Superconductivity in pictures

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How to measure superconductivity

Electrical resistance of some metals drops to zero below a certain temperature which is called "critical temperature" (H. K. O. 1911)

How to observe superconductivity
1. Take Nb (niobium) wire
2. Connect to a voltmeter and a current source
3. Immerse into helium Dewar (T=4.2 K boiling point)
4. Measure electrical resistance (R) versus the temperature (T)

<table>
<thead>
<tr>
<th>T (K)</th>
<th>R (Ohm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>A</td>
</tr>
<tr>
<td>300</td>
<td>A</td>
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Heike Kamerling Onnes
Meissner effect – the key signature of superconductivity
Importance of superconductivity: Qubits for quantum computers are made of superconductors
Magnetic levitation

Levitation is the process by which an object is held aloft, without mechanical support, in a stable position.
Magnetic levitation train
Discovery of the supercurrent, known now as proximity effect, in SNS junctions

Superconductivity of Contacts with Interposed Barriers

HANS MEISSNER
Department of Physics, The Johns Hopkins University, Baltimore, Maryland
(Received August 25, 1959)

Resistance vs current diagrams and “Diagrams of State” have been obtained for 63 contacts between crossed wires of tin. The wires were plated with various thicknesses of the following metals: copper, silver, gold, chromium, iron, cobalt, nickel, and platinum. The contacts became superconducting, or showed a noticeable decrease of their resistance at lower temperatures if the plated films were not too thick. The limiting thicknesses were about $35 \times 10^{-4}$ cm for Cu, Ag, and Au; $7.5 \times 10^{-4}$ cm for Pt, $4 \times 10^{-4}$ cm for Cr, and less than $2 \times 10^{-4}$ cm for the ferromagnetic metals Fe, Co, and Ni. The investigation was extended to measurements of the resistance of contacts between crossed wires of copper or gold plated with various thicknesses of tin. Simultaneous measurements of the (longitudinal) resistance of the tin-plated gold or copper wires showed that these thin films of tin do not become superconducting for thicknesses below certain minimum values. These latter findings are in agreement with previous measurements at Toronto. The measurements at Toronto usually were believed to be unreliable because films of tin evaporated onto quartz substrates can be superconducting at thicknesses as small as $1.6 \times 10^{-6}$ cm. It is now believed that just as superconducting electrons can drift into an adjoining normal conducting layer and make it superconducting, normal electrons can drift into an adjoining superconducting layer and prevent superconductivity.

![Graph](image1)

**Fig. 1.** Resistance vs current diagram of cobalt-plated contact Co 4, representative of diagrams type A.

![Graph](image2)

**Fig. 2.** Resistance vs current diagram of silver-plated contact Ag 2, representative of diagrams type B.
Explanation of the supercurrent in SNS junctions --- Andreev reflection

Andreev reflection

A.F. Andreev, 1964
Non-Abelian Majorana Modes in Vortices

Non-Abelian Statistics of Half-Quantum Vortices in \( p \)-Wave Superconductors

D. A. Ivanov

Institut für Theoretische Physik, ETH-Hönggerberg, CH-8093 Zürich, Switzerland
(Received 17 May 2000)

Excitation spectrum of a half-quantum vortex in a \( p \)-wave superconductor contains a zero-energy Majorana fermion. This results in a degeneracy of the ground state of the system of several vortices. From the properties of the solutions to Bogoliubov–de Gennes equations in the vortex core we derive the non-Abelian statistics of vortices identical to that for the Moore-Read (Pfaffian) quantum Hall state.

Total number of quantum states for \( N \) pairs of such Majorana vortices is:

\[ 2^N \]

Example \( N=100 \)

\[ 2^{100} = 10^{30} \]
Majorana modes in a vortex

Theory: Vortex in the nano-hole contains Majorana states (gap is about $T_c$)


Schematic of the array (22x22 islands)

- Orange arrow: external magnetic field
- Red arrow: magnetic field that penetrates the array
- Yellow film: Bi$_2$Se$_3$ topological insulator
- Blue block: Niobium square island
Resistance of the sample as a function of magnetic field at 300 mK (zoom into lower fields)
Searching for an explanation: Little-Parks effect (’62)
The basic idea: magnetic field induces non-zero vector-potential, which produces non-zero superfluid velocity, thus reducing the Tc.
Superconducting vortices produced by magnetic field

Quantum trapping

magnetic field penetrates in the form of quantum flux tubes
Measuring nanowires within GHz resonators. Detection of individual phase slips.

A. Belkin et al, Appl. Phys. Lett. 98, 242504 (2011)
Resonators used to detect single phase slips (SPS) and double phase slips (DPS)

A. Belkin et al, PRX 5, 021023 (2015)

\[ T = 360 \text{ mK} \]
\[ f = f_0(H=0) \]
Phase gradiometers templated by DNA

SQUIDs, or Superconducting Quantum Interference Devices, invented in 1964 by Robert Jaklevic, John Lambe, Arnold Silver, and James Mercereau of Ford Scientific Laboratories, are used to measure extremely small magnetic fields. They are currently the most sensitive magnetometers known, with the noise level as low as 3 fT·Hz⁻¹/₂. While, for example, the Earth magnet field is only about 0.0001 Tesla, some electrical processes in animals produce very small magnetic fields, typically between 0.000001 Tesla and 0.000000001 Tesla. SQUIDs are especially well suited for studying magnetic fields this small.

Measuring the brain’s magnetic fields is even much more difficult because just above the skull the strength of the magnetic field is only about 0.3 picoTesla (0.0000000000003 Tesla). This is less than a hundred-millionth of Earth’s magnetic field. In fact, brain fields can be measured only with the most sensitive magnetic-field sensor, i.e. with the superconducting quantum interference device, or SQUID.
Vortices introduce electrical resistance to otherwise superconducting materials

Magnetic field creates vortices--
Vortices cause dissipation (i.e. a non-zero electrical resistance)!

The order parameter:
\[ \Psi = |\Psi| \exp(-i\varphi) \]

Vortex core: normal, not superconducting; diameter \( \xi \sim 10 \text{ nm} \)
DC transport measurement schematic

Phase slip events are shown as red dots
Transport properties: Little’s Phase Slip

\[ \Delta(x) = |\Delta(x)| \exp[i\phi] \]

Two types of phase slips (PS) can be expected:
1. The usual, thermally activated PS (TAPS)
2. Quantum phase slip (QPS)

How to use voltage to determine the rate of phase slips?

Phase evolution: \( \frac{d\phi}{dt} = \frac{2eV}{\hbar} \)

Simplified derivation:
1. From Schrödinger equation: \( i\hbar \frac{d\Psi}{dt} = E\Psi \)
2. The solution is: \( \Psi = \exp(-iEt/\hbar) \)
3. The phase of the wavefunction is \( \phi = Et/\hbar \)
Superconducting electrons form pairs, so the energy is: \( E = 2eV \)
(here \( V \) is the electric potential or voltage)

Thus the resulting equation is: \( \frac{d\phi}{dt} = \frac{2eV}{\hbar} \)
Superconductivity: very basic introduction

Electrical resistance is zero only if current is not too strong.

$\begin{align*}
V \text{ (voltage)} \\
I \text{ (current)}
\end{align*}$

Superconducting regime

$I_c$

Normal state--Ohms law
Search for QPS at high bias currents, by measuring the fluctuations of the switching current.

Slope = $R_N = 3.28k\Omega$
Fabrication of nanowires

Method of Molecular Templating

Si/ SiO$_2$/SiN substrate with undercut

~ 0.5 mm Si wafer
500 nm SiO$_2$
60 nm SiN
Width of the trenches ~ 50 - 500 nm

HF wet etch for ~10 seconds to form undercut

TEM image of a wire shows amorphous morphology.
Nominal MoGe thickness = 3 nm

Schematic picture of the pattern
Nanowire + Film Electrodes used in transport measurements
Measurement Scheme

Circuit Diagram

Sample mounted on the $^3$He insert.
Tony Bollinger's sample-mounting procedure in winter in Urbana

Procedure (~75% Success)
- Put on gloves

- Put grounded socket for mounting in vise with grounded indium dot tool connected
- Spray high-backed black chair all over and about 1 m square meter of ground with anti-static spray

  - DO NOT use green chair
  - Not sure about short-backed black chairs

- Sit down
- Spray bottom of feet with anti-static spray

- Plant feet on the ground. **Do not move your feet again for any reason until mounting is finished.**
- Mount sample
- Keep sample in grounded socket until last possible moment
- Test samples in dipstick at ~1 nA
Possible Origin of Quantum Phase Slips

Origin of quantum phase slips

He3

He4

R (T)

T (K)

QPS on

QPS off
Dichotomy in nanowires: Evidence for superconductor-insulator transition (SIT)

\[ R = \frac{V}{I} \quad I \sim 3 \text{ nA} \]

The difference between samples is the amount of the deposited Mo79Ge21.

\[ R_{\text{sheet}} = 100 - 400 \, \Omega \]

Can the insulating behavior be due to Anderson localization of the BCS condensate?

Useful Expression for the Free Energy of a Phase Slip

“Arrhenius-Little” formula for the wire resistance:

\[ R_{AL} \approx R_N \exp[-\Delta F(T)/k_B T] \]

\[ \Delta F = (8\sqrt{2}/3)(H_c(T)^2/8\pi)(A\xi(T)) \]

\[ \frac{\Delta F(0)}{k_B T_c} = \sqrt{6} \frac{\hbar I_c(0)}{2e k_B T_c} = 0.83 \frac{R_q L}{R_n_\xi(0)} = 0.83 \frac{R_q}{R_\xi(0)} \]

Quantum limit to phase coherence in thin superconducting wires

M. Tinkham\textsuperscript{a)} and C. N. Lau
Physics Department, Harvard University, Cambridge, Massachusetts 02138
Linearity of the Schrödinger’s equation

Suppose $\Psi_1$ is a valid solution of the Schrödinger equation:

$$i\hbar \frac{\partial \psi_1}{\partial t} = \frac{\partial^2 \psi_1}{\partial x^2} + U(x)\psi_1$$

And suppose that $\Psi_2$ is another valid solution of the Schrödinger equation:

$$i\hbar \frac{\partial \psi_2}{\partial t} = \frac{\partial^2 \psi_2}{\partial x^2} + U(x)\psi_2$$

Then $(\Psi_1 + \Psi_2)/\sqrt{2}$ is also a valid solution, because:

$$i\hbar \frac{\partial (\psi_1 + \psi_2)}{\partial t} = \frac{\partial^2 (\psi_1 + \psi_2)}{\partial x^2} + U(x)(\psi_1 + \psi_2)$$

The state $(\Psi_1 + \Psi_2)/\sqrt{2}$ is a new combined state which is called “quantum superposition” of state (1) and (2)
Quantum tunneling is possible since quantum superpositions of states are possible.

George Gamow
(He also developed Big Bang theory)
Schrödinger cat –
the ultimate macroscopic quantum phenomenon

E. Schrödinger, Naturwiss. 23 (1935), 807.
Schrödinger cat – thought experiment

Hans Geiger

Geiger counter
What sort of tunneling we will consider?

- Red color represents some strong current in the superconducting wire loop
- Blue color represents no current or a much smaller current in the loop
Previous results relate loops with insulating interruptions (SQUIDs)

-Red color represents some strong current in the superconducting loop
-Blue color represents no current or very little current in the superconducting loop

\(I_s\) - Insulating gap
Leggett’s prediction for macroscopic quantum tunneling (MQT) in SQUIDs

Macroscopic Quantum Systems and the Quantum Theory of Measurement

A. J. LEGGETT

School of Mathematical and Physical Sciences
University of Sussex, Brighton BN1 9QH

(Received August 27, 1980)

It is this property which makes a SQUID the most promising candidate to date for observing macroscopic quantum tunnelling; if it should ever become possible to observe macroscopic quantum coherence, the low entropy and consequent lack of dissipation will be absolutely essential.21)
MQT report by Kurkijarvi and collaborators (1981)

Decay of the Zero-Voltage State in Small-Area, High-Current-Density Josephson Junctions

FIG. 1. Measured distribution for $T = 1.6$ K for small high-current-density junction. The solid line is a fit by the CL theory for $R = 20 \, \Omega$, $C = 8 \, fF$, and $i_{c\Phi} = 310.5 \, \mu A$. The inset is $U(\phi)$ for $x = 0.8$ with barrier $\Delta E$.

FIG. 2. Measured distribution widths $\sigma$ vs $T$ for two junctions with current sweep of $\sim 400 \, \mu A$/sec. Curve $a$ is lower current density junction data and curve $b$ is higher density junction data. The traces adjacent to the plots are the corresponding $I-V$ characteristics at 4.2 K. The scales are the same for both traces.
Types of Qubit

Quantum state:

$$|\psi> = A^*|0> + B^*|1>$$

$$A^2 + B^2 = 1$$

A and B are complex numbers
Quantization of electrical circuits

The quantized $LC$ oscillator

Hamiltionian:

$$\hat{H}_{LC} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L}$$

Capacitive term Inductive term

Canonically conjugate variables:

$$\hat{\Phi} = \text{Flux through the inductor.}$$
$$\hat{Q} = \text{Charge on capacitor plate.}$$

$$[\hat{\Phi}, \hat{Q}] = i\hbar$$

Discrete energy spectrum of the LC-circuit

Correspondence with simple harmonic oscillator

\[
\hat{H}_{LC} = \frac{\Phi^2}{2L} + \frac{\dot{Q}^2}{2C}
\]

\[
\left[ \Phi, \dot{Q} \right] = i\hbar
\]

\[
\hat{H}_{SHO} = \frac{k\dot{X}^2}{2} + \frac{\dot{P}^2}{2m}
\]

\[
\left[ \dot{X}, \dot{P} \right] = i\hbar
\]

Correspondence:

\[
\Phi \leftrightarrow \dot{X} \quad L \leftrightarrow \frac{1}{k} \quad \omega = \frac{1}{\sqrt{LC}} \leftrightarrow \sqrt{\frac{k}{m}}
\]

\[
\dot{Q} \leftrightarrow \dot{P} \quad C \leftrightarrow m
\]

Solve using ladder operators:

\[
\hat{a} = \left( \frac{\hat{Q}}{Q_{zpf}} - i \frac{\hat{\Phi}}{\Phi_{zpf}} \right)
\]

\[
\Phi_{zpf} = \sqrt{2\hbar Z}
\]

\[
Q_{zpf} = \sqrt{2\hbar / Z}
\]

\[
Z = \omega_L = \frac{1}{\omega C} = \sqrt{L/C}
\]

\[
\hat{H}_{LC} = \hbar \omega \left( \hat{a}^\dagger \hat{a} + \frac{1}{2} \right)
\]

\[
[\hat{a}_r, \hat{a}^\dagger_r] = 1
\]

Non-harmonicity is the key factor

The Josephson junction

\[ I = I_c \sin \left( \frac{2\pi \Phi}{\Phi_0} \right) \]

\[ \Phi_0 = \frac{h}{2e} \]

flux quantum

\[ V = \Phi_0 \]

\[ V = I \]

S superconductor-insulator-superconductor tunnel junction

\[ I_c = \frac{\pi \Delta}{2e R} \]

\[ E_{\text{stored}} = E_J \left( 1 - \cos \left( \frac{2\pi \Phi}{\Phi_0} \right) \right) \]

\[ E_J = \frac{I_c \Phi_0}{2\pi} \]

Josephson Energy

For small flux: \[ E_J = \frac{\Phi^2}{2L_J} + \text{const} \]

Josephson inductance \[ L_J = \frac{\Phi_0}{2\pi I_c} \]

\[ \Phi_0 \]

Non-harmonicity is the key factor
How transmons are measured

IN

Attenuator 20 dB

3 dB Attenuator 4K

Low Noise Amplifier 6-20 GHz, 30 dB 1K 1 dB

Circulator 8-12 GHz

Heat Exchange 150mK

Isolator 8-12 GHz

Mixing Chamber 50mK

Attenuator 3 dB

SS Power Filter

Low Pass Filter DC - 12 GHz

SS-SS Cable

Cu-Cu Cable

NbTi-SS Cable

Microwave Switch

Superconducting Solenoid

Cavity 1

Cavity 2
How transmons are measured

Transmission versus frequency for dispersive measurement. $\omega_r$ denotes the resonant frequency without the dispersive shift. Depending on the qubit state, the cavity frequency is pulled by $\pm g^2/\Delta$. 
Transmon Meissner Qubit in Cu 3D cavity

Vortices (i.e., their cores)

J. Ku, Z. Yoscovits, A. Levchenko, J. Eckstein, and A. Bezryadin,
Decoherence and radiation-free relaxation in Meissner transmon qubit coupled to Abrikosov vortices,
Magnetic field effects
Magnetic field effects

Theor. effective area, $A_{eff}^{th}(\mu m^2)$

Experimental Effective Area, $A_{eff}^{ex}(\mu m^2)$

(c)

Heterodyne Voltage (mV)

Magnetic Field (G)

(1) $\Delta B_{ex}$

(II)
Examples quantum time-domain oscillation: Ramsey fringe
Conclusions

- Superconductivity allows us to test fundamental quantum phenomena, for example the macroscopic quantum tunneling and macroscopic quantum coherence.

- Superconductivity has been used to design many types of useful devices. The examples considered are the SQUIDS and the qubits for quantum computers.