the same info in more convenient form, but would never be needed, because everything knowable about the event would be determined by the preceding events in the light cone.

What about in a world where things are not completely predictable on the basis of anything? The Strong form would mean that one could find within the past light cone enough information to obtain a much predictive accuracy as possible about an event.

When we say "Nothing can travel faster than the speed of light", what do we mean?

We know that no ordinary mass can go faster, because that would require infinite energy. We know that no conserved quantity can go faster, because then it would not be conserved in some reference frames. If we believe that causation must go forward in time, then we know that no ”information” can go faster than c, because that would allow backwards-in-time causation. (What happens if you can send info backward? Say you send your grandma info that somebody much cuter than your grandpa was about to move into her neighborhood. Then you aren’t born. Then the info doesn’t get sent.)