Physics 419
Lecture 10: Special Relativity
Feb. 25, 2021

1 Themes

- Simultaneity
- Experimental confirmation of special relativity
- Twin paradox
- Unification of electricity and magnetism

2 Simultaneity

In special relativity, at the same time and at the same place are relativised. Since we all have to agree on the speed of light, nothing other than length/time, we cannot agree on the time or distance if we are moving with a constant speed relative to some other frame. As a result, we do not tell the same story about two events which appear to happen at the same time in another frame. See previous lecture.

3 Experimental confirmation of SR

Why is time dilation interesting? The principle of relativity tells us that K' (the person who owns the moving clock) will reverse the roles of the two clocks. K will say that the clock owned by K is ticking slowly.

There is no non-arbitrary way to answer the question, "Who is right?" (Here, there's not even agreement about who's running faster, much less about who's right.) As a consequence, there is no absolute time.

The size of the effect is the same as for Lorentz time dilation, but it is conceptually quite different. For Lorentz there was a preferred frame (the ether), and all clocks moving through the ether ran slowly with respect to absolute time. Now there is no preferred frame. Each person says that clocks moving w.r.t him are running slowly.

Just to be sure you understand ...

The reason K and K' disagree about the clocks is that although they see the light taking different paths, they both say the light is travelling at c relative to themselves. It is NOT a matter of them being too dumb to correct for the travel time of the light- the corrections disagree because they disagree about the relative speed of the light wrt to different objects.

Are there any “real” effects of all this?

Consider cosmic ray muons, produced from high-energy radiation from the sun or other solar systems consisting mainly of protons, are produced in the upper atmosphere, 25-50 km above the ground. These particles only live about 2 microseconds, in which time they can only travel a few hundred meters at the speed of light. Nevertheless, a large number of them make it to the ground. How is this possible?

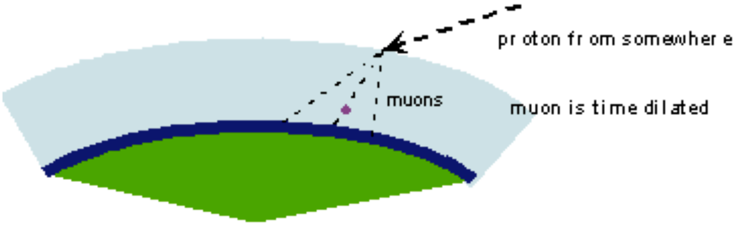

Figure 1: Cosmic ray muons: Time dilation

The solution is time dilation. Cosmic ray muons travel at a speed about 99% that of the speed of light, that is, v/c=.99. Hence, the time dilation factor is about 7.09. Hence, if a muon at rest decays in 2 microseconds, one traveling at v=.99c decays in 2 times 7.09 or 14.18 microseconds. In this time, muons can very nearly reach the ground.

In order for SR to be self consistent, all clocks must be affected the same way. Thus, time dilation is not a property of any specific mechanism, but of time itself.

What does the muon see? From its point of view, there is no time dilation, but the atmosphere (which is moving) is Lorentz contracted. In the 2 μs muon lifetime, the earth, traveling close to c, has time to collide with the muon because it starts close by.

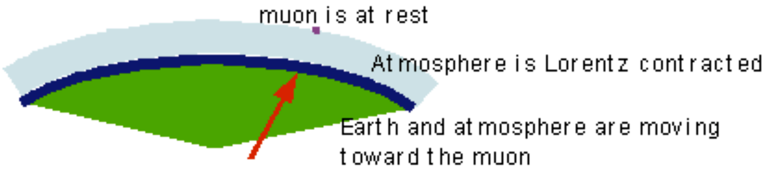

Figure 2: Cosmic ray muons: distance contraction

Both observers agree the muon will reach the surface. They disagree about why this is so, i.e. they disagree about the values of some particular quantities used in the calculation leading to the conclusion about whether the event occurs.

4 Twin paradox

Suppose Alice and Beth are twins. Alice sets off in her rocket so fast that the time dilation factor becomes 10. She travels away from Earth for 10 years, as measured by Beth, who has remained on Earth. Alice then turns around and returns to Earth at the same rapid pace.

When Alice returns home, Beth has aged 20 years. How much has Alice aged?

There appears to be a paradox. According to the Lorentz transformation, during the time Alice is traveling:

Beth says: I measure Alice's clock to be running slow by a factor of ten, so she has aged only two years.

Alice says: My clock is fine. I measure Beth's clock to be running slow by a factor of ten, so she has aged only 2 years.

They start and end standing right next to each other, so a direct comparison of clocks is possible. Who is correct?

The answer is that Alice, the twin who turned around, has aged less. The situation is not symmetrical, because in order to return to Earth, Alice must have accelerated. Our descriptions of how things looked to different observers (Lorentz transformations) so far do not describe accelerated observers, so we only know how things look to Beth. Of course Alice must agree that Beth is older, when they now stand side-by side. Now we can put together a conclusion about how Beth must have looked to Alice while Alice was accelerating. While turning back (accelerating toward earth) , Alice must observe Beth's clock to be running fast, not slow.

In other words, this is not a paradox at all but just a reminder that the SR transformations only work between reference frames which are not accelerating (AT LEAST with respect to each other, leaving aside the question of absolute acceleration.) But you can also see that from SR we can draw conclusions about how things MUST look to accelerating observers.

5 Symmetry of Special Relativity

Special Relativity is a symmetry of the laws of physics. It says that if you look at some set of events from any one of a collection of different reference frames in constant relative motion, the laws needed to describe the events don't change. Galilean relativity said the same thing. But Maxwell's laws didn't obey Galilean relativity. That's because G's relativity made assumptions about how some particular quantities (distance, time, mass) changed between different frames (i.e. that they didn't change) , and those assumptions turned out to be wrong.

In Special Relativity

Moving clocks run slowly, but each person thinks his clock is at rest. Moving rulers are Lorentz contracted along the direction of motion, but each person thinks his ruler is at rest.

Another invariant feature is whether or not two objects are at the same place at the same time. Can you think of an argument why you would expect that to be invariant?

From Galileo, we have that

$$\begin{aligned} x &\rightarrow x - vt \\ t &\rightarrow t \end{aligned} \tag{1}$$

From special relativity, we find that

$$\begin{aligned} x &\rightarrow \gamma(x - vt) \equiv x', \quad \gamma = 1/\sqrt{1-(v/c)^2} \\ t &\rightarrow \gamma(t - vx/c^2) \equiv t'. \end{aligned} \tag{2}$$

Both types of transformation are INVERTIBLE. That is, if you transform to a primed frame using relative velocity v, the transform using relative velocity -v, you get back the original coordinates. That's obvious for Galilean transforms, and you can easily check it for Lorentz transforms. That's one of the properties that make the transforms part of what's called a GROUP of symmetries. More importantly, since you use the SAME TYPE OF TRANSFORM either way, you can't get a clue as to which frame is the "proper" one.

Important note: even if there were no such thing as light, or anything else that traveled at speed c relative to other stuff, all of the essential points of relativity are contained in the new rules for converting between coordinate systems. Our arguments about "what would observer A see by using light" are not essential, just convenient paths toward these transformations. The key point is NOT about practical limitations on observations, but rather what sets of variables different observers have to use to get nature to obey the same simple laws.

According to Galileo, if I measure an object to have velocity v_1 and you have velocity v_2, then you will measure the object to have velocity $v_1' = v_1 - v_2$. Not so according to special relativity. Let us define

$\beta = v/c$. The velocity is

$$\beta_1' = \frac{\beta_1 - \beta_2}{1 - \beta_1\beta_2} \tag{3}$$

Plug in any combination of velocities you want and you will never exceed $\beta = 1$ that is $v = c$.

Example:

I measure a proton to be moving 0.75 c. You are moving 0.75 c in the opposite direction (i.e., not with the proton). You will not measure 1.5 c for the protons speed, but rather 0.96 c.

6 Unification

Einstein's one simple postulate solves a lot of problems. Consider the magnetic force on a moving charge due to the electric current in an electrically neutral wire (no electric field):

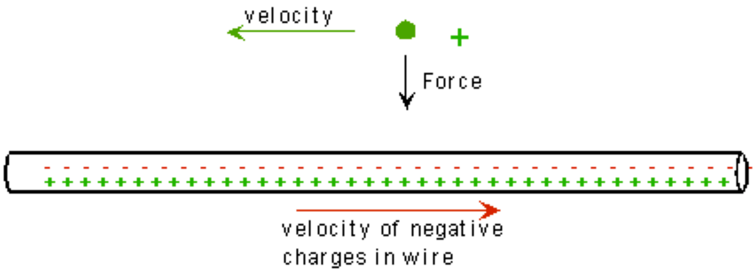

Figure 3: Neutral wire with a positive charge $+q$ located above it. The charge is moving in the same direction as are the electrons in the wire. The question is what force does the wire experience?

In the frame of the wire, the charge q, since it is moving must experience a magnetic field. Now consider the frame of the charge. It is at rest and the wire is neutral and hence it experiences no force. The electrons experience a Lorentz contraction. As a result, their charge density (charge per unit length) is not the same as the positive background. Hence, the $+q$ charge on the outside does not exactly see a neutral wire. Hence, there must be a force. So what was a magnetic force in one frame becomes an electrical force in another.