

Superconductivity in pictures

Alexey Bezryadin

*Department of Physics
University of Illinois at Urbana-Champaign*



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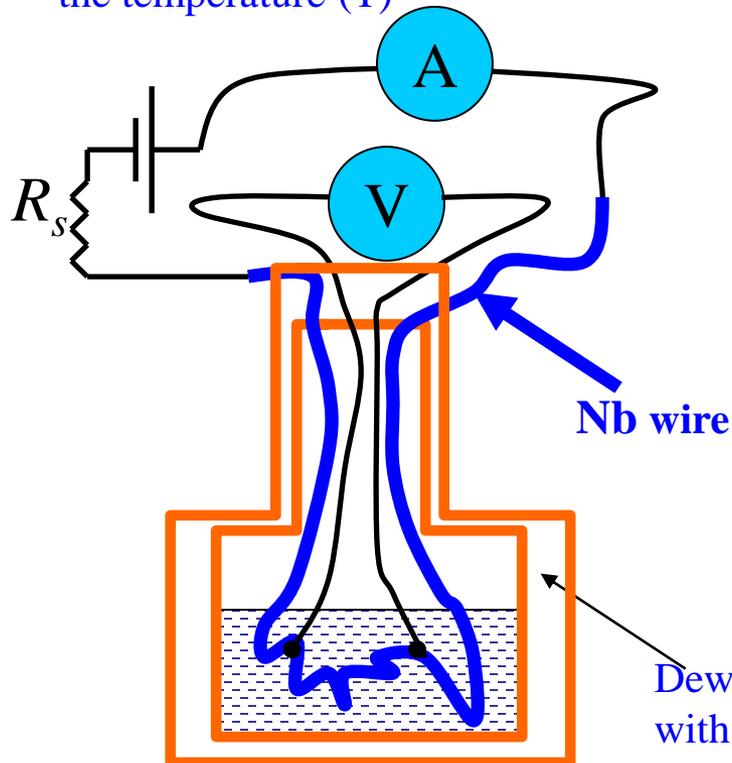


How to measure superconductivity

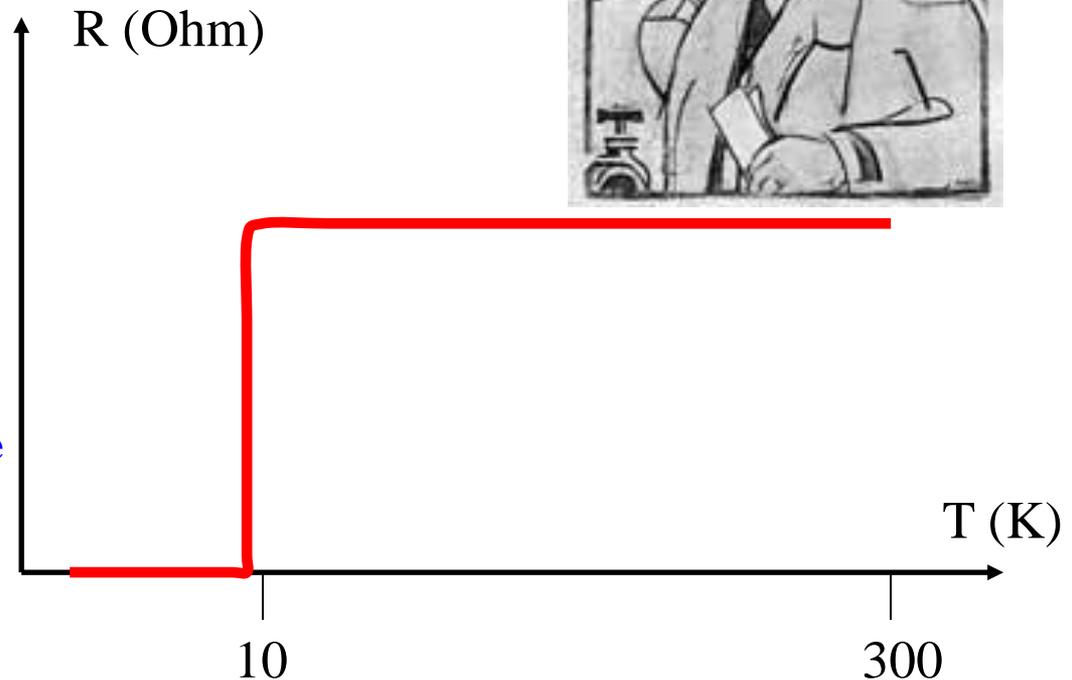
Electrical resistance of some metals drops to zero below a certain temperature which is called "critical temperature" (H. K. O. 1911)

How to observe superconductivity

1. Take Nb (niobium) wire
2. Connect to a voltmeter and a current source
3. Immerse into helium Dewar (T=4.2 K boiling point)
4. Measure electrical resistance (R) versus the temperature (T)



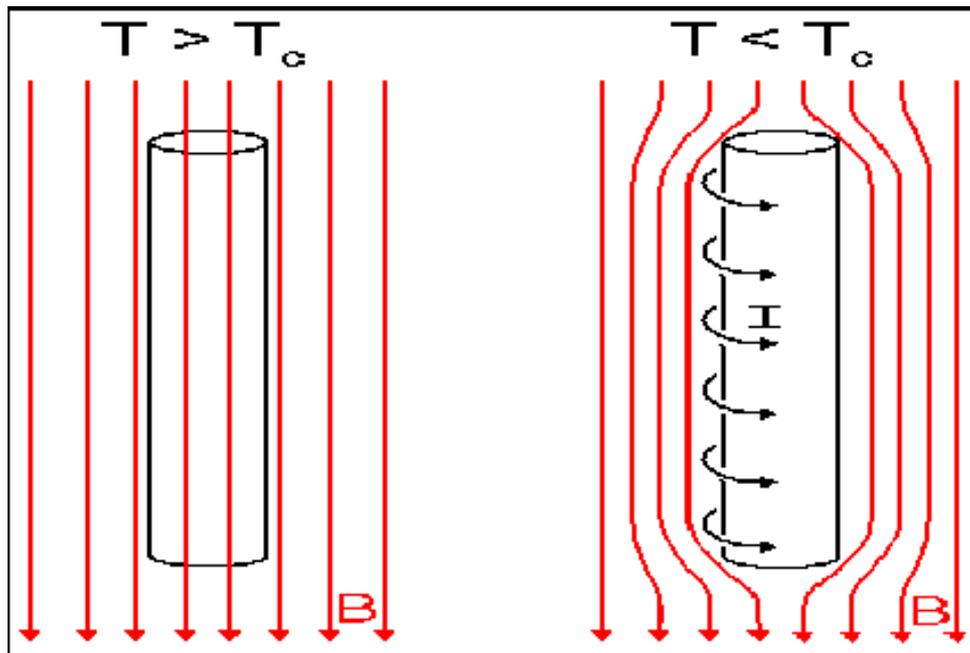
Heike Kamerling Onnes



Dewar with liquid Helium (4.2K)



Meissner effect – the key signature of superconductivity



BCS – Bardeen Cooper Schrieffer theory of superconductivity

Formula	T_c (K)	H_c (T)	Type	BCS
Elements				
Al	1.20	0.01	I	yes
Cd	0.52	0.0028	I	yes
Diamond:B	11.4	4	II	yes
Ga	1.083	0.0058	I	yes
Hf	0.165		I	yes
α -Hg	4.15	0.04	I	yes
β -Hg	3.95	0.04	I	yes
In	3.4	0.03	I	yes
Ir	0.14	0.0016 ^[7]	I	yes
α -La	4.9		I	yes
β -La	6.3		I	yes
Mo	0.92	0.0096	I	yes
Nb	9.26	0.82	II	yes
Os	0.65	0.007	I	yes

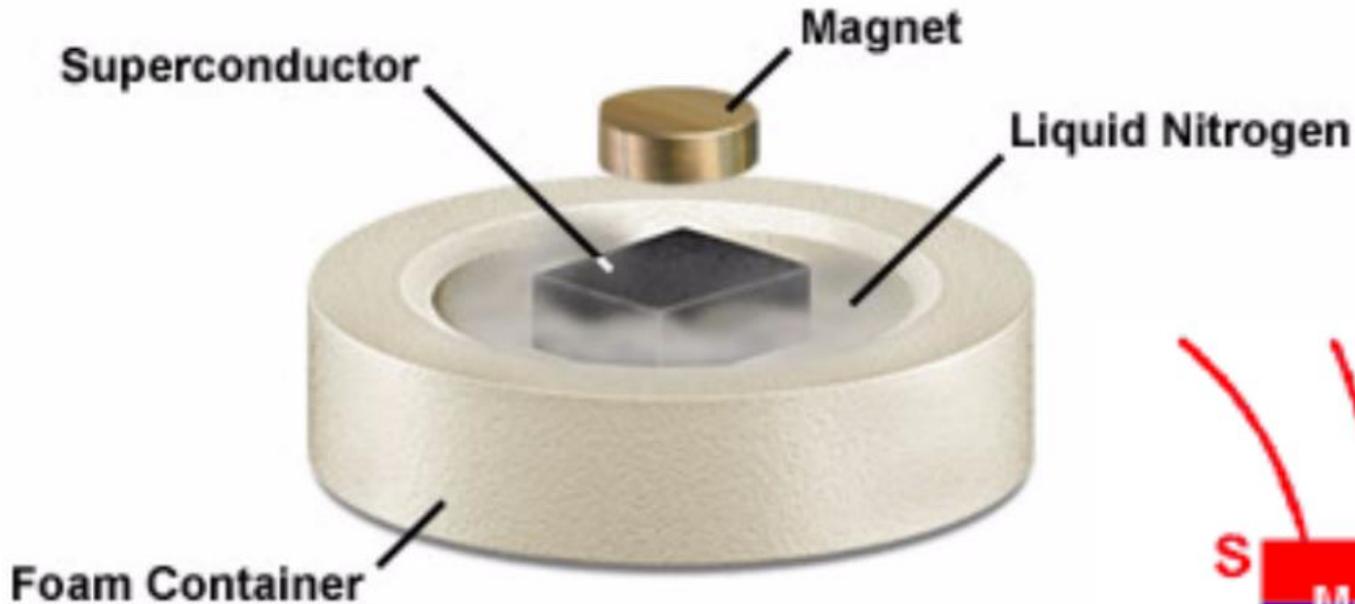
Importance of superconductivity: Qubits and modern quantum computers are made of superconductors

IBM Q

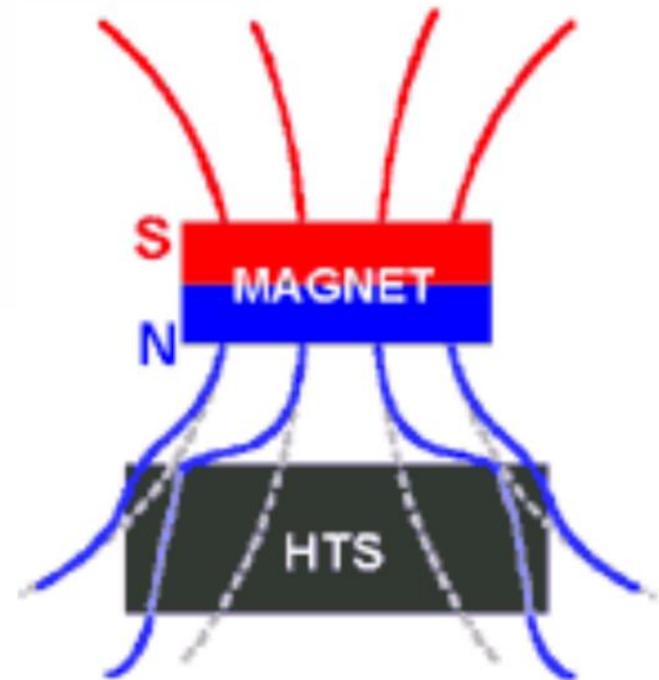


Magnetic levitation

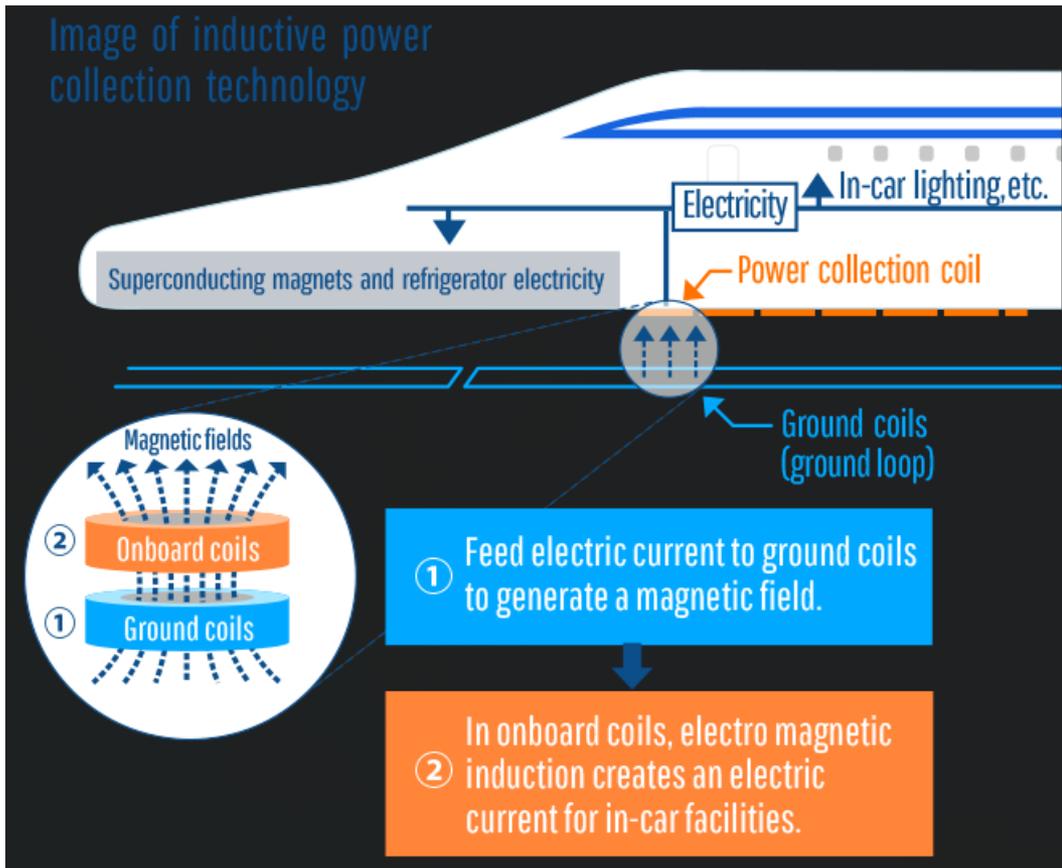
The Meissner Effect



Levitation is the process by which an object is held aloft, without mechanical support, in a stable position.



Magnetic levitation train

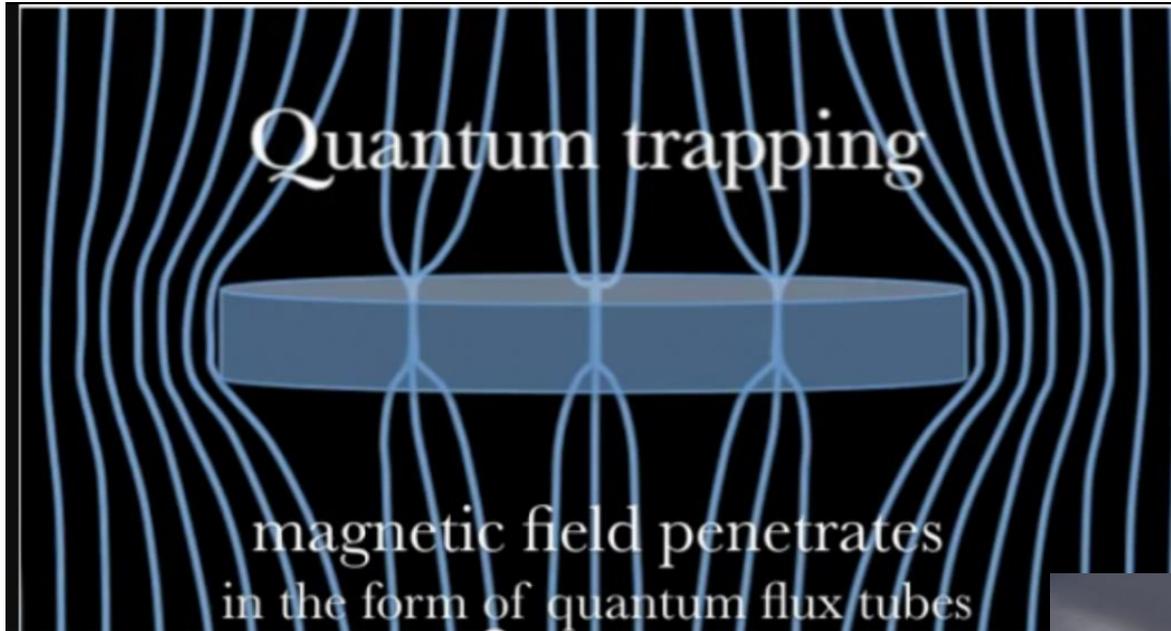


blogs.wsj.com

U.S. Transportation Secretary Foxx
Rides on Japan's Maglev Train - Jap...

The SCMaglev system uses an [electrodynamic suspension](#) (EDS) system. The train's [bogies](#) have [superconducting](#) magnets installed, and the guideways contain two sets of metal coils.

Superconducting vortices produced by magnetic field

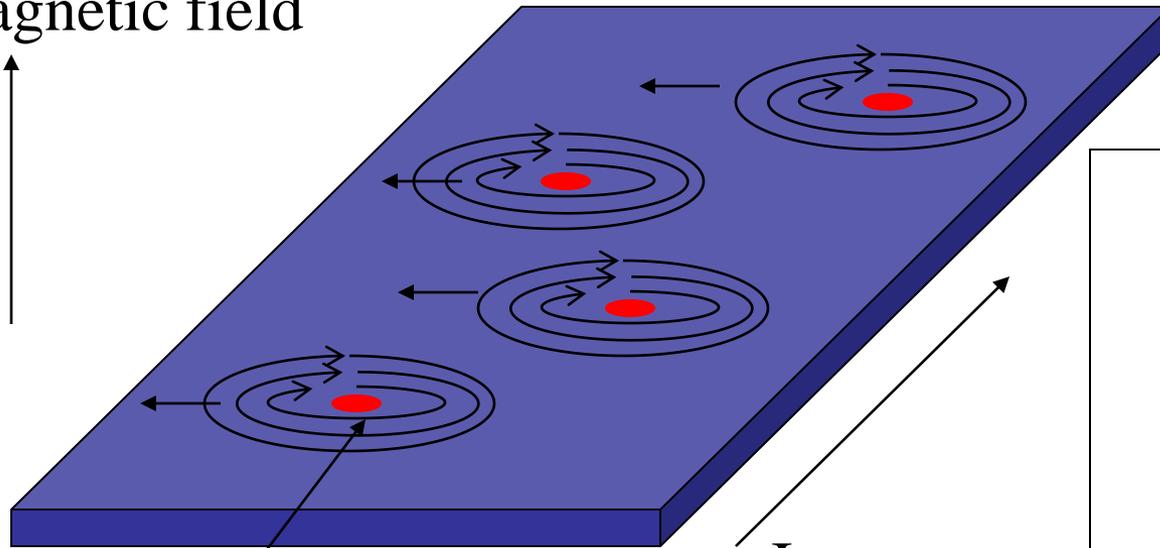


Vortices or flux tubes are quantized tubes carrying magnetic field into superconductor. They also introduce electrical resistance to otherwise superconducting materials.

Magnetic field creates vortices--

Vortices cause dissipation (i.e. a non-zero electrical resistance)!

B -magnetic field



The order parameter:

$$\Psi = |\Psi| \exp(-i\phi)$$

amplitude

phase

Vortex core (red): normal, not superconducting; diameter $\xi \sim 10$ nm

The current is extended to a scale called “lambda”, which is larger than ξ in type II superconductors (such as thin films of any material)

Reminder: single electron in empty space

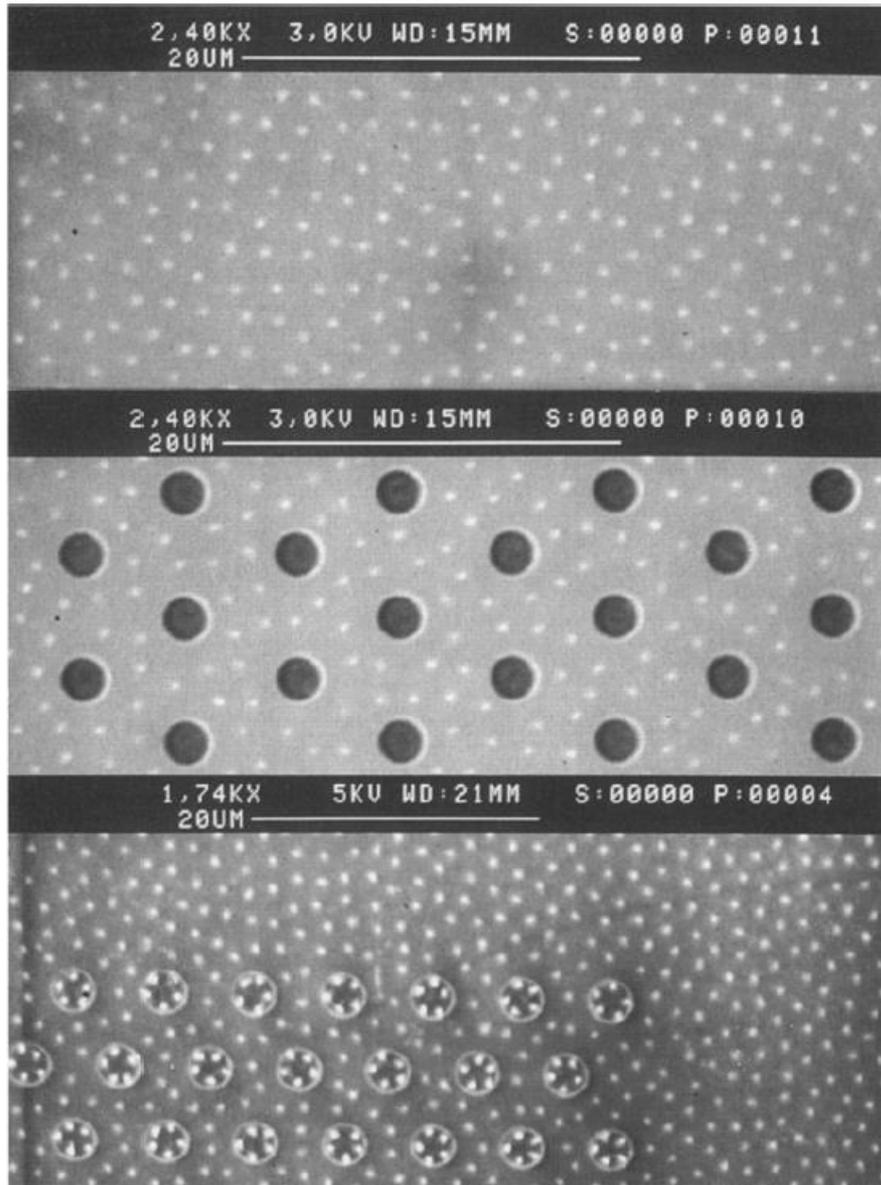
Wave function: $\Psi = |\Psi| \exp(ikx)$

Wave number: $k = 2\pi/\lambda$

General form: $\Psi = |\Psi| \exp(-i\phi)$

Phase: $\phi = kx$

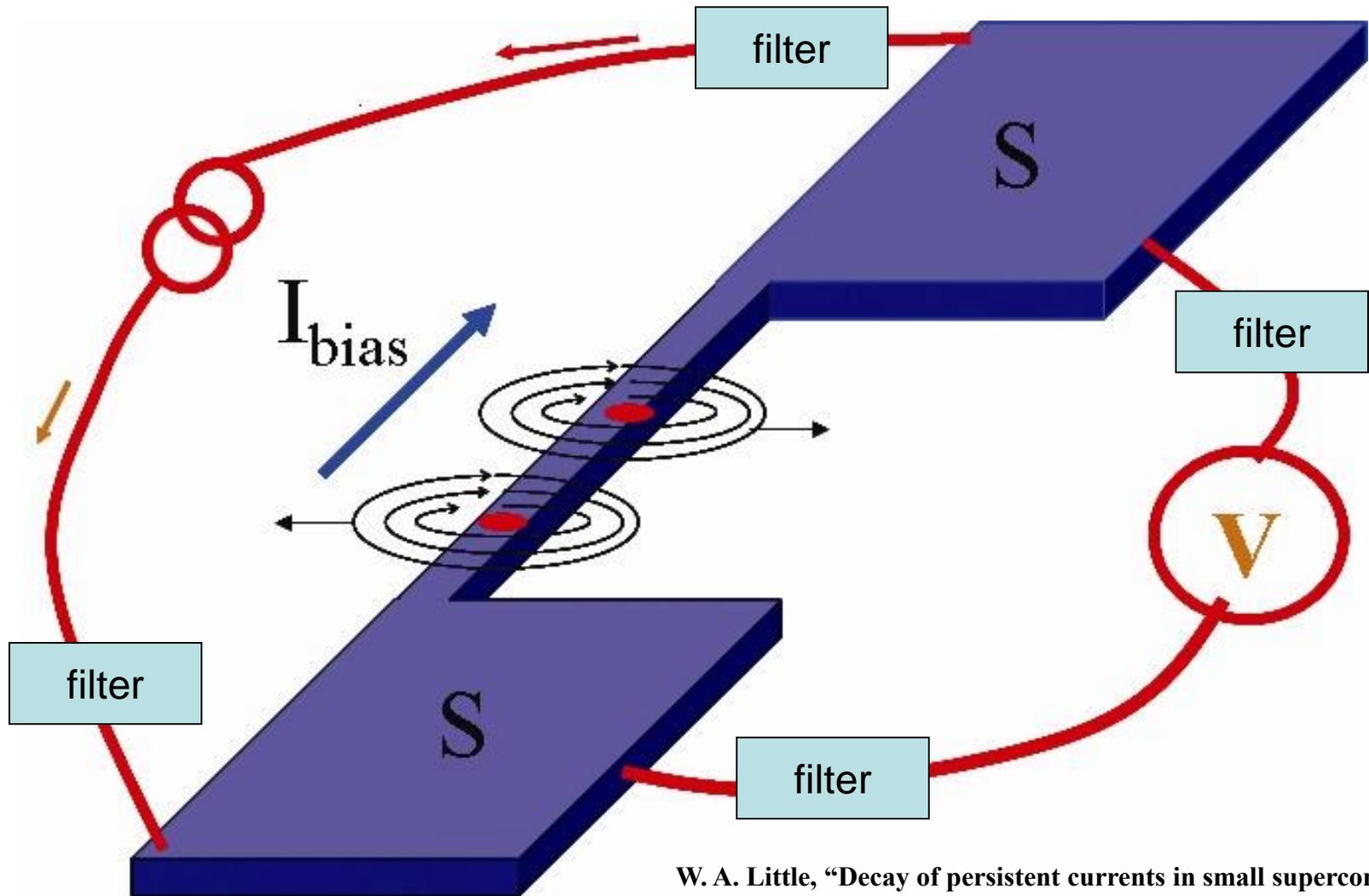
Vortices in Nb films with through and blind holes



A. Bezryadin and B. Pannetier
“Role of Edge Superconducting States in Trapping of Multi-Quanta Vortices by Microholes. Application of the Bitter Decoration Technique”,
J. Of Low Temp. Phys., V.102, p.73 (1996).

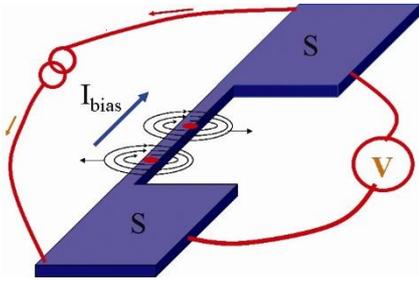
DC transport measurement schematic to detect phase slips

Phase slip events or crossing vortices are shown as red dots



W. A. Little, "Decay of persistent currents in small superconductors",
Physical Review, V.156, pp.396-403 (1967).

How to use voltage to determine the rate of phase slips?



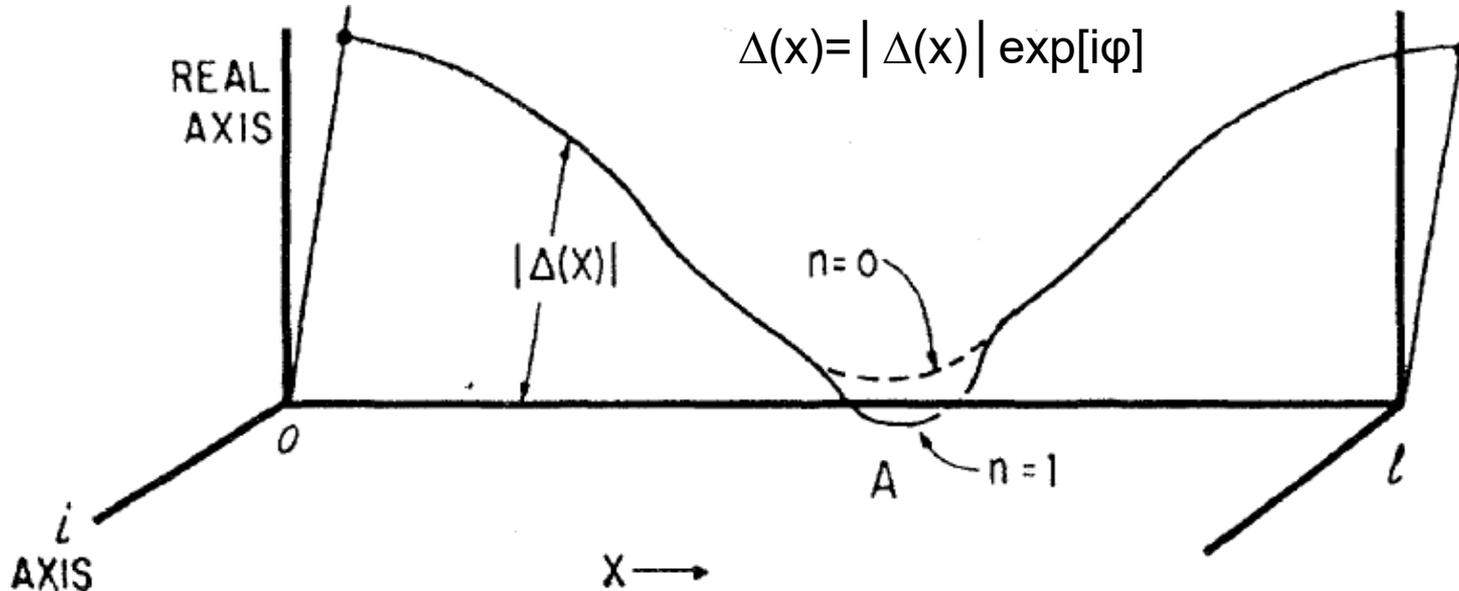
Phase evolution equation: $d\phi/dt = 2eV/\hbar$

Simplified derivation:

1. From Schrödinger equation: $i\hbar(d\Psi/dt)=E \Psi$
2. The solution is: $\Psi=\exp(-iEt/\hbar)$ (here E is the energy)
3. The phase of the wavefunction is $\phi=Et/\hbar$
4. The energy is defined by the electric potential (voltage), V as follows: $E=2eV$. Note that the effective charge of superconducting electrons is $2e$, where “e” is the charge of one electron. Such superconducting electron pairs are called Cooper pairs.

Thus, the resulting equation is: $d\phi/dt = 2eV/\hbar$

Transport properties: Little's Phase Slip



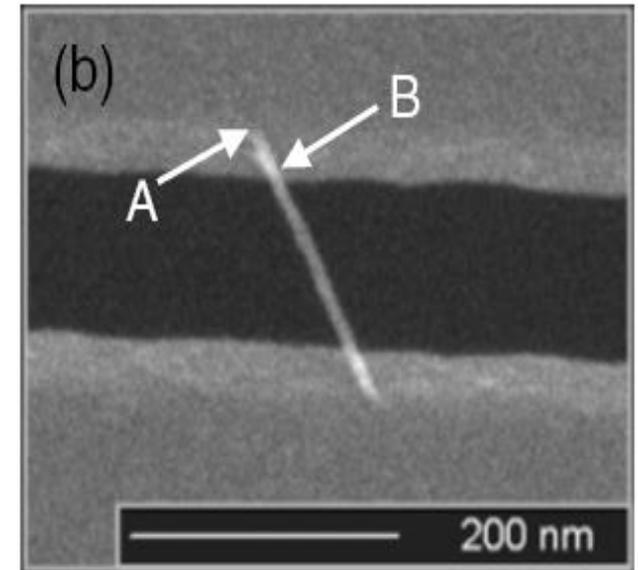
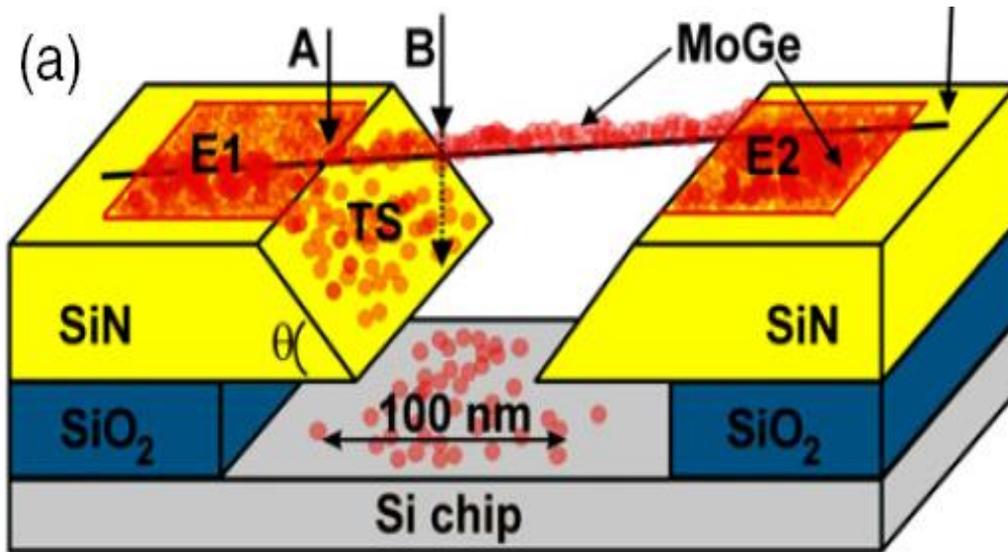
W. A. Little, "Decay of persistent currents in small superconductors", *Physical Review*, V.156, pp.396-403 (1967).

Two types of phase slips (PS) can be expected:

1. The usual, thermally activated PS (TAPS)
2. Quantum phase slip (QPS)

Fabrication of nanowires

Method of Molecular Templating



Si/ SiO₂/SiN substrate with undercut

~ 0.5 mm Si wafer

500 nm SiO₂

60 nm SiN

Width of the trenches ~ 50 - 500 nm

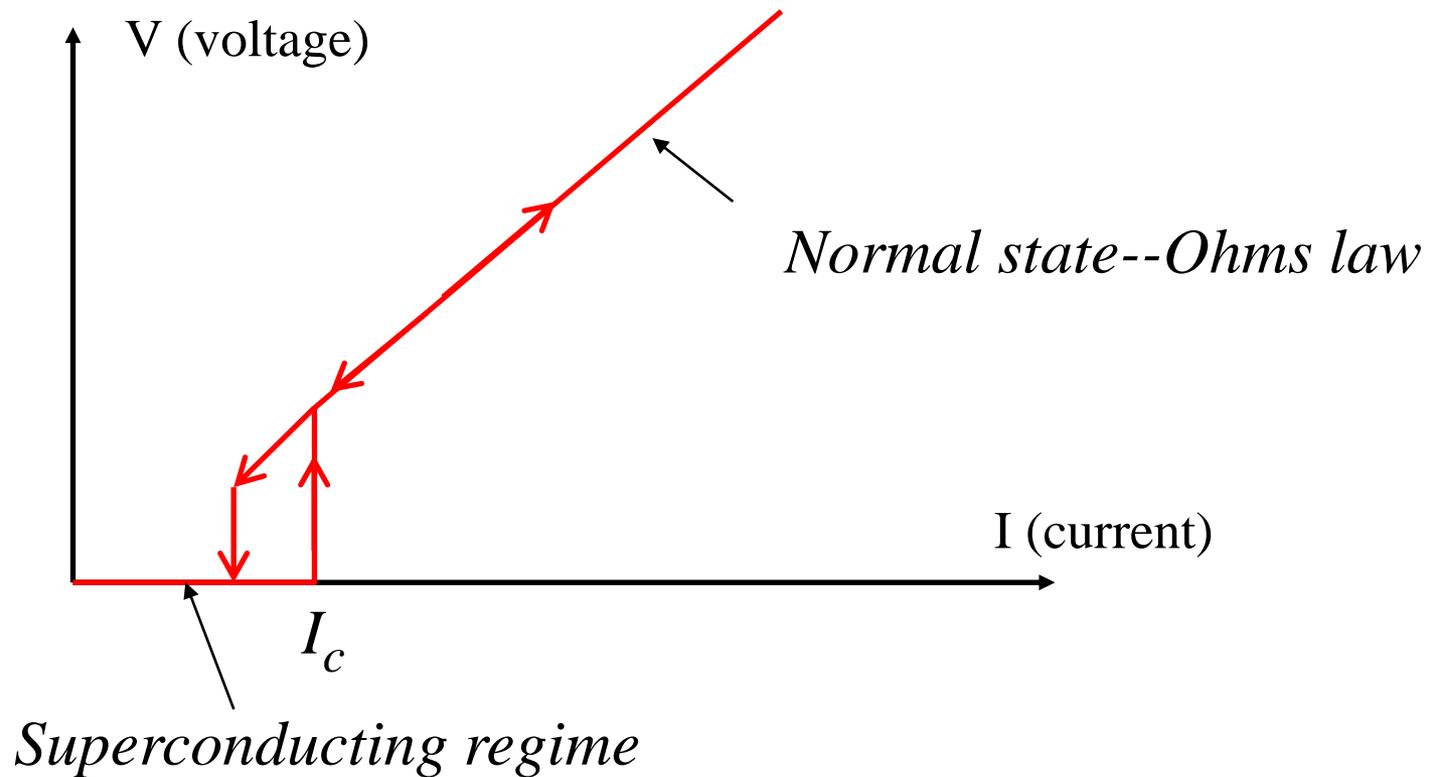
**HF wet etch for ~10 seconds
to form undercut**

Bezryadin, Lau, Tinkham, *Nature* **404**, 971 (2000)



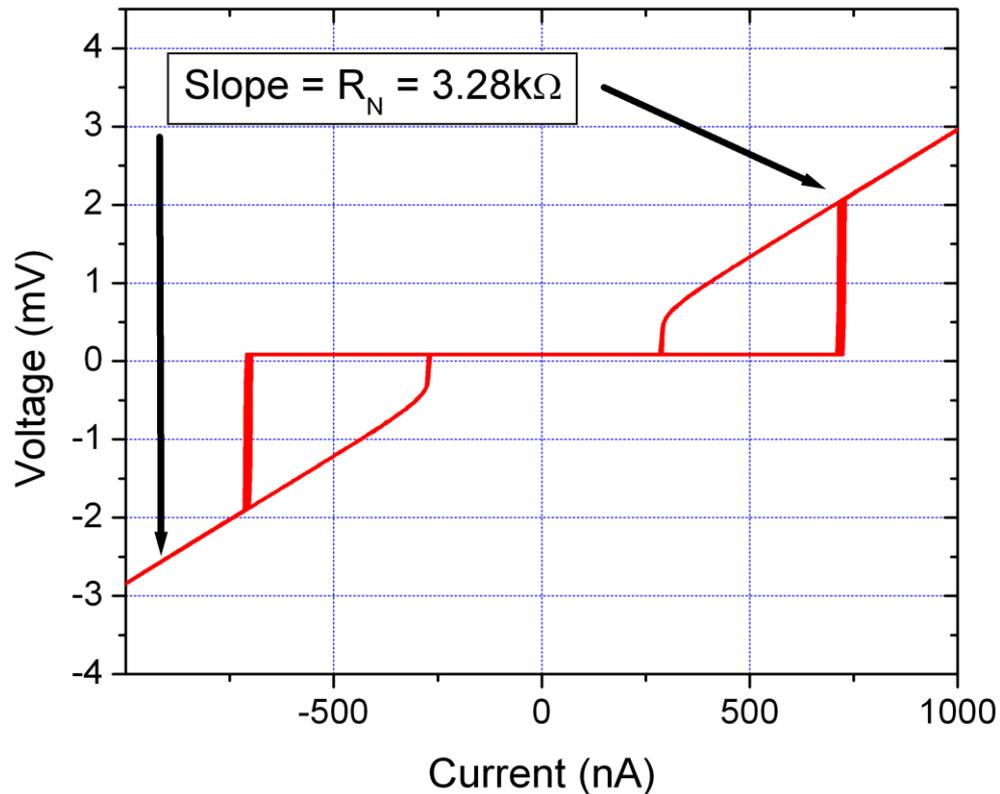
Expected voltage-current curve

Electrical resistance is zero only if current is not too strong

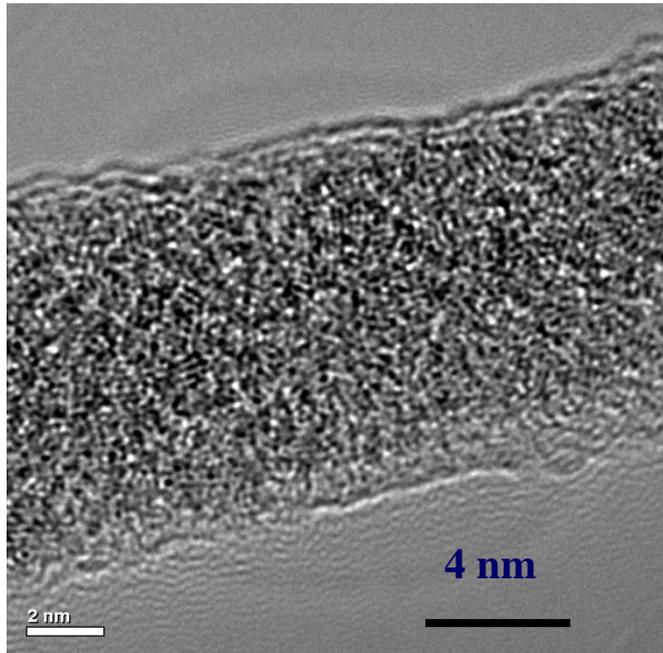


Experimental voltage-current curve.

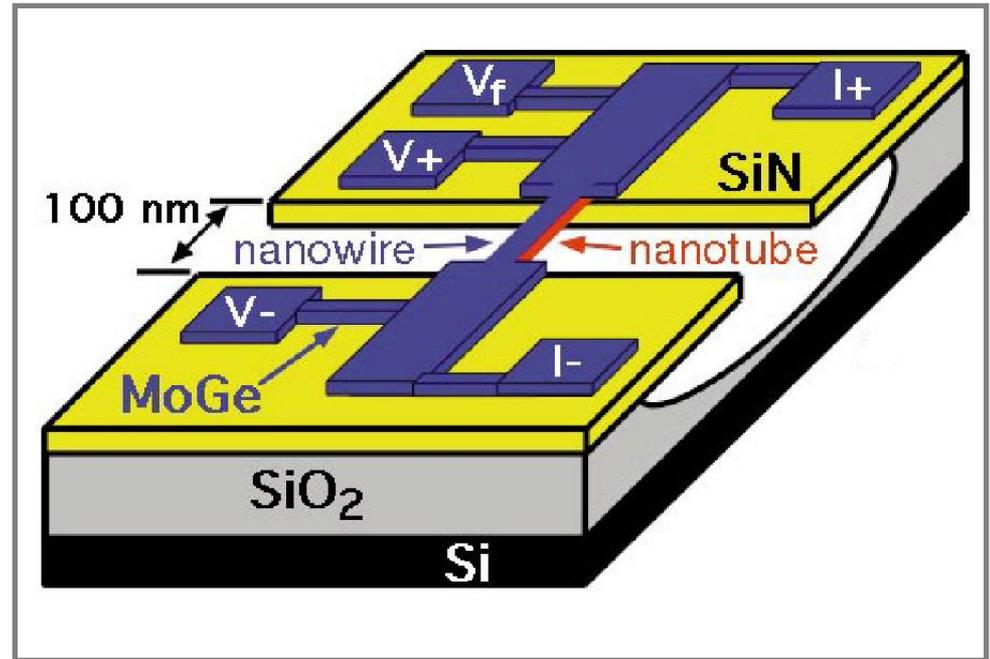
Evidence of Little's phase slips: the curve was measured many times and each time we detect a different critical current.



Sample Fabrication



**TEM image of a wire shows amorphous morphology.
Nominal MoGe thickness = 3 nm**



**Schematic picture of the pattern
Nanowire + Film Electrodes used in
transport measurements**



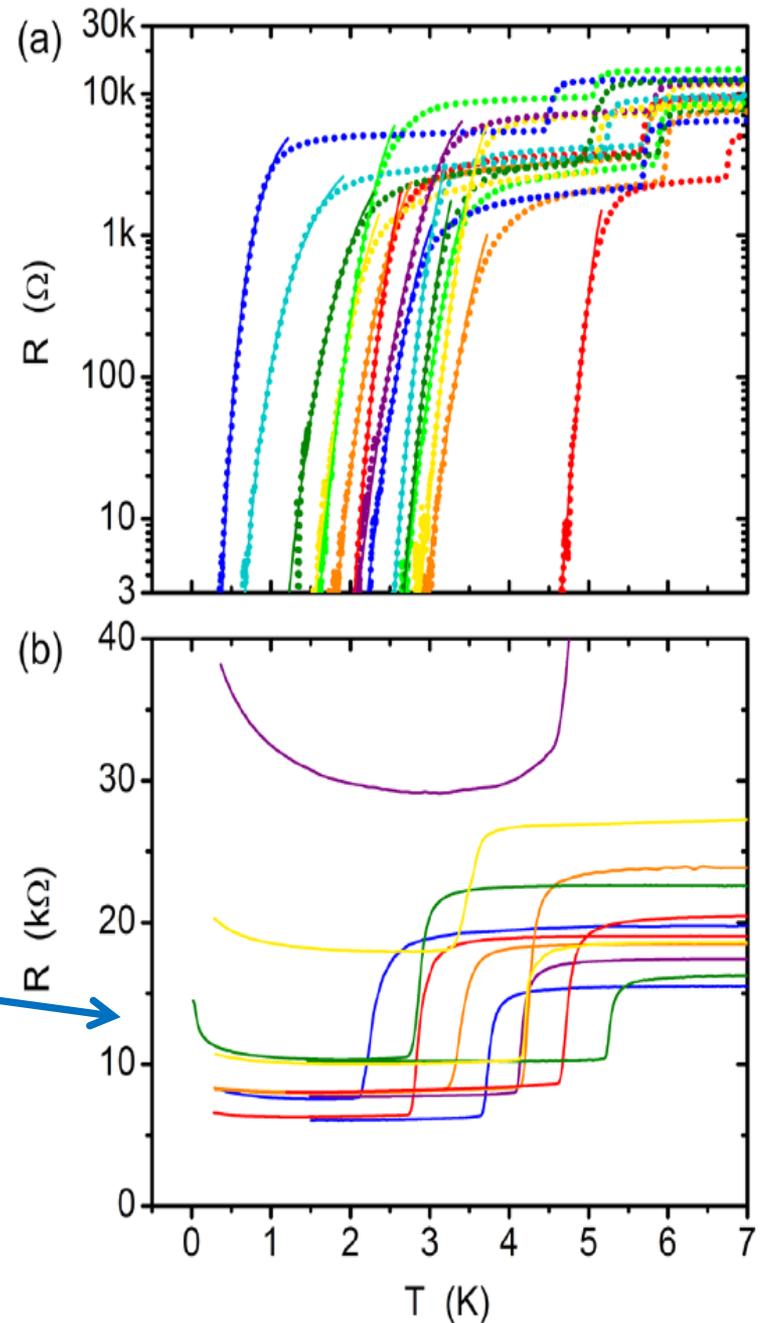
Dichotomy in nanowires: Evidence for superconductor- insulator transition (SIT)

$$R=V/I \quad I \sim 3 \text{ nA}$$

The difference between samples is the amount of the deposited Mo₇₉Ge₂₁.

$$R_{\text{sheet}} = 100 - 400 \ \Omega$$

Can the insulating behavior be due to Anderson localization of the BCS condensate?



Bollinger, Dinsmore, Rogachev, Bezryadin,
Phys. Rev. Lett. **101**, 227003 (2008)



Linearity of the Schrödinger's equation



Suppose Ψ_1 is a valid solution of the Schrödinger equation:

$$i\hbar \frac{\partial \psi_1}{\partial t} = \frac{\partial^2 \psi_1}{\partial x^2} + U(x)\psi_1$$

And suppose that Ψ_2 is another valid solution of the Schrödinger equation:

$$i\hbar \frac{\partial \psi_2}{\partial t} = \frac{\partial^2 \psi_2}{\partial x^2} + U(x)\psi_2$$

Then $(\Psi_1 + \Psi_2)/\sqrt{2}$ is also a valid solution, because:

$$i\hbar \frac{\partial (\psi_1 + \psi_2)}{\partial t} = \frac{\partial^2 (\psi_1 + \psi_2)}{\partial x^2} + U(x)(\psi_1 + \psi_2)$$

The state $(\Psi_1 + \Psi_2)/\sqrt{2}$ is a new combined state which is called “quantum superposition” of state (1) and (2)

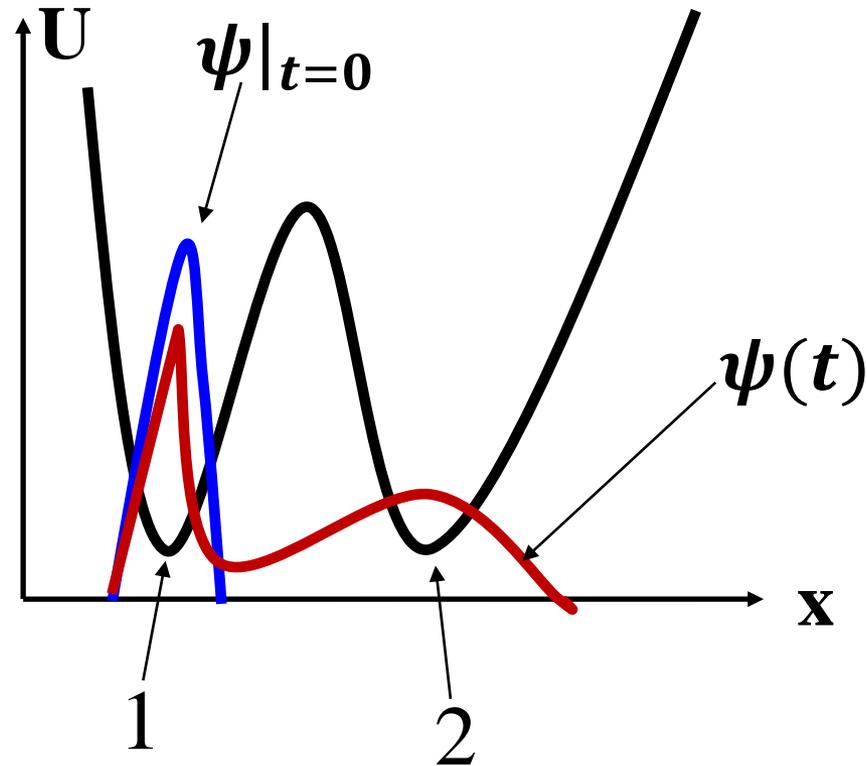


Quantum tunneling



George Gamow

(He also developed
Big Bang theory)

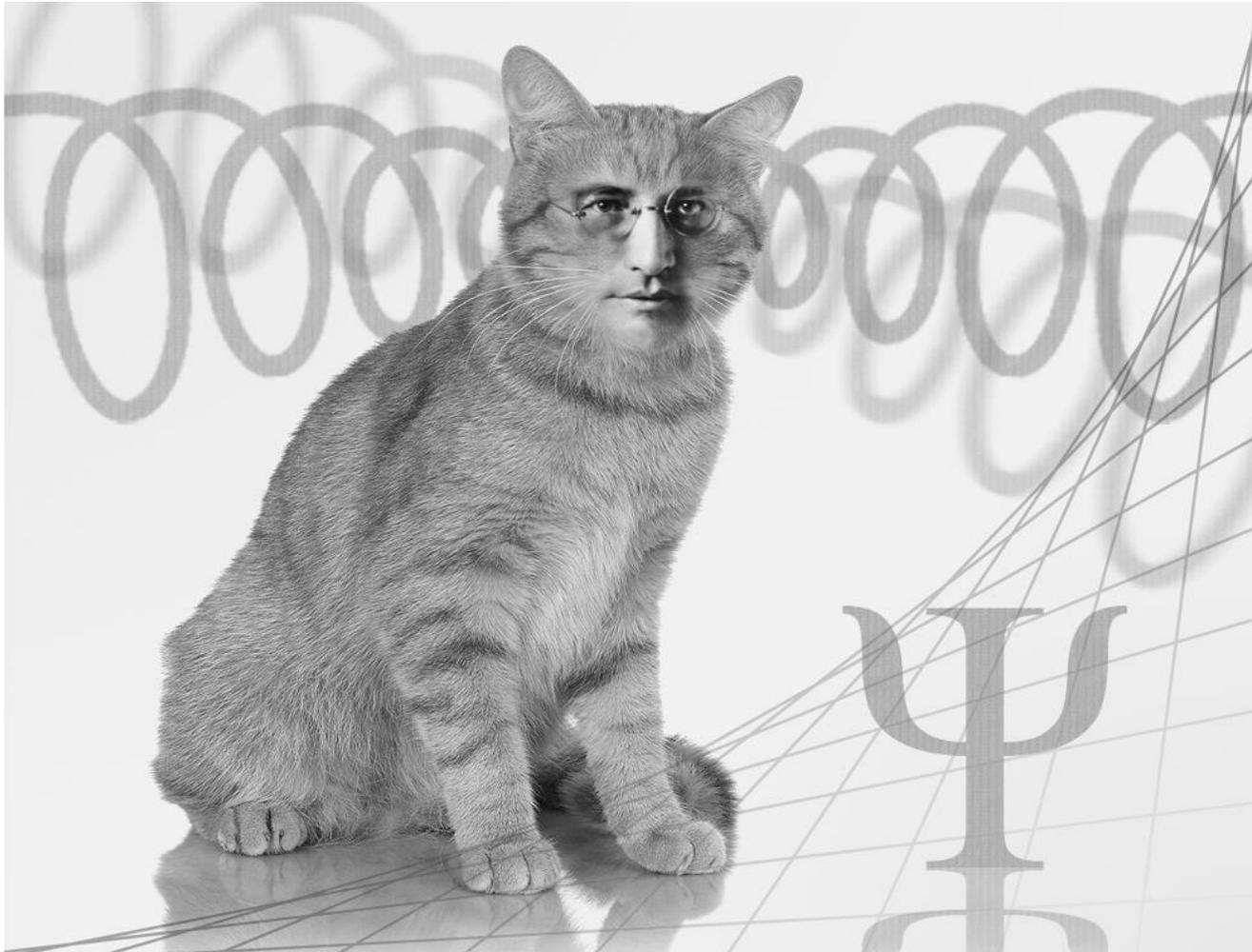


**Quantum tunneling is possible
since quantum superpositions of
states are possible.**

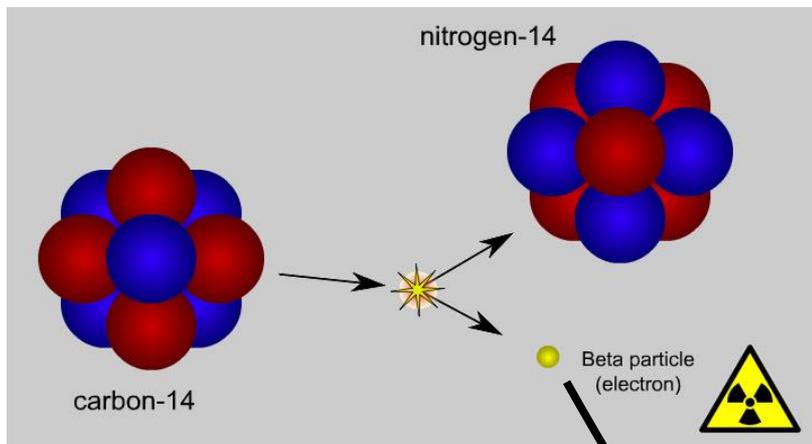


Schrödinger cat – the ultimate macroscopic quantum phenomenon

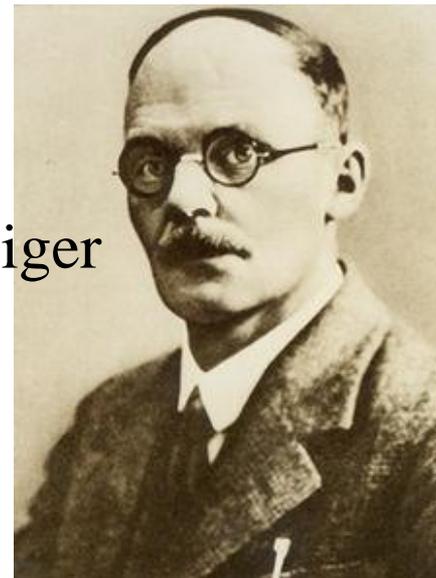
E. Schrödinger, Naturwiss. **23** (1935), 807.



Schrödinger cat – thought experiment



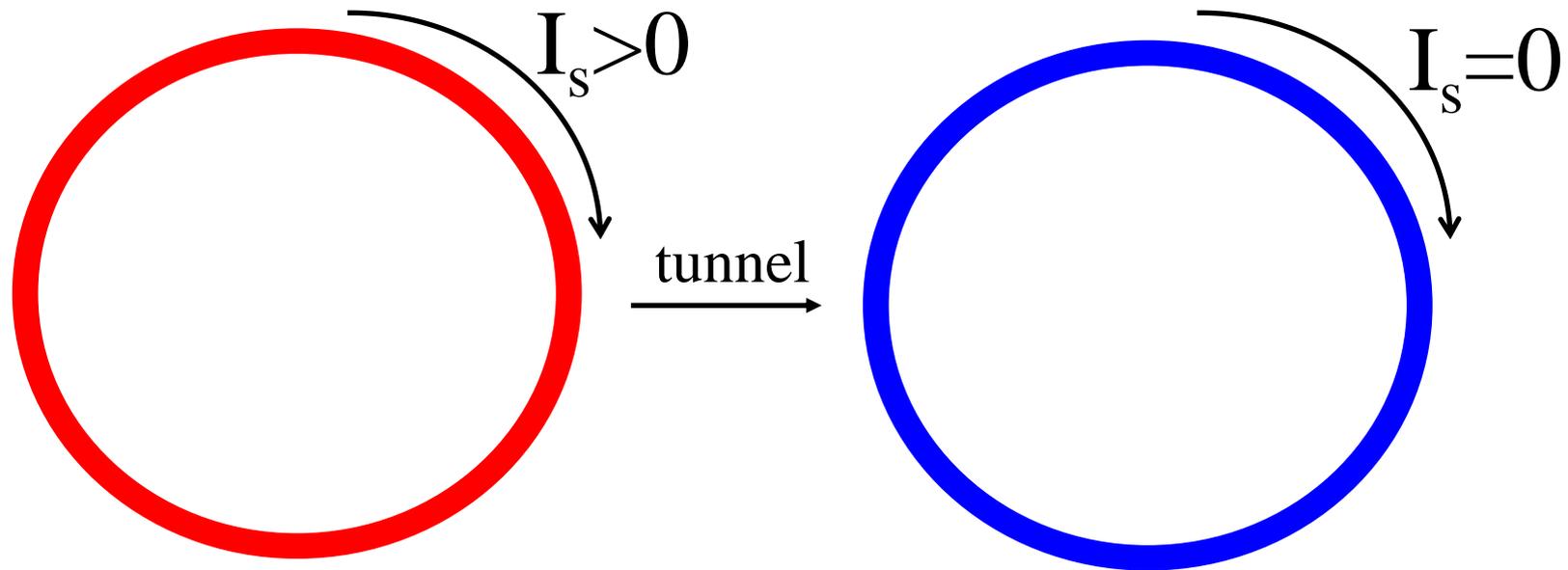
Hans Geiger



Geiger counter



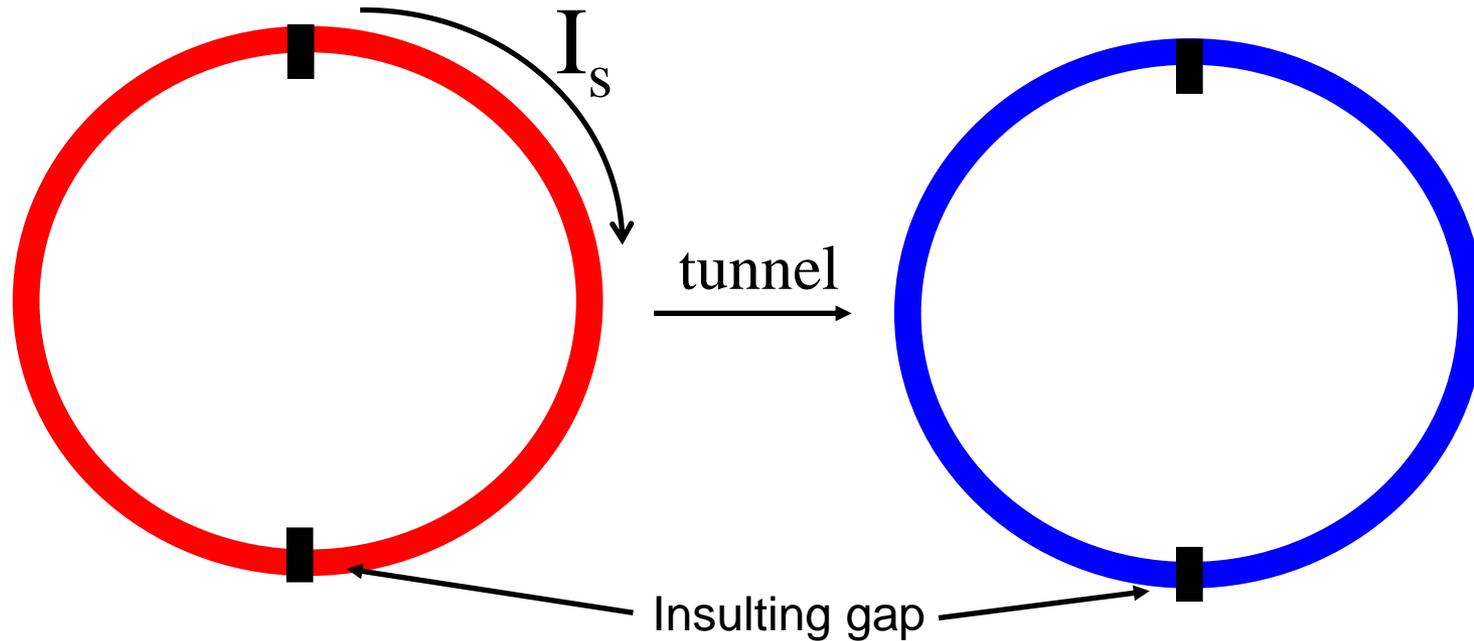
What sort of tunneling we will consider?



- Red color represents some strong current in the superconducting wire loop
- Blue color represents no current or a much smaller current in the loop



Previous results relate loops with insulating interruptions (SQUIDS)



- Red color represents some strong current in the superconducting loop
- Blue color represents no current or very little current in the superconducting loop



Leggett's prediction for macroscopic quantum tunneling (MQT) in SQUIDs

80

Supplement of the Progress of Theoretical Physics, No. 69, 1980

Macroscopic Quantum Systems and the Quantum Theory of Measurement

A. J. LEGGETT

*School of Mathematical and Physical Sciences
University of Sussex, Brighton BN1 9QH*

(Received August 27, 1980)

It is this property which makes a SQUID the most promising candidate to date for observing macroscopic quantum tunnelling; if it should ever become possible to observe macroscopic quantum *coherence*, the low entropy and consequent lack of dissipation will be absolutely essential.²¹⁾



MQT report by Kurkijarvi and collaborators (1981)

VOLUME 47, NUMBER 9

PHYSICAL REVIEW LETTERS

31 AUGUST 1981

Decay of the Zero-Voltage State in Small-Area, High-Current-Density Josephson Junctions

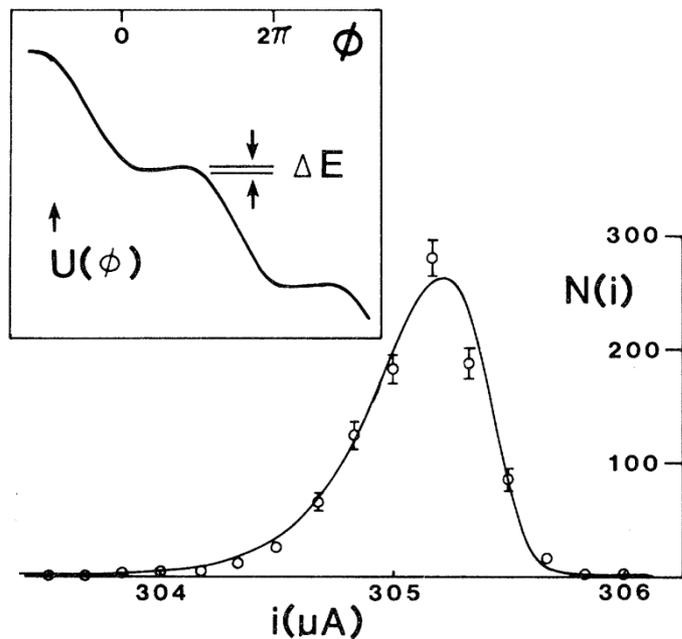


FIG. 1. Measured distribution for $T = 1.6$ K for small high-current-density junction. The solid line is a fit by the CL theory for $R = 20 \Omega$, $C = 8$ fF, and $i_{\text{CFF}} = 310.5 \mu\text{A}$. The inset is $U(\phi)$ for $x = 0.8$ with barrier ΔE .

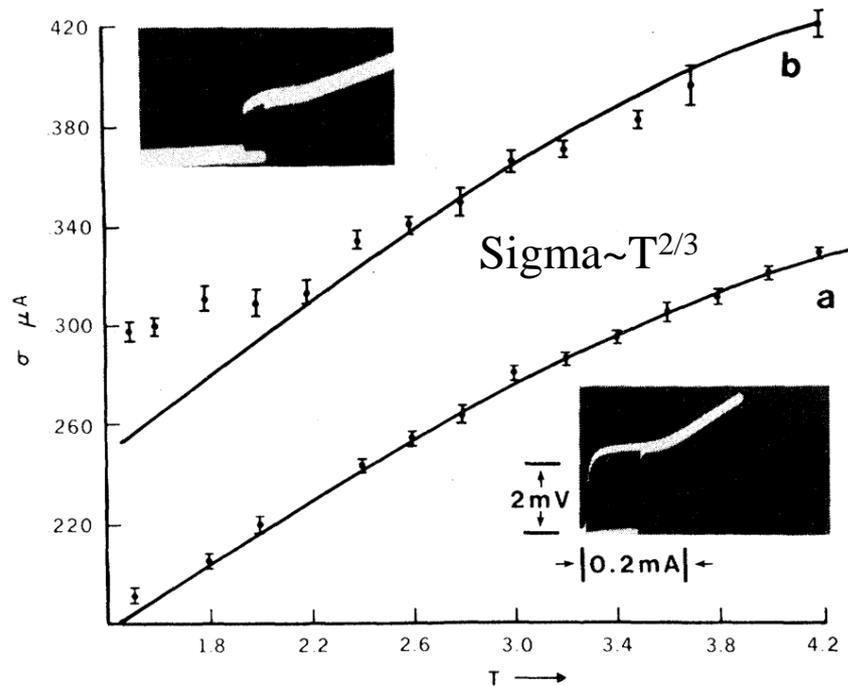
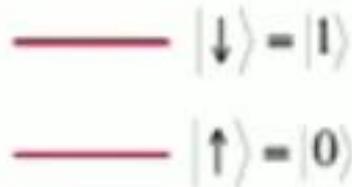


FIG. 2. Measured distribution widths σ vs T for two junctions with current sweep of $\sim 400 \mu\text{A}/\text{sec}$. Curve a is lower current density junction data and curve b is higher density junction data. The traces adjacent to the plots are the corresponding I - V characteristics at 4.2 K. The scales are the same for both traces.



TM

Types of Qubit



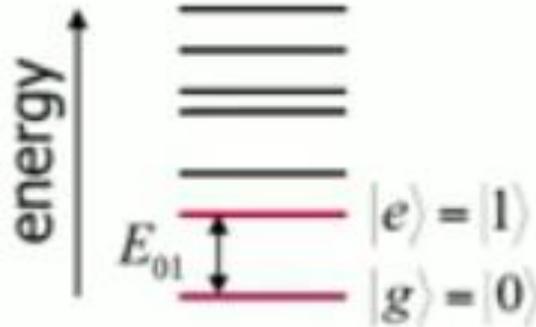
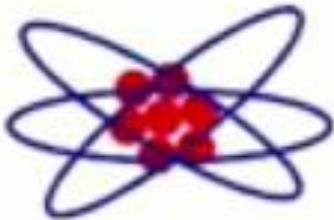
single spin-1/2

Quantum state:

$$|\psi\rangle = A^*|0\rangle + B^*|1\rangle$$

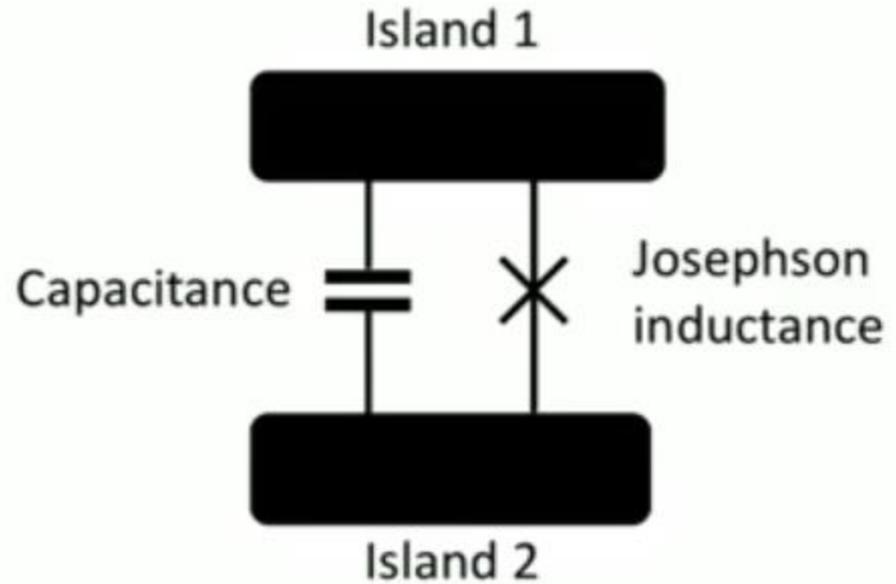
$$A^2 + B^2 = 1$$

A and B are
complex numbers



single atom

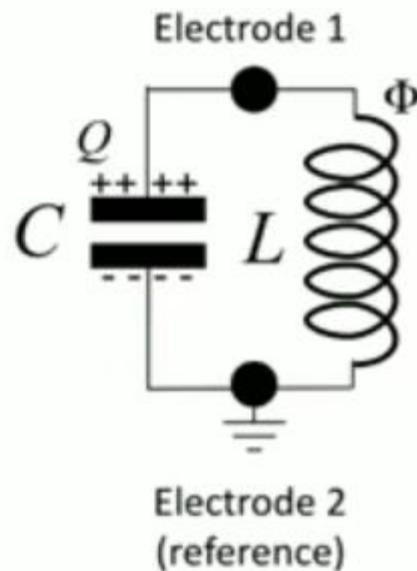
Transmon Qubit



Theory of transmons: J. Koch et al., Phys. Rev. A **76**, 042319 (2007).

Quantization of electrical circuits

The quantized LC oscillator



Hamiltonian:

$$\hat{H}_{LC} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L}$$

Capacitive term Inductive term

Canonically conjugate variables:

$\hat{\Phi}$ = Flux through the inductor.

\hat{Q} = Charge on capacitor plate.

$$[\hat{\Phi}, \hat{Q}] = i\hbar$$

Discrete energy spectrum of the LC-circuit

Correspondence with simple harmonic oscillator

$$\hat{H}_{\text{LC}} = \frac{\hat{\Phi}^2}{2L} + \frac{\hat{Q}^2}{2C}$$

$$[\hat{\Phi}, \hat{Q}] = i\hbar$$

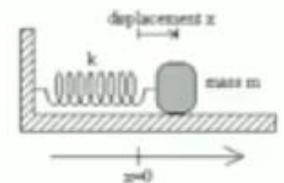
$$\hat{H}_{\text{SHO}} = \frac{k\hat{X}^2}{2} + \frac{\hat{P}^2}{2m}$$

$$[\hat{X}, \hat{P}] = i\hbar$$

Correspondence:

$$\begin{aligned} \hat{\Phi} &\leftrightarrow \hat{X} & L &\leftrightarrow \frac{1}{k} \\ \hat{Q} &\leftrightarrow \hat{P} & C &\leftrightarrow m \end{aligned}$$

$$\omega = \frac{1}{\sqrt{LC}} \leftrightarrow \sqrt{\frac{k}{m}}$$



Solve using ladder operators:

$$\hat{a} = \left(\frac{\hat{Q}}{Q_{\text{zpf}}} - i \frac{\hat{\Phi}}{\Phi_{\text{zpf}}} \right)$$

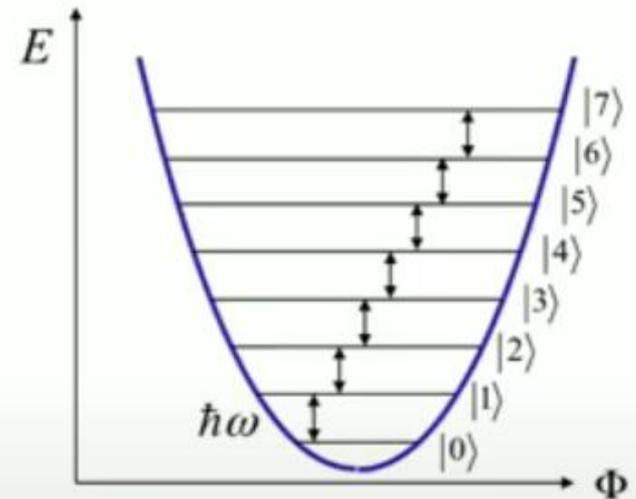
$$\Phi_{\text{zpf}} = \sqrt{2\hbar Z}$$

$$Q_{\text{zpf}} = \sqrt{2\hbar / Z}$$

$$\hat{a}^\dagger = \left(\frac{\hat{Q}}{Q_{\text{zpf}}} + i \frac{\hat{\Phi}}{\Phi_{\text{zpf}}} \right)$$

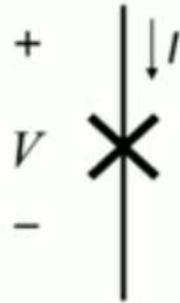
$$Z = \omega L = \frac{1}{\omega C} = \sqrt{\frac{L}{C}}$$

$$\hat{H}_{\text{LC}} = \hbar\omega \left(\hat{a}^\dagger \hat{a} + \frac{1}{2} \right) \quad [\hat{a}_r, \hat{a}_r^\dagger] = 1$$



Non-harmonicicity is the key factor

The Josephson junction



$$I = I_c \sin\left(2\pi \frac{\Phi}{\Phi_0}\right)$$

$$\Phi_0 = \frac{h}{2e}$$

flux quantum

$$V = \dot{\Phi}$$

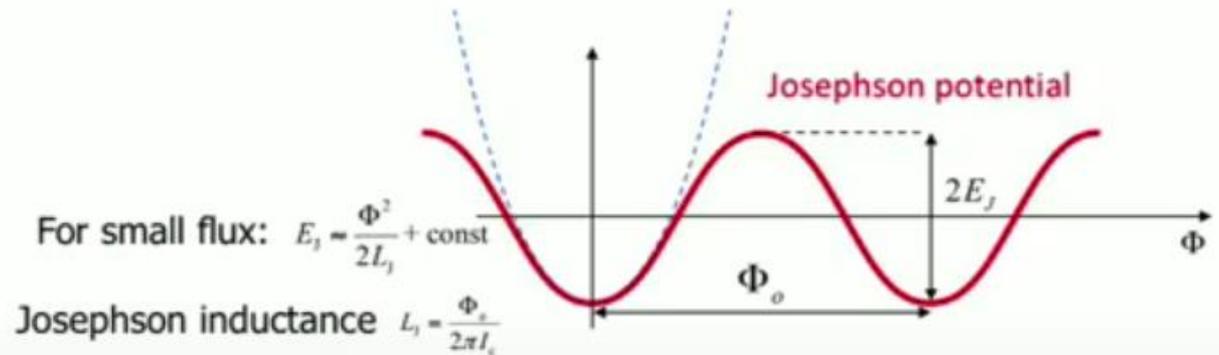


S superconductor-
I insulator-
S superconductor
tunnel junction

$$I_c = \frac{\pi \Delta}{2e R}$$

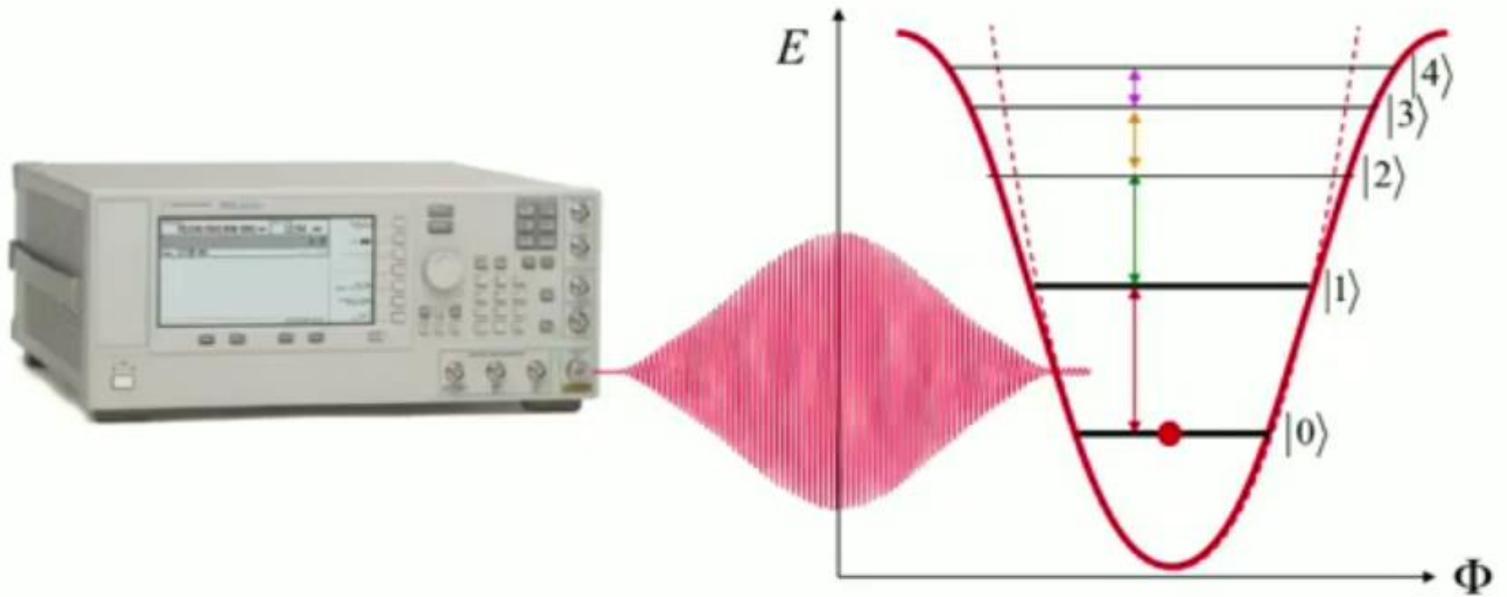
$$E_{\text{stored}} = E_J \left(1 - \cos\left(2\pi \frac{\Phi}{\Phi_0}\right)\right)$$

$$E_J = \frac{I_c \Phi_0}{2\pi} \quad \text{Josephson Energy}$$

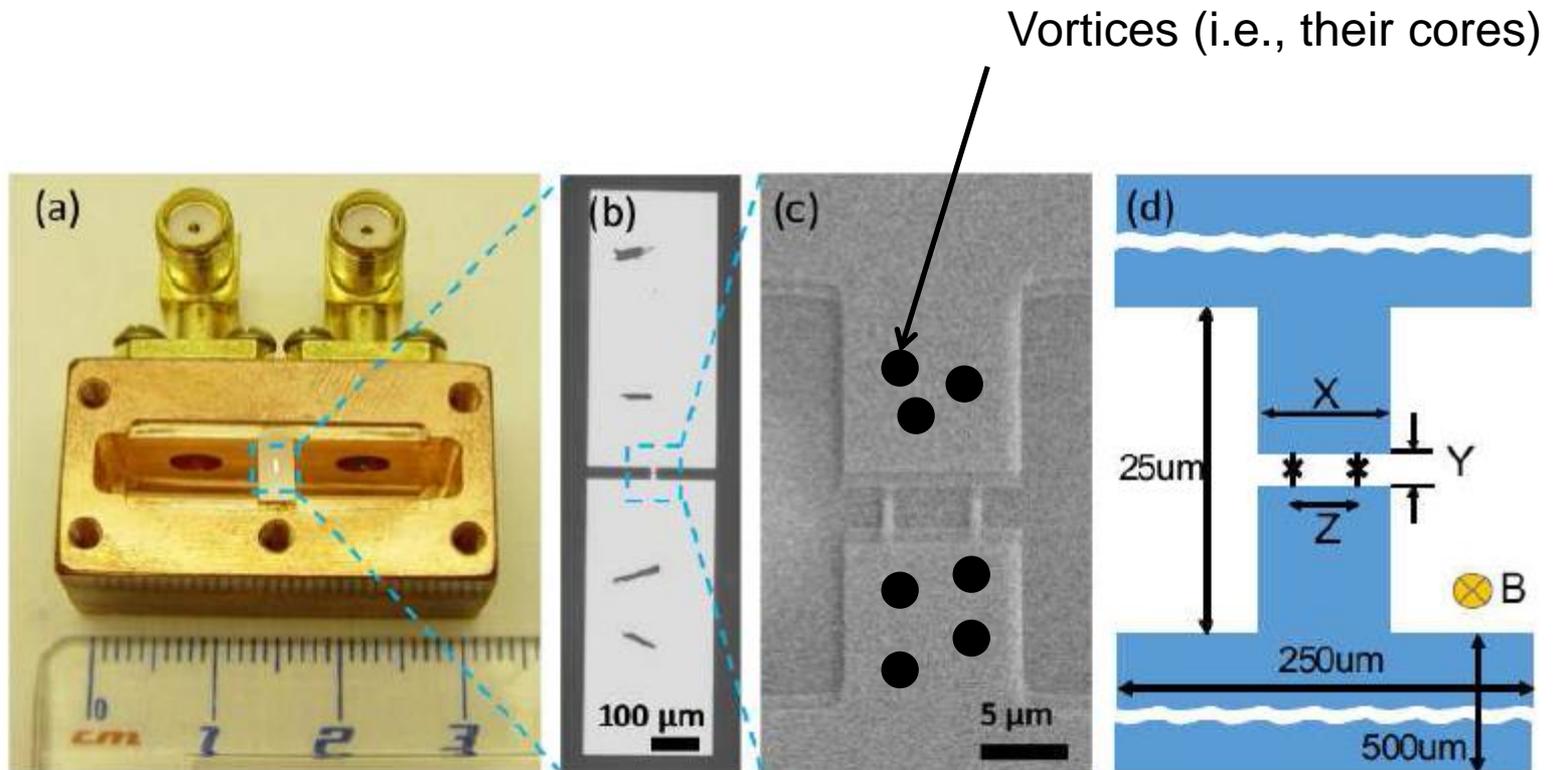


Non-harmonicicity is the key factor

Transmon energy spectrum

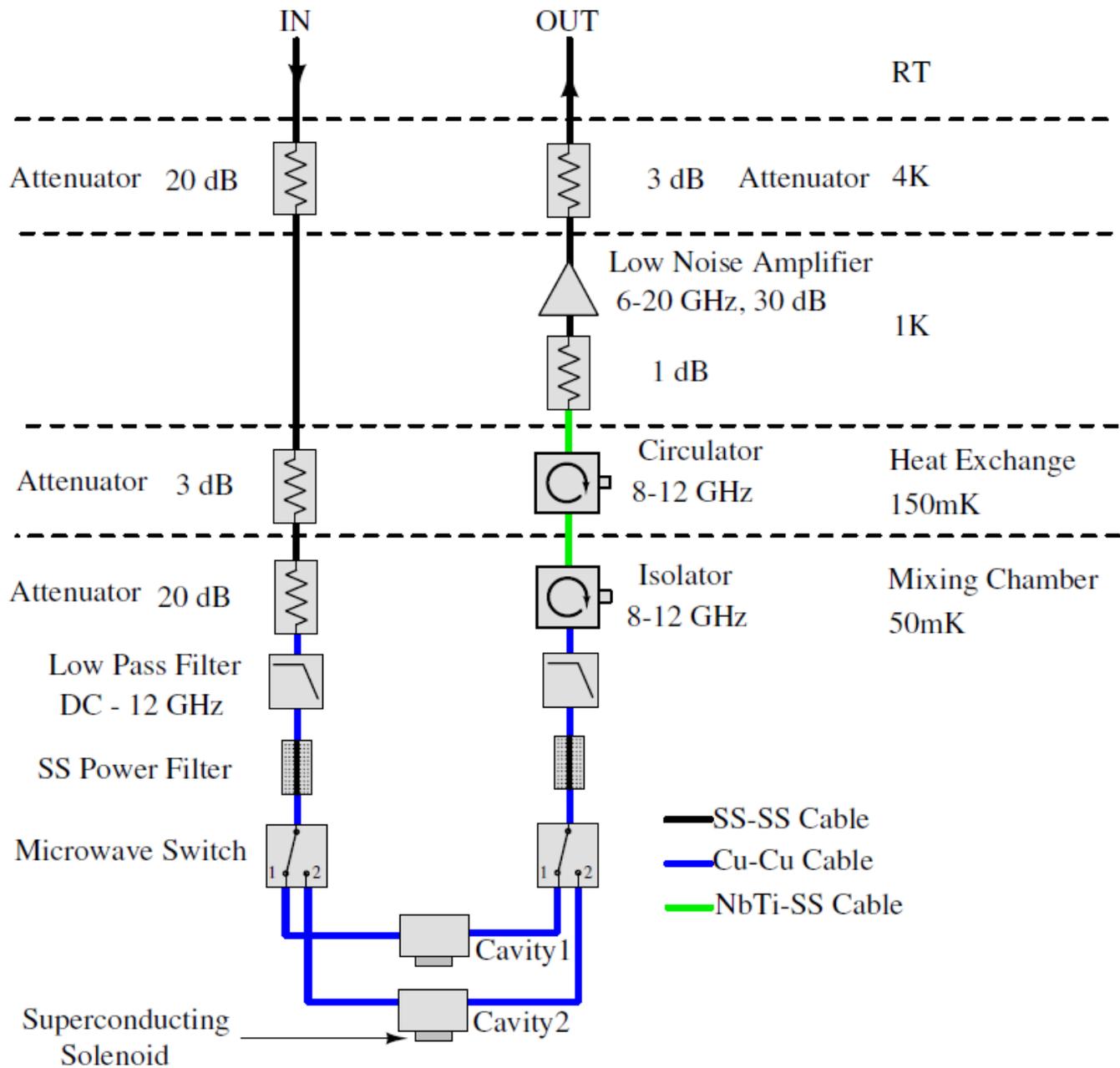


How to measure superconducting qubits

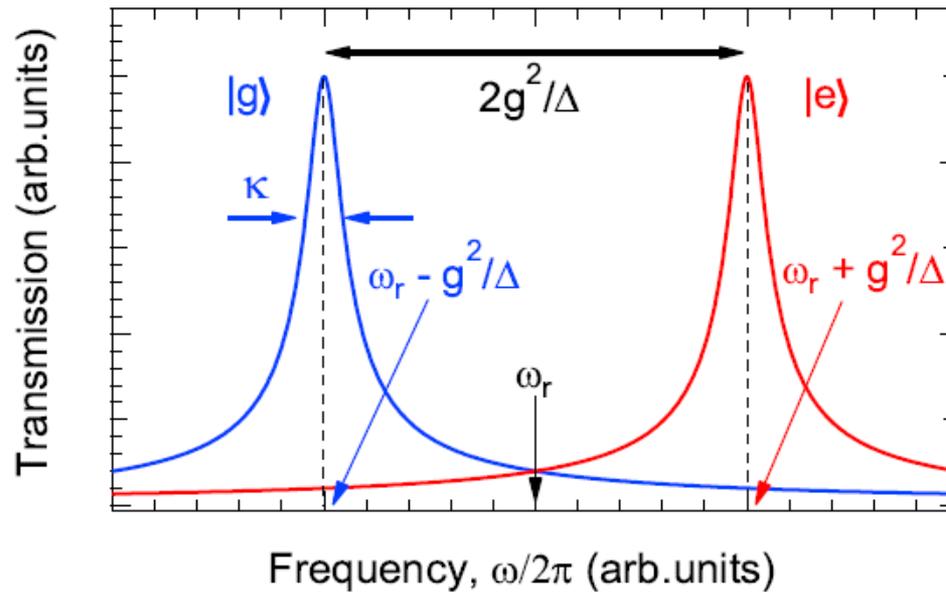


J. Ku, Z. Yoscovits, A. Levchenko, J. Eckstein, and A. Bezryadin,
Decoherence and radiation-free relaxation in Meissner transmon qubit coupled to Abrikosov vortices,
Physical Review B **94**, 165128(1-14) (2016).

How transmons
are
measured

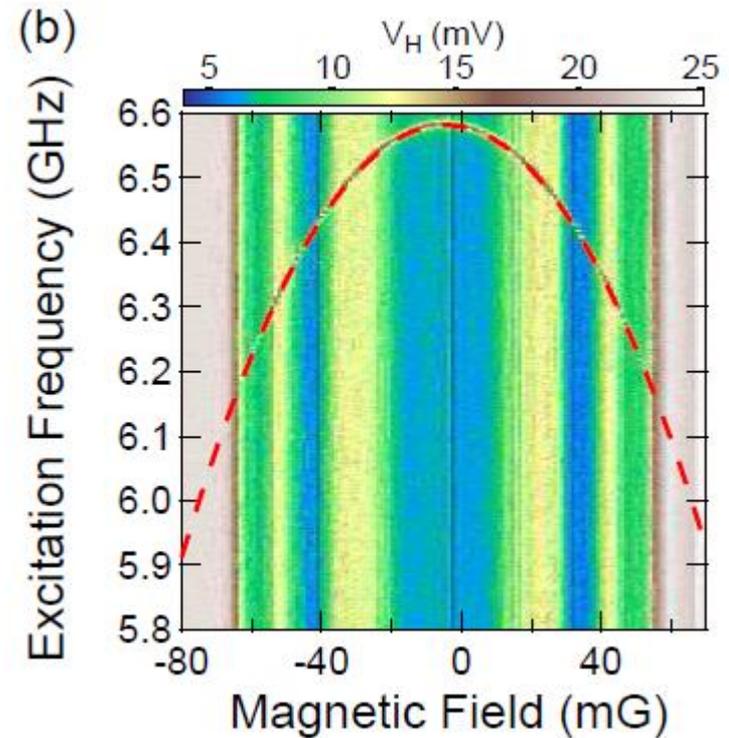
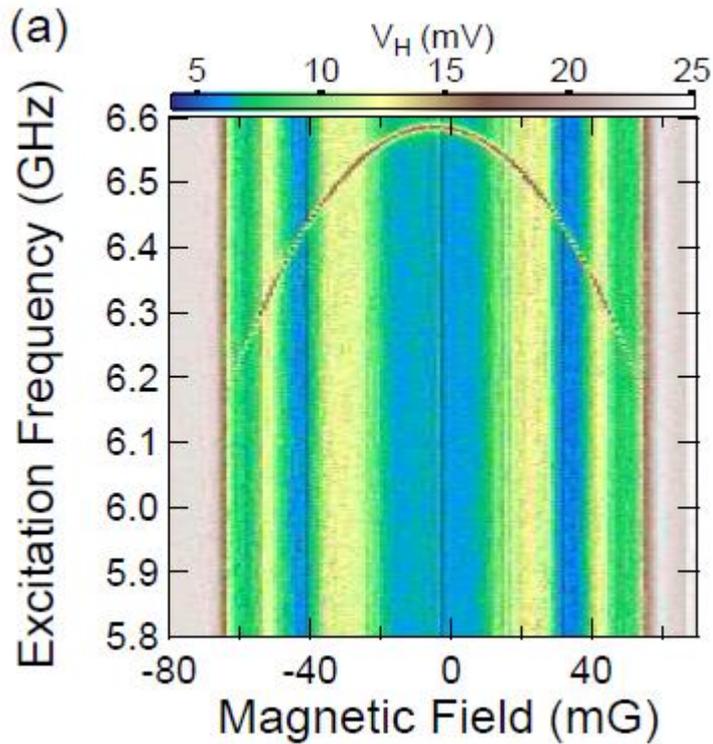


How transmons are measured

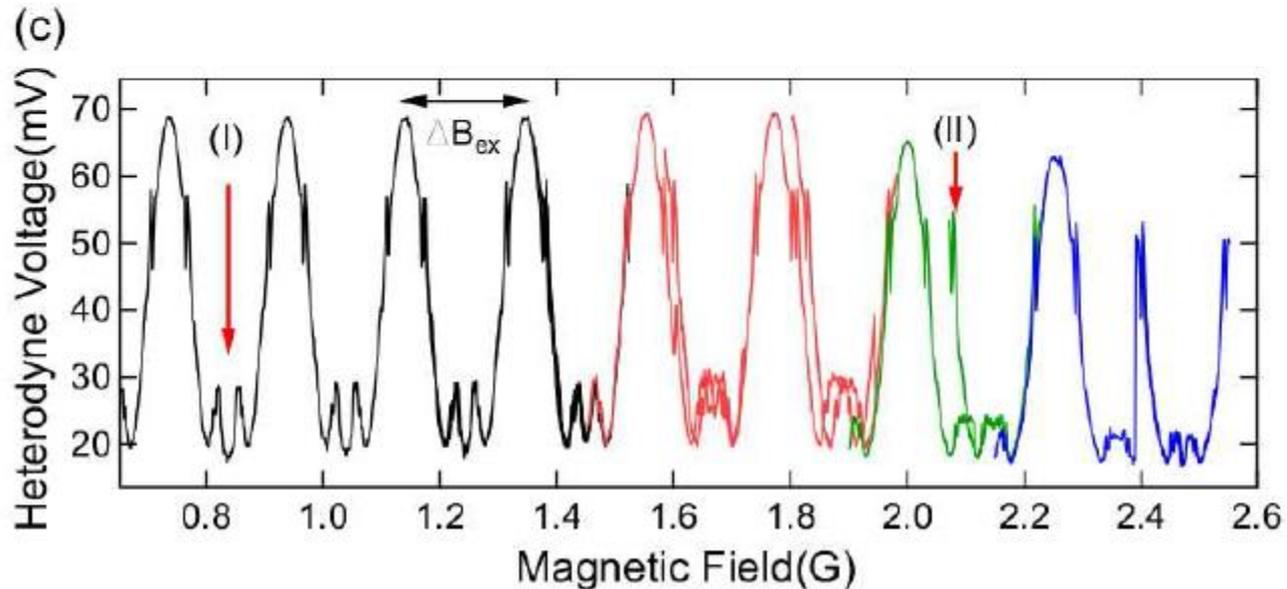
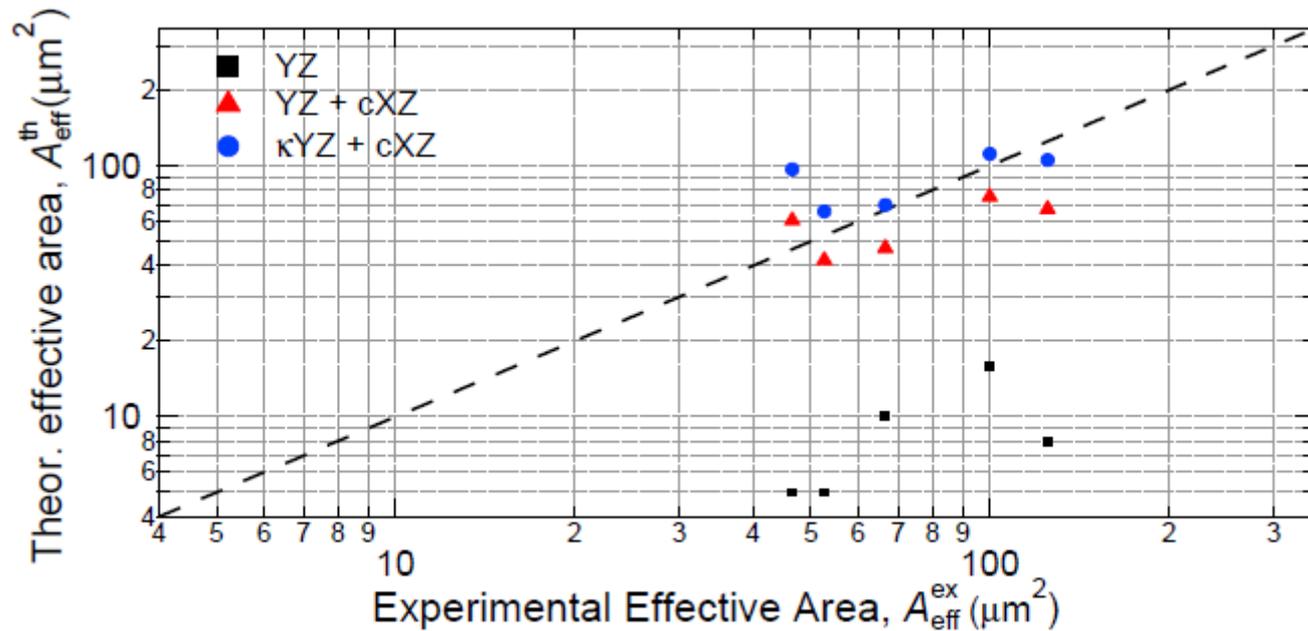


Transmission versus frequency for dispersive measurement. ω_r denotes the resonant frequency without the dispersive shift. Depending on the qubit state, the cavity frequency is pulled by $\pm g^2/\Delta$.

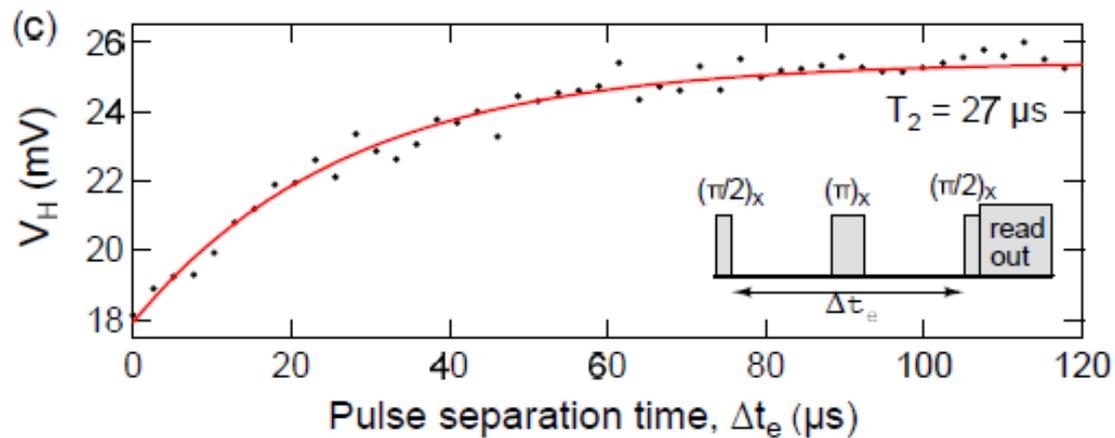
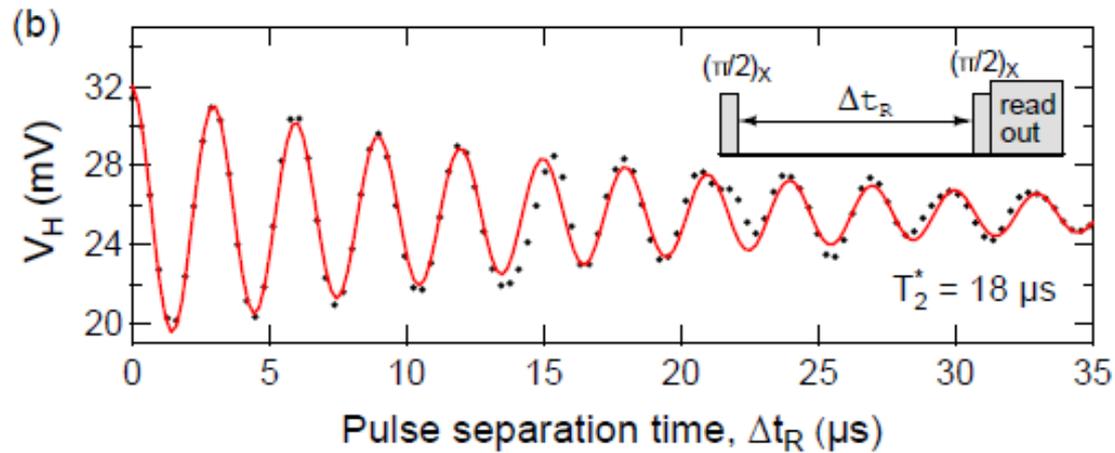
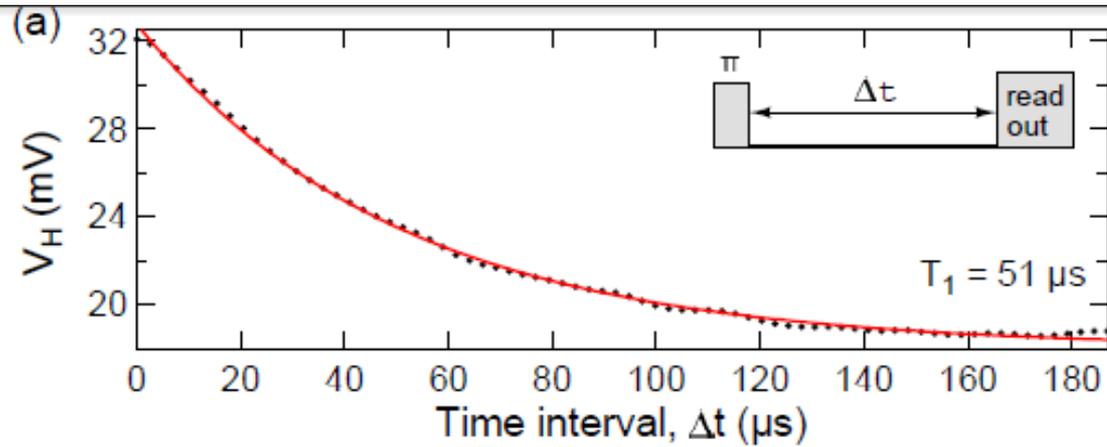
Magnetic field effects



Magnetic field effects

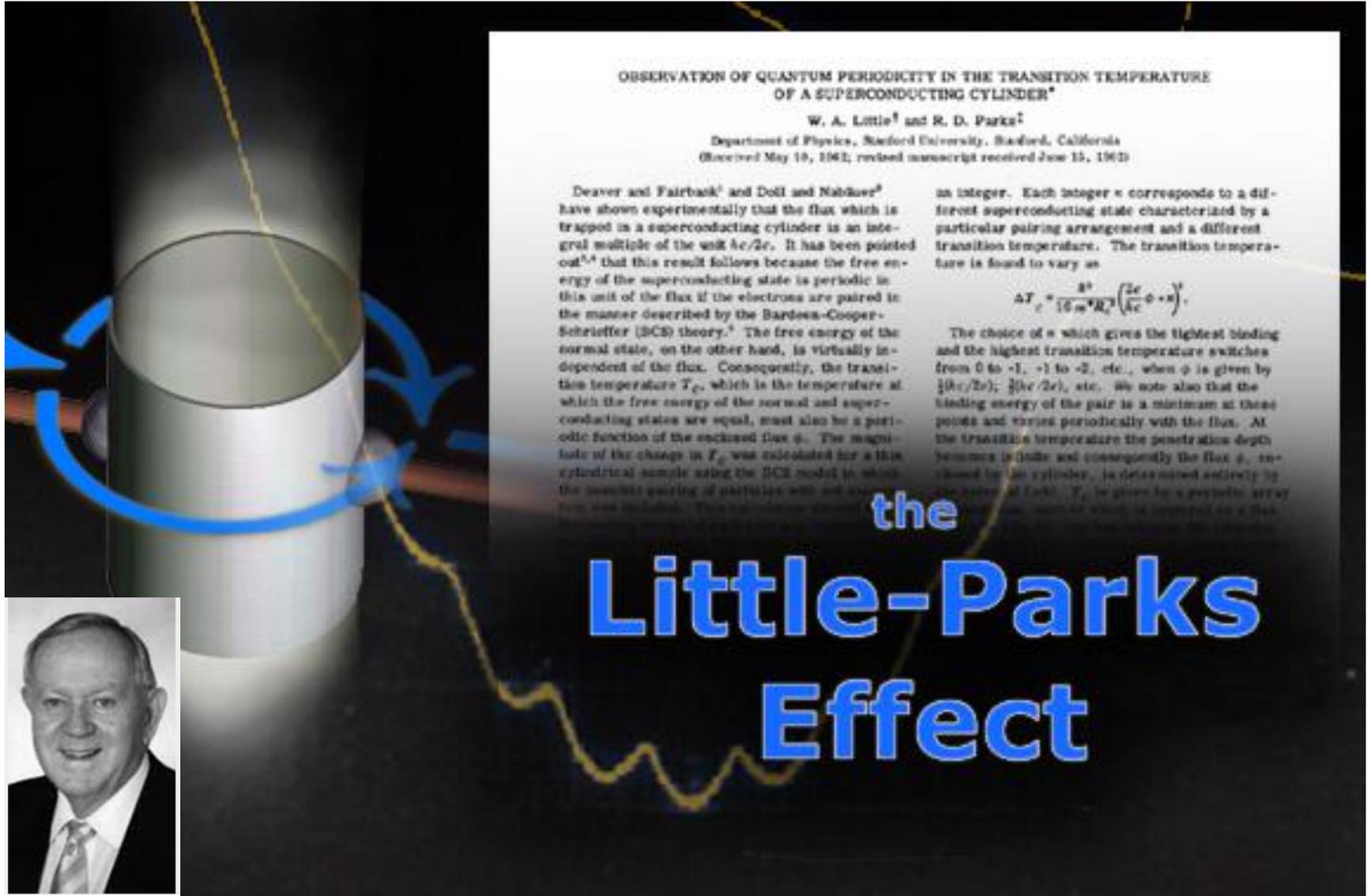


Examples quantum
time-domain oscillation:
Ramsey fringe



Searching for an explanation: Little-Parks effect ('62)

The basic idea: magnetic field induces non-zero vector-potential, which produces non-zero superfluid velocity, thus reducing the T_c .



OBSERVATION OF QUANTUM PERIODICITY IN THE TRANSITION TEMPERATURE OF A SUPERCONDUCTING CYLINDER*

W. A. Little[†] and R. D. Parks[‡]

Department of Physics, Stanford University, Stanford, California
(Received May 19, 1962; revised manuscript received June 15, 1962)

Deaver and Fairbank¹ and Doll and Nabauer² have shown experimentally that the flux which is trapped in a superconducting cylinder is an integral multiple of the unit $hc/2e$. It has been pointed out^{3,4} that this result follows because the free energy of the superconducting state is periodic in this unit of the flux if the electrons are paired in the manner described by the Bardeen-Cooper-Schrieffer (BCS) theory.⁵ The free energy of the normal state, on the other hand, is virtually independent of the flux. Consequently, the transition temperature T_c , which is the temperature at which the free energy of the normal and superconducting states are equal, must also be a periodic function of the enclosed flux ϕ . The magnitude of the change in T_c was calculated for a thin cylindrical sample using the BCS model in which the possible pairing of particles with net spin zero was included. The calculation showed that T_c is an integer. Each integer n corresponds to a different superconducting state characterized by a particular pairing arrangement and a different transition temperature. The transition temperature is found to vary as

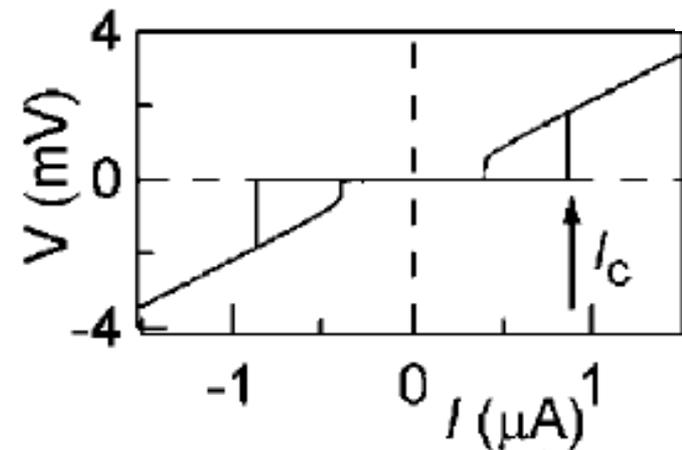
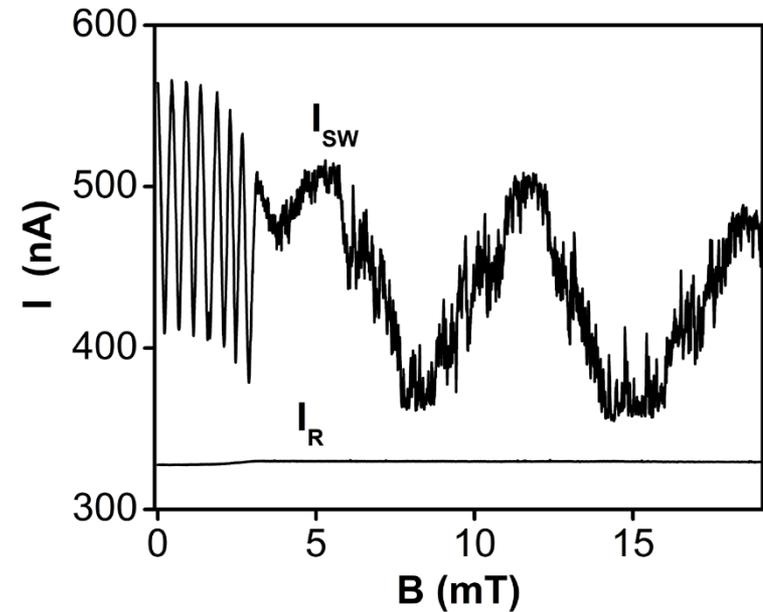
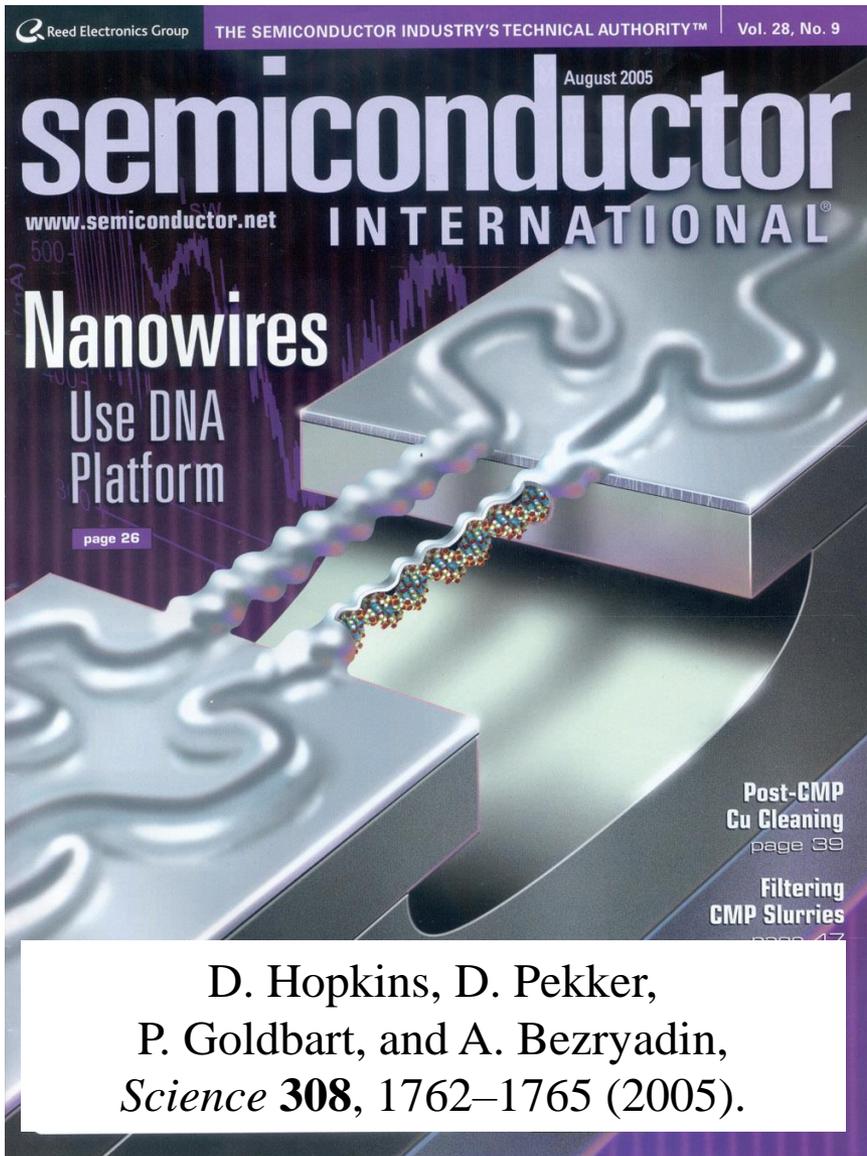
$$\Delta T_c = \frac{8^3}{15\pi^2 R^2} \left(\frac{2e}{hc} \phi + n \right)^2.$$

The choice of n which gives the tightest binding and the highest transition temperature switches from 0 to -1, -1 to -2, etc., when ϕ is given by $\frac{1}{2}(hc/2e)$; $\frac{3}{2}(hc/2e)$, etc. We note also that the binding energy of the pair is a minimum at these points and varies periodically with the flux. At the transition temperature the penetration depth becomes infinite and consequently the flux ϕ , enclosed by the cylinder, is determined entirely by the external field. T_c is given by a periodic array of peaks and valleys, the period of which is constant at a flux

the
**Little-Parks
Effect**



Phase gradiometers templated by DNA



SQUID – superconducting quantum interference device

SQUID helmet project at Los Alamos



Magnetic field scales:

Earth field: $\sim 1\text{G}$

Fields inside animals:
 $\sim 0.01\text{G}-0.00001\text{G}$

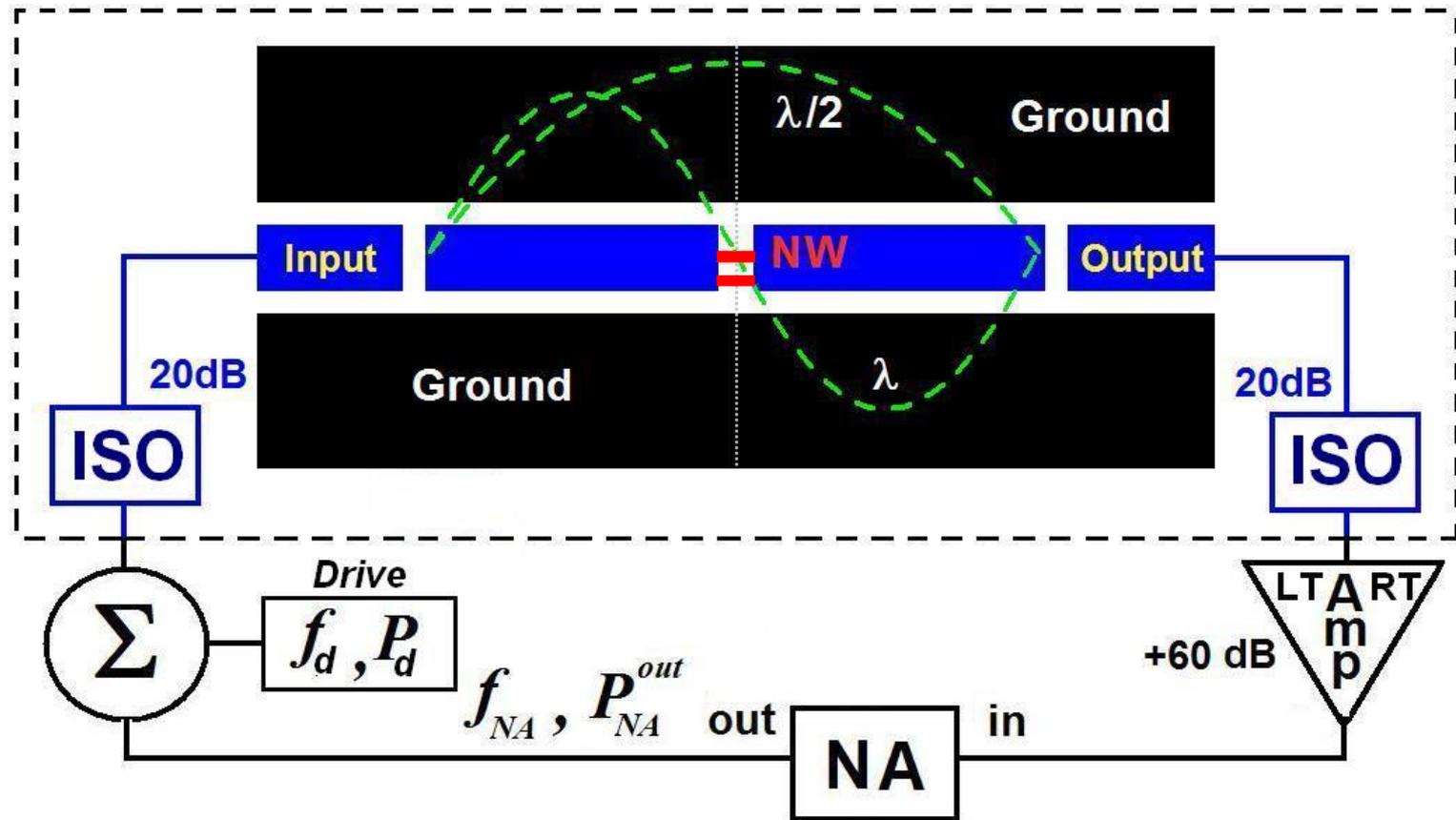
Fields on the **human brain**:
 $\sim 0.3\text{nG}$

This is less than a hundred-millionth of the Earth's magnetic field.

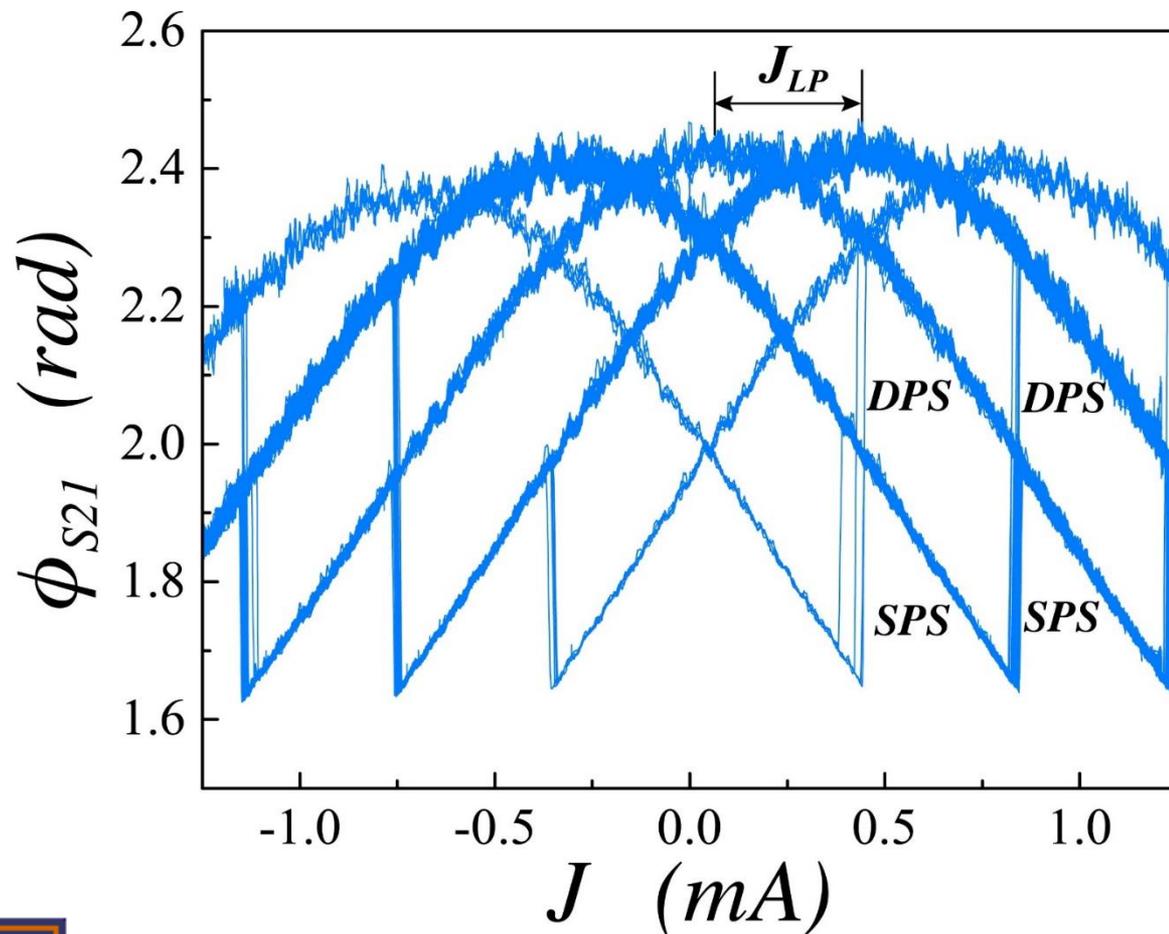
SQUIDs, or Superconducting Quantum Interference Devices, invented in 1964 by Robert Jaklevic, John Lambe, Arnold Silver, and James Mercereau of Ford Scientific Laboratories, are used to measure extremely small magnetic fields. They are currently the most sensitive magnetometers known, with the noise level as low as $3\text{ fT}\cdot\text{Hz}^{-1/2}$. While, for example, the Earth magnet field is only about 0.0001 Tesla, some electrical processes in animals produce very small magnetic fields, typically between 0.000001 Tesla and 0.000000001 Tesla. SQUIDs are especially well suited for studying magnetic fields this small.

Measuring the brain's magnetic fields is even much more difficult because just above the skull the strength of the magnetic field is only about 0.3 picoTesla (0.000000000000003 Tesla). This is less than a hundred-millionth of Earth's magnetic field. In fact, brain fields can be measured only with the most sensitive magnetic-field sensor, i.e. with the superconducting quantum interference device, or SQUID.

Measuring nanowires within GHz resonators. Detection of individual phase slips.



Resonators used to detect single phase slips (SPS) and double phase slips (DPS)



$T = 360$ mK
 $f = f_0(H=0)$



Discovery of the supercurrent, known now as proximity effect, in SNS junctions

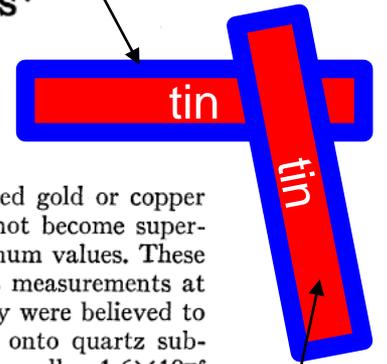
Superconductivity of Contacts with Interposed Barriers*

HANS MEISSNER†

Department of Physics, The Johns Hopkins University, Baltimore, Maryland

(Received August 25, 1959)

Non-superconductor (normal metal, i.e., Ag)



Resistance vs current diagrams and "Diagrams of State" have been obtained for 63 contacts between crossed wires of tin. The wires were plated with various thicknesses of the following metals: copper, silver, gold, chromium, iron, cobalt, nickel, and platinum. The contacts became superconducting, or showed a noticeable decrease of their resistance at lower temperatures if the plated films were not too thick. The limiting thicknesses were about 35×10^{-6} cm for Cu, Ag, and Au; 7.5×10^{-6} cm for Pt, 4×10^{-6} cm for Cr, and less than 2×10^{-6} cm for the ferromagnetic metals Fe, Co, and Ni. The investigation was extended to measurements of the resistance of contacts between crossed wires of copper or gold plated with various thicknesses of tin. Simultaneous measurements

of the (longitudinal) resistance of the tin-plated gold or copper wires showed that these thin films of tin do not become superconducting for thicknesses below certain minimum values. These latter findings are in agreement with previous measurements at Toronto. The measurements at Toronto usually were believed to be unreliable because films of tin evaporated onto quartz substrates can be superconducting at thicknesses as small as 1.6×10^{-6} cm. It is now believed that just as superconducting electrons can drift into an adjoining normal conducting layer and make it superconducting, normal electrons can drift into an adjoining superconducting layer and prevent superconductivity.

Superconductor (tin)

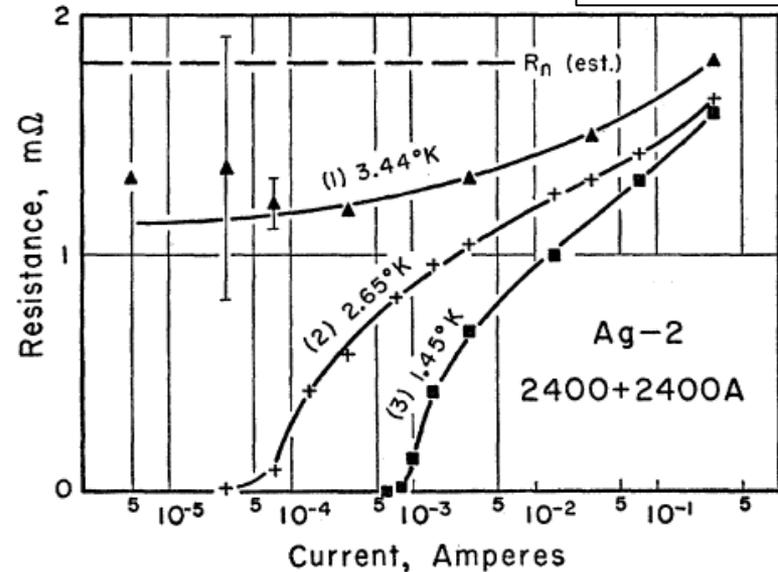
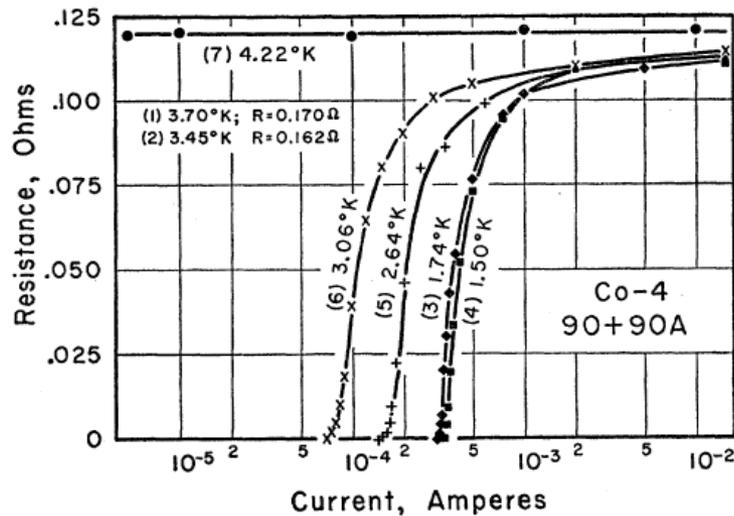
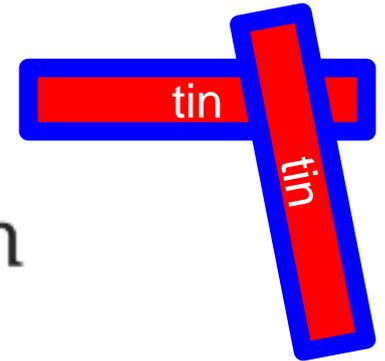


FIG. 1. Resistance vs current diagram of cobalt-plated contact Co 4, representative of diagrams type A.

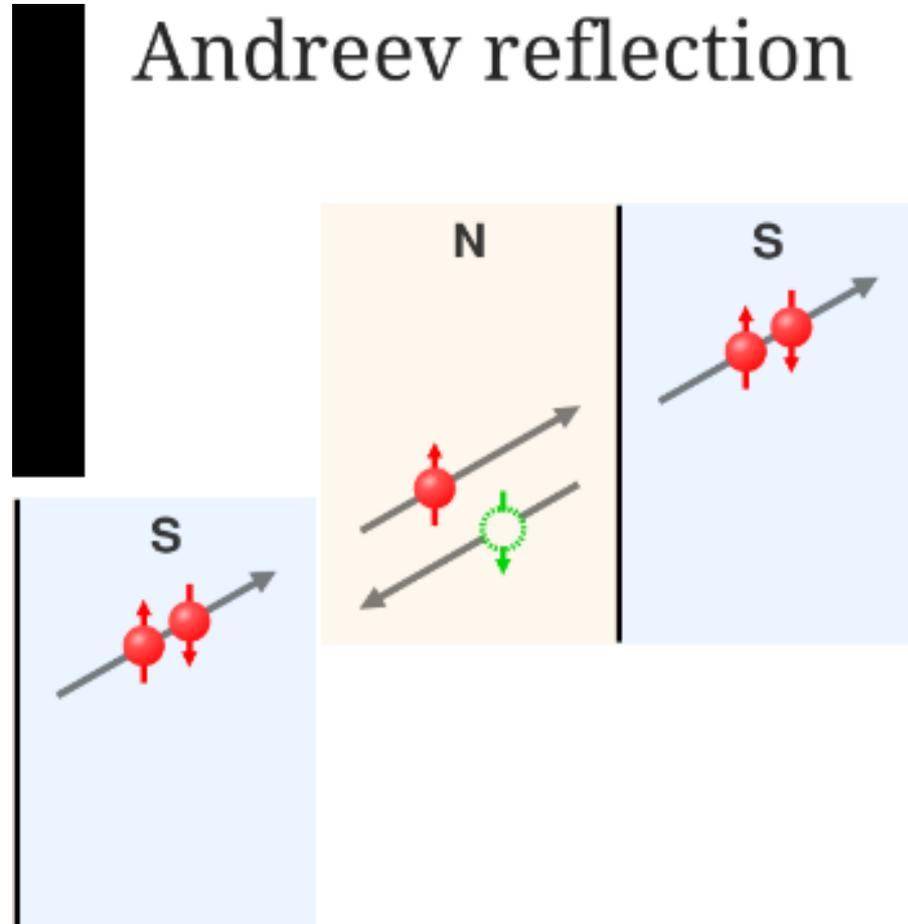
FIG. 2. Resistance vs current diagram of silver-plated contact Ag 2, representative of diagrams type B.



Explanation of the supercurrent in SNS junctions --- Andreev reflection



www.kapitza.ras.ru
www.kapitza.ras.ru/~andreev/afan...



A.F. Andreev, 1964



TM

Non-Abelian Majorana Modes in Vortices

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Non-Abelian Statistics of Half-Quantum Vortices in p -Wave Superconductors

D. A. Ivanov

Institut für Theoretische Physik, ETH-Hönggerberg, CH-8093 Zürich, Switzerland

(Received 17 May 2000)

Excitation spectrum of a half-quantum vortex in a p -wave superconductor contains a zero-energy Majorana fermion. This results in a degeneracy of the ground state of the system of several vortices. From the properties of the solutions to Bogoliubov–de Gennes equations in the vortex core we derive the non-Abelian statistics of vortices identical to that for the Moore-Read (Pfaffian) quantum Hall state.



FIG. 3. Elementary braid interchange of two vortices.

Total number of quantum states
for N pairs of such Majorana vortices is:

$$2^N$$

Example $N=100$

$$2^{100}=10^{30}$$

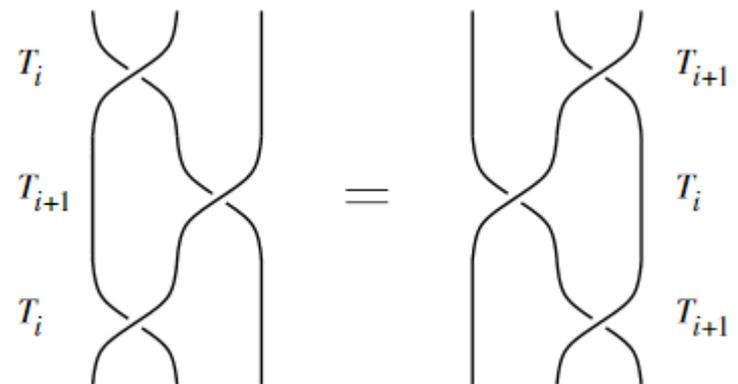
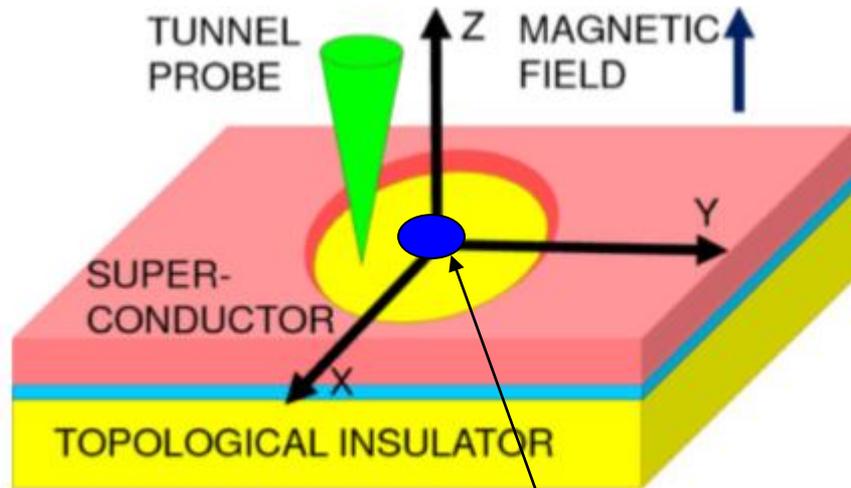


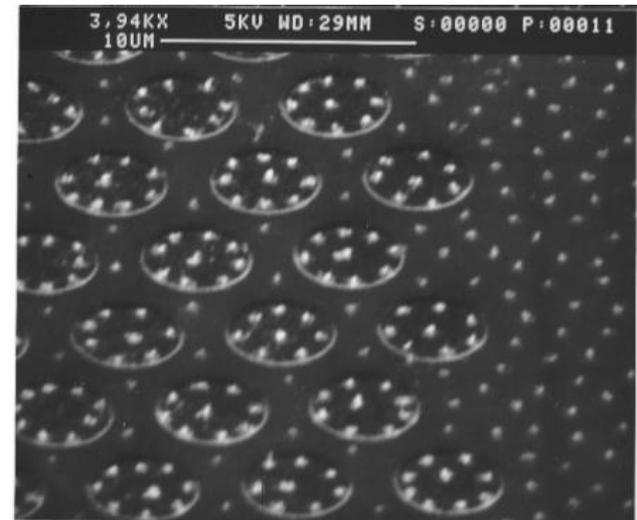
FIG. 2. Defining relation for the braid group: $T_i T_{i+1} T_i = T_{i+1} T_i T_{i+1}$.



Majorana modes in a vortex



Theory: Vortex in the nano-hole contains Majorana states (gap is about T_c)



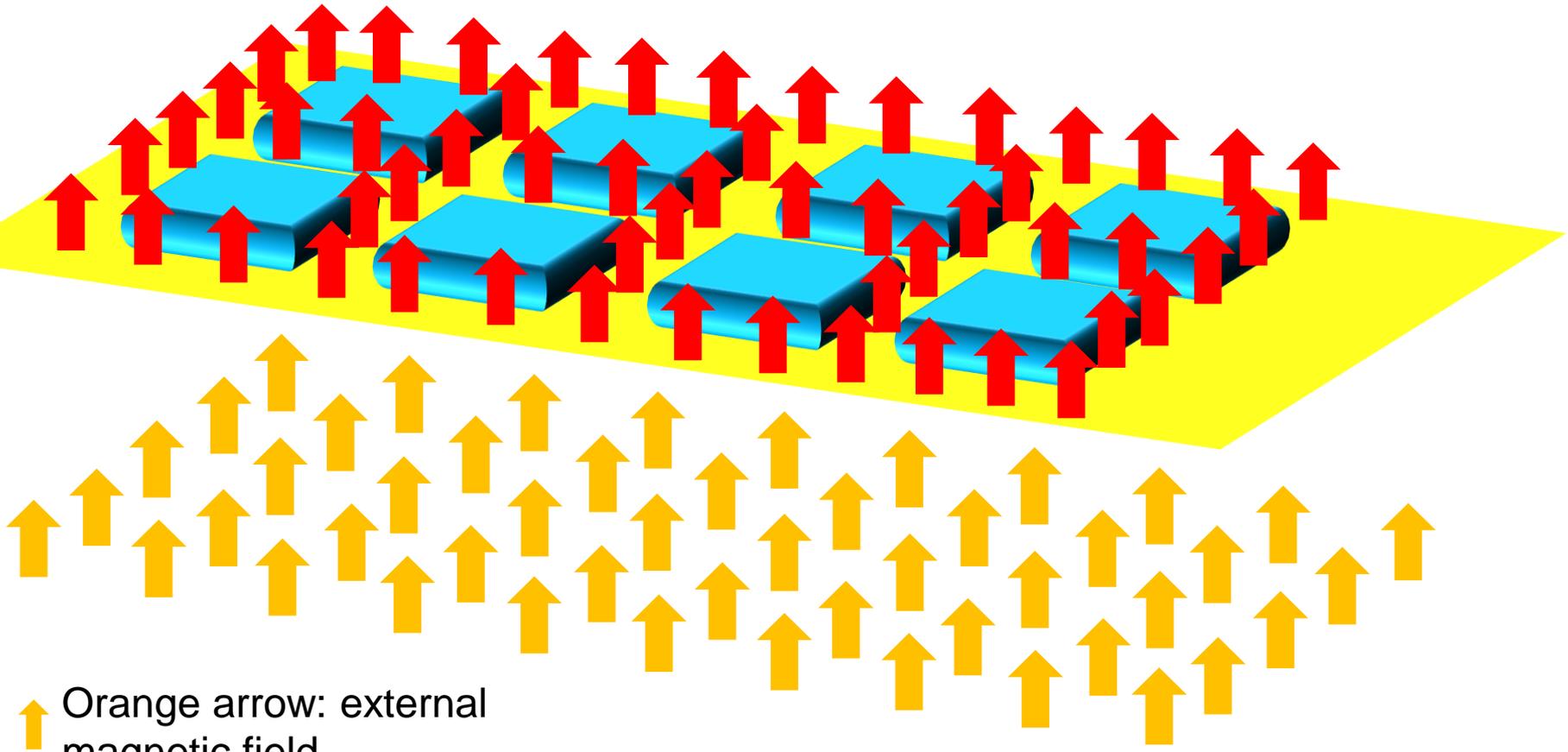
A. Bezryadin, Yu. Ovchinnikov, B. Pannetier, PRB 53, 8553 (1996)

R.S. Akzyanov, A.V. Rozhkov, A.L.Rakhmanov, and F. Nori, PRB 89, 085409 (2014)

PHYSICAL REVIEW B 84, 075141 (2011)



Schematic of the array (22x22 islands)



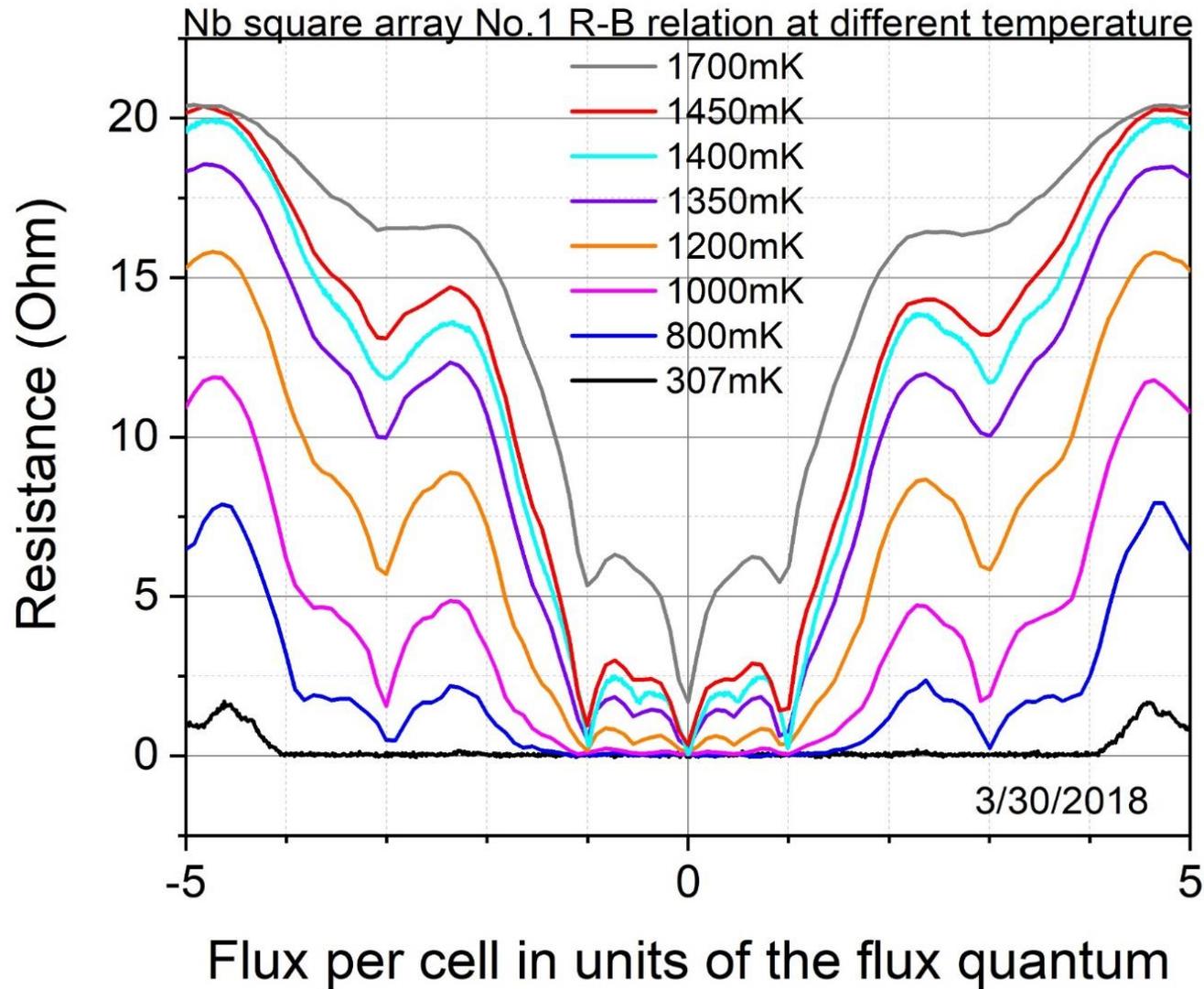
↑ Orange arrow: external magnetic field

↑ Red arrow: magnetic field that penetrates the array

Yellow film: Bi₂Se₃ topological insulator

Blue block: Niobium square island

Resistance of the sample as a function of magnetic field at 300 mK
(zoom into lower fields)



Useful Expression for the Free Energy of a Phase Slip

“Arrhenius-Little” formula for the wire resistance:

$$R_{AL} \approx R_N \exp[-\Delta F(T)/k_B T]$$

$$\Delta F = (8\sqrt{2}/3)(H_c(T)^2/8\pi)(A\xi(T))$$

$$\frac{\Delta F(0)}{k_B T_c} = \sqrt{6} \frac{\hbar I_c(0)}{2ek_B T_c} = 0.83 \frac{R_q L}{R_n \xi(0)} = 0.83 \frac{R_q}{R_{\xi(0)}}$$

Quantum limit to phase coherence in thin superconducting wires

M. Tinkham^{a)} and C. N. Lau

Physics Department, Harvard University, Cambridge, Massachusetts 02138



Conclusions

- Superconductivity allows us to test fundamental quantum phenomena: quantum coherence, quantum superposition, macroscopic quantum tunneling.
- Superconductors have been used to create strong and stable magnetic fields, in levitating trains for example.
- Superconducting quantum interference devices enabled researches to measure very small magnetic fields, such as those produced by human brain.
- Superconductors are used to build qubits, which are the building blocks of quantum computers.



Tony Bollinger's sample-mounting procedure in winter in Urbana

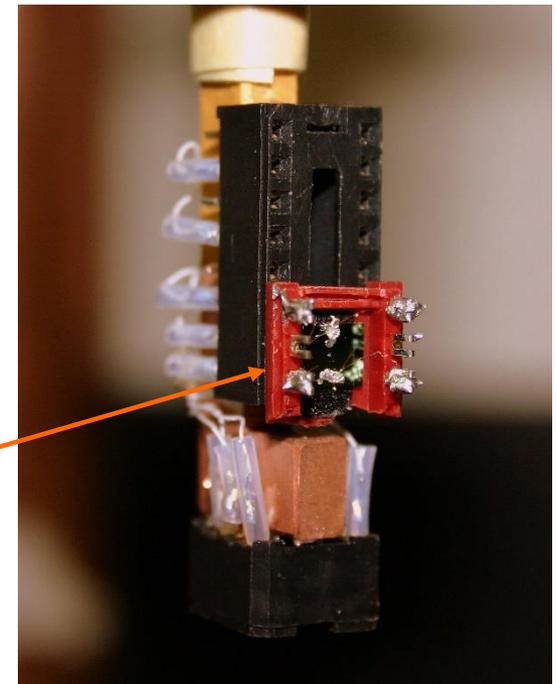
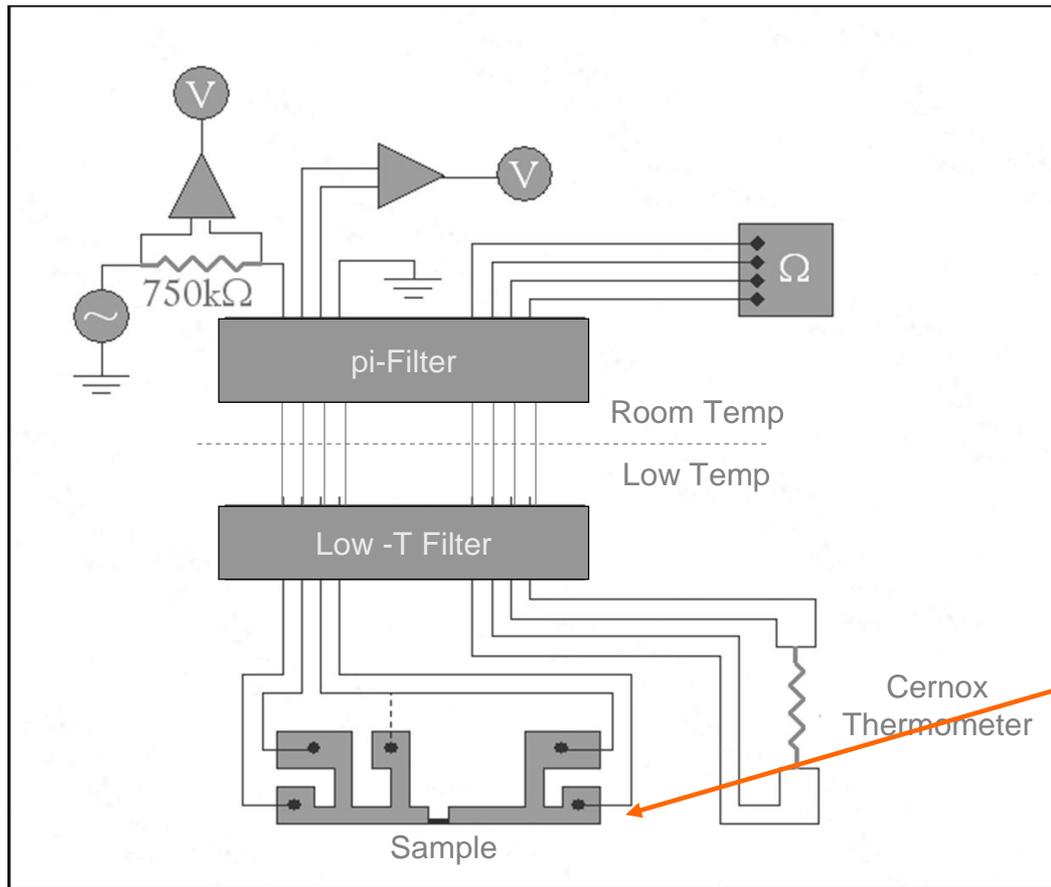


Procedure (~75% Success)

- Put on gloves
- Put anti static wrist strap
- Put grounded socket for mounting in vise with grounded indium dot tool connected
- Spray high-backed black chair all over and about 1 m square meter of ground with anti-static spray
 - DO NOT use green chair
 - Not sure about short-backed black chairs
- Sit down
- Spray bottom of feet with anti-static spray
- Plant feet on the ground. ***Do not move your feet again for any reason until mounting is finished.***
- Mount sample
- Keep sample in grounded socket until last possible moment
- Test samples in dipstick at ~1 nA



Measurement Scheme



Circuit Diagram

Sample mounted on the ^3He insert.



Possible Origin of Quantum Phase Slips

