Superconductivity in pictures

Alexey Bezryadin

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University of Illinois at Urbana-Champaign

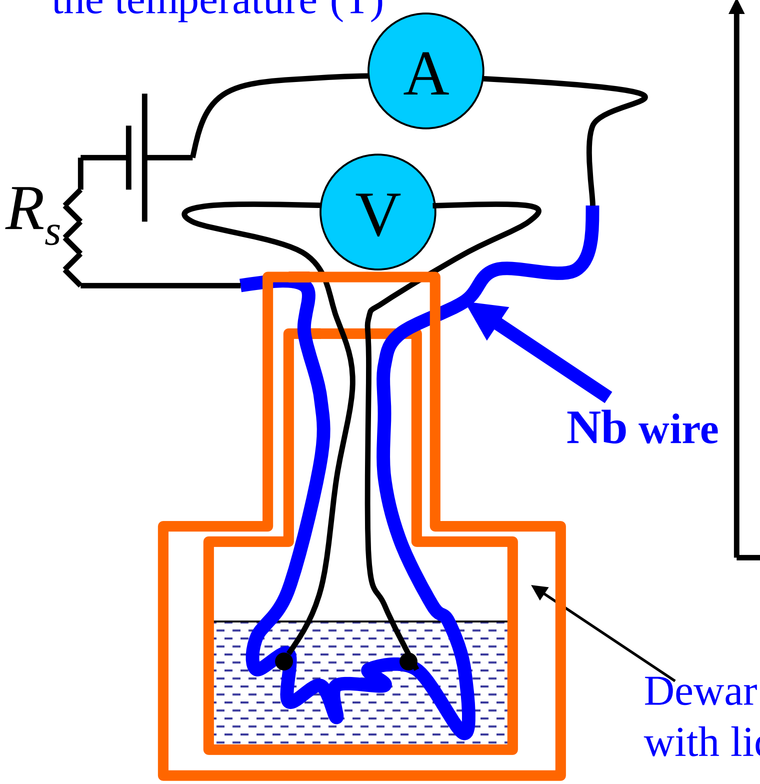
How to measure superconductivity

Electrical resistance of some metals drops to zero below a certain temperature which is called "critical temperature“ (H. K. O. 1911)

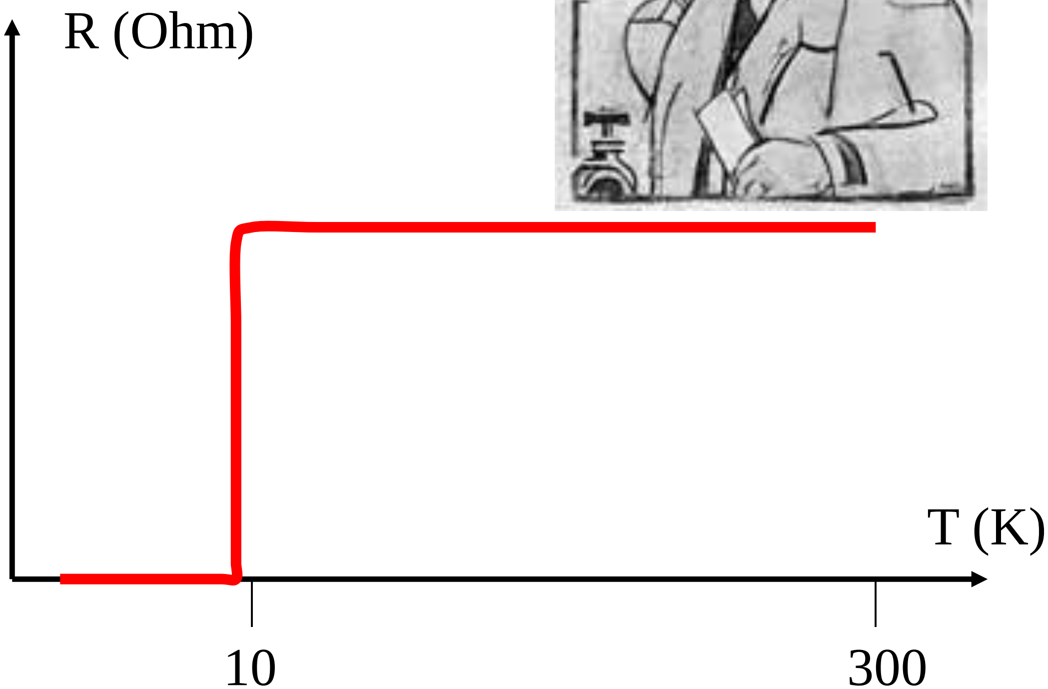
How to observe superconductivity

1. Take Nb (niobium) wire
2. Connect to a voltmeter and a current source
3. Immerse into helium Dewar (T=4.2 K boiling point)
4. Measure electrical resistance (R) versus the temperature (T)

Heike Kamerling Onnes



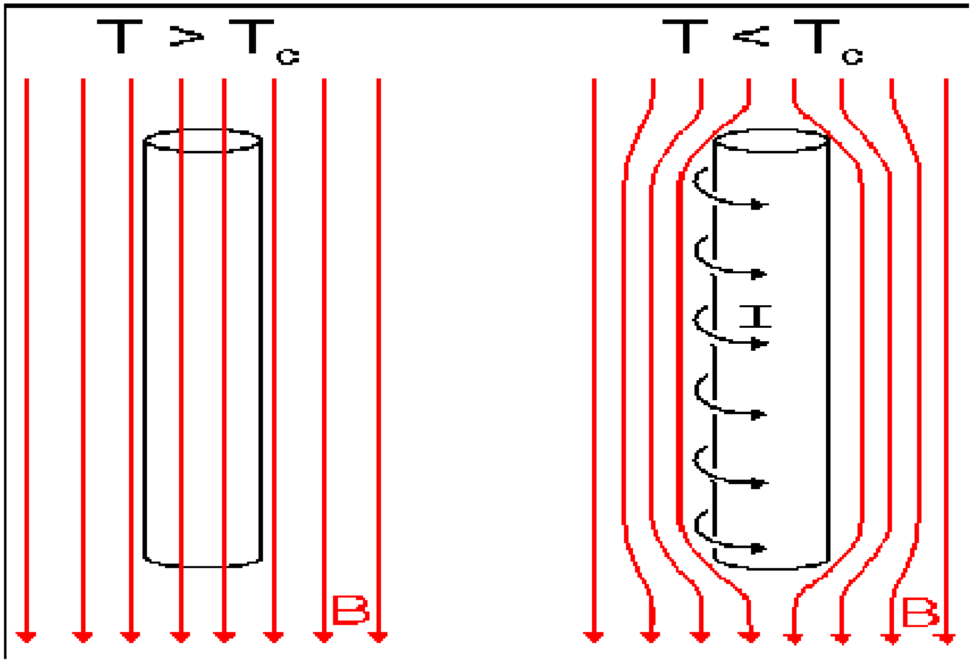
Dewar
with liquid Helium (4.2K)



Importance of superconductivity: Qubits for quantum computers are made of superconductors

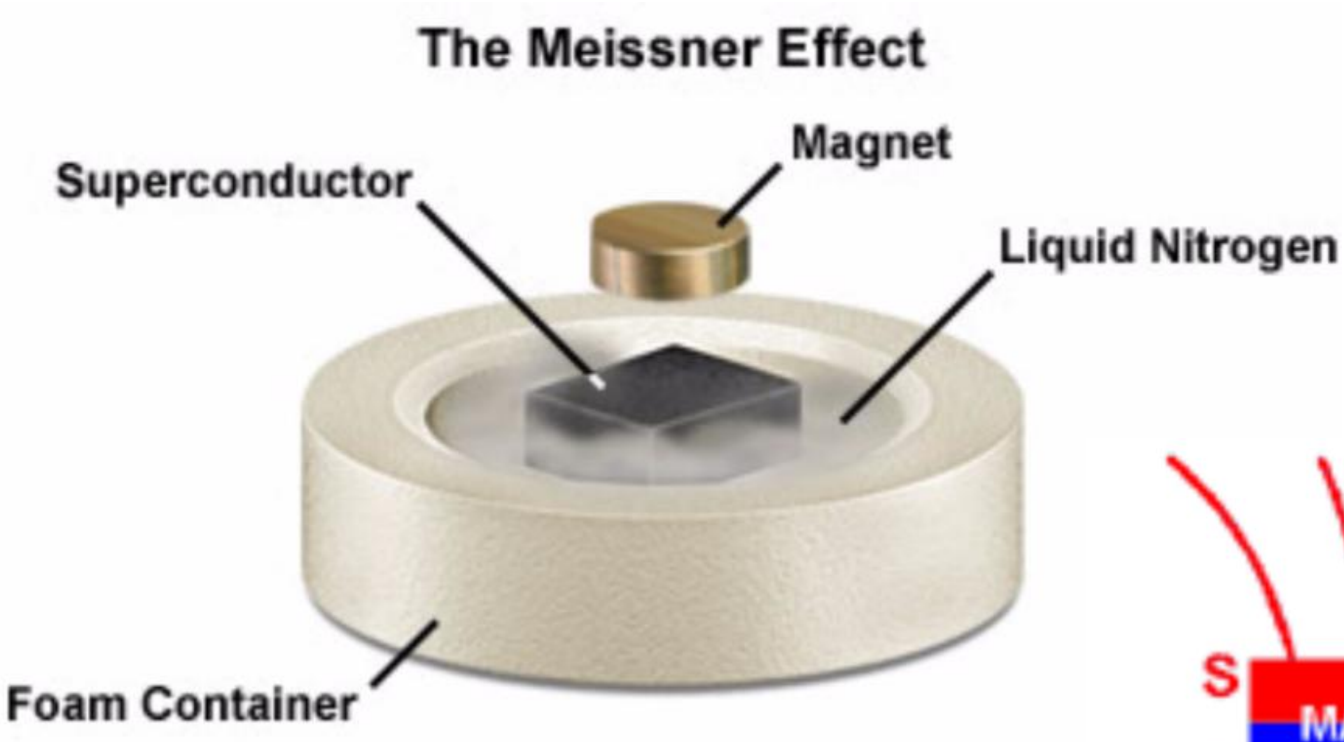
IBM Q

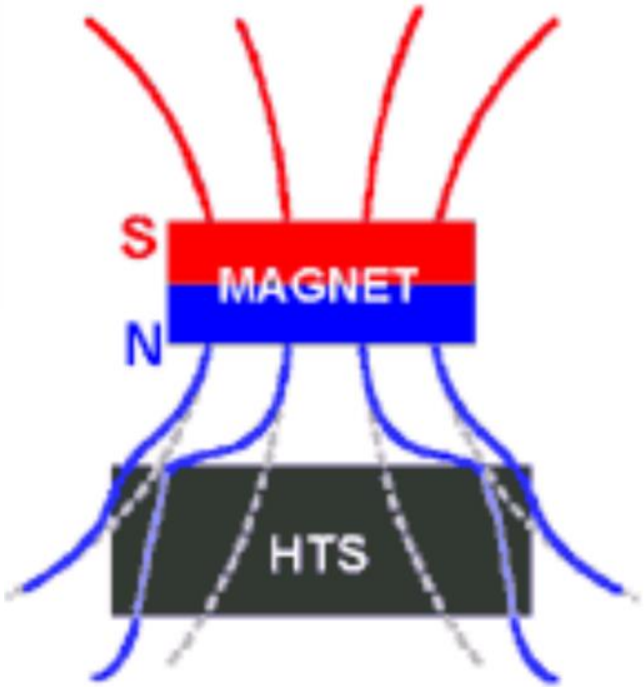
Meissner effect – key signature of superconductivity



Formula	T_c (K)	H_c (T)	Type	BCS
Elements				
Al	1.20	0.01	I	yes
Cd	0.52	0.0028	I	yes
Diamond:B	11.4	4	II	yes
Ga	1.083	0.0058	I	yes
Hf	0.165		I	yes
α-Hg	4.15	0.04	I	yes
β-Hg	3.95	0.04	I	yes
In	3.4	0.03	I	yes
Ir	0.14	0.0016[7]	I	yes
α-La	4.9		I	yes
β-La	6.3		I	yes
Mo	0.92	0.0096	I	yes
Nb	9.26	0.82	II	yes
Os	0.65	0.007	I	yes

Magnetic levitation



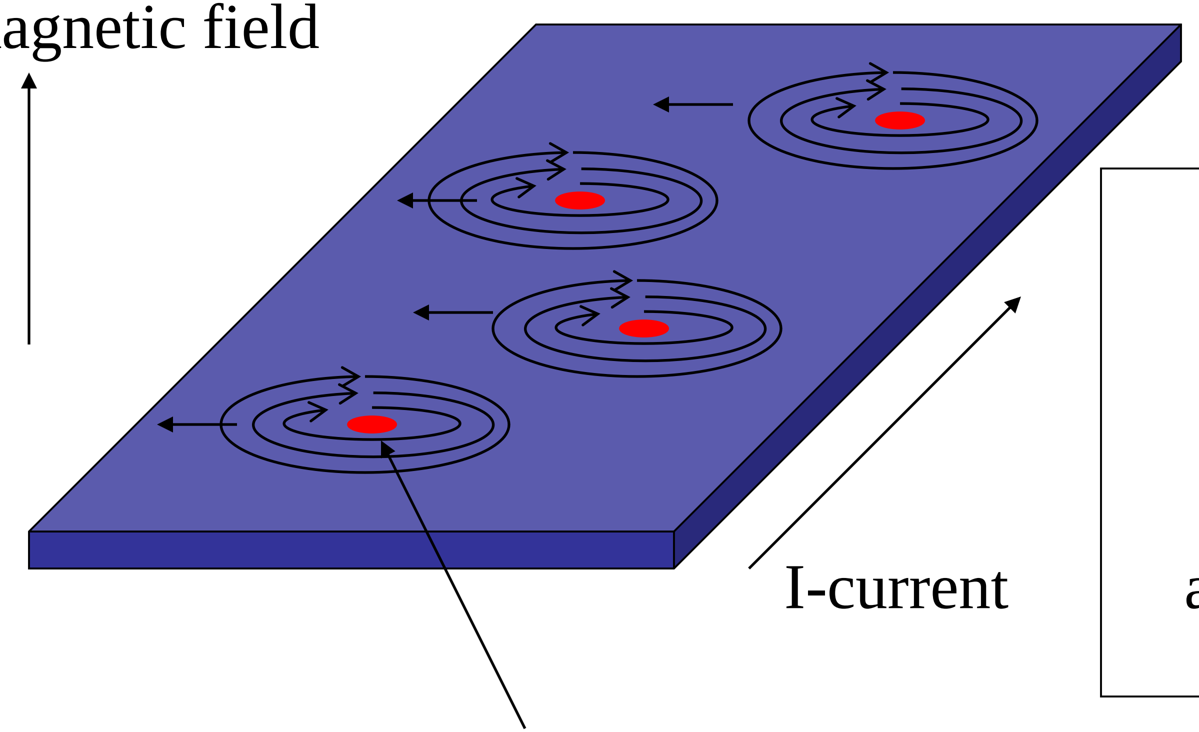


Levitation is the process by which an object is held aloft, without mechanical support, in a stable position.

Vortices introduce electrical resistence to otherwise superconducting materials

Magnetic field creates vortices--

Vortices cause dissipation (i.e. a non-zero electrical resistance)!



The order parameter:

$$\boxed{\Psi = |\Psi| \exp(-i\varphi)}$$

amplitude

phase

Vortex core: normal, not superconducting; diameter ξ~10 nm

Quantum operator for the momentum of an electron (zero magnetic field)

The momentum operator in quantum mechanics, $\hat{p}_x$, in one dimension is given by:

$$\hat{p}_x = -i\hbar\nabla_x = -i\hbar\frac{d}{dx}.$$

a. Show that the momentum operator can be written as: $\hat{p}_x = \frac{\hbar}{i}\nabla_x$

b. Given the wave function:

$$\Psi(x,t) = Ae^{-ikx}e^{iwt},$$

find the eigenvalue for momentum. Show all work in a step-by-step manner.

Quantum operator for the momentum of an electron (non-zero magnetic field)

$$H = \frac{1}{2m}\left(\vec{p} - q\vec{A}\right)^2 + q\varphi$$

$$\vec{p} = m\vec{v} + q\vec{A}$$

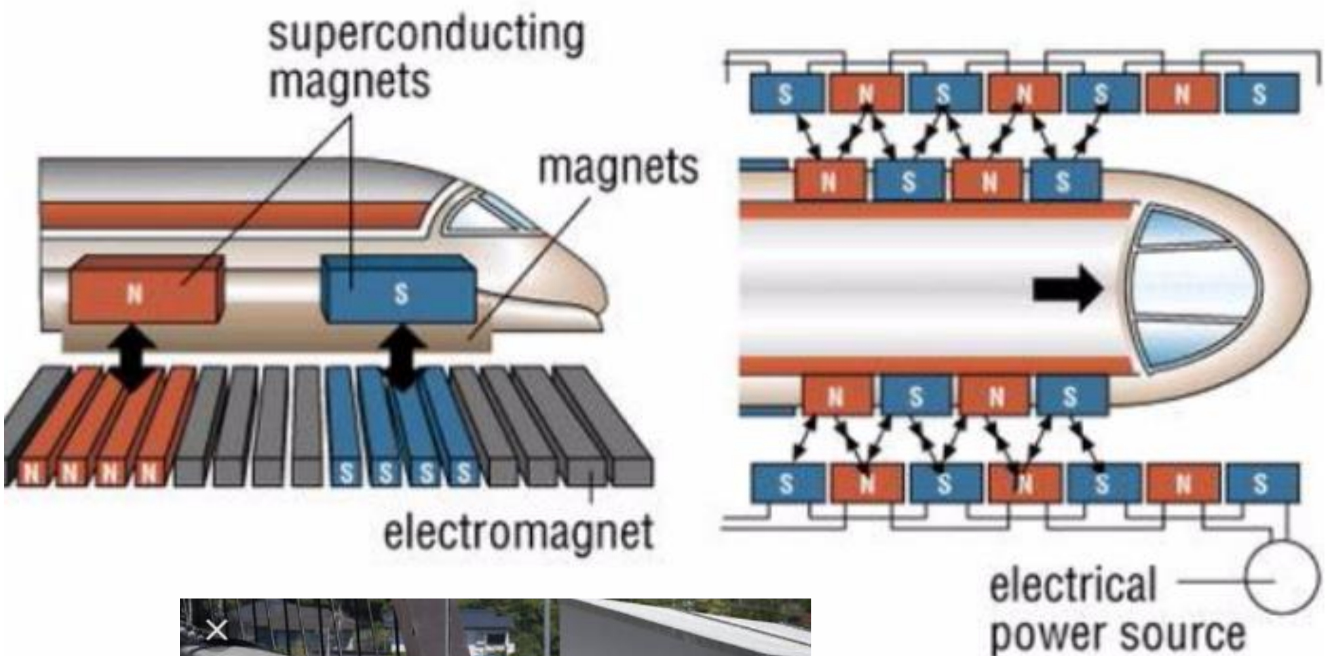
$$\hat{p}_x = -i\hbar\nabla_x = -i\hbar\frac{d}{dx} \quad =0$$

v~A

B=curl(A)

B is the magnetic field

Magnetic levitation train



blogs.wsj.com

U.S. Transportation Secretary Foxx Rides on Japan's Maglev Train - Jap...

PHYSICAL REVIEW VOLUME 117, NUMBER 3 FEBRUARY 1, 1960

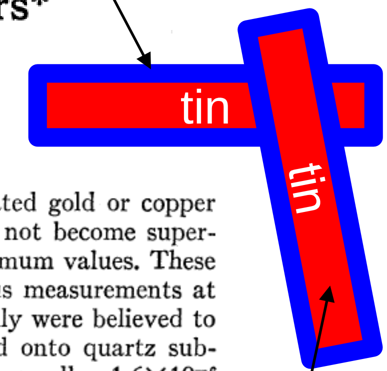
Discovery of the supercurrent, known now as proximity effect, in SNS junctions

Superconductivity of Contacts with Interposed Barriers*

HANS MEISSNER†
Department of Physics, The Johns Hopkins University, Baltimore, Maryland

(Received August 25, 1959)

Non-superconductor (normal metal, i.e., Ag)



Superconductor (tin)

Resistance *vs* current diagrams and "Diagrams of State" have been obtained for 63 contacts between crossed wires of tin. The wires were plated with various thicknesses of the following metals: copper, silver, gold, chromium, iron, cobalt, nickel, and platinum. The contacts became superconducting, or showed a noticeable decrease of their resistance at lower temperatures if the plated films were not too thick. The limiting thicknesses were about 35×10^{-6} cm for Cu, Ag, and Au; 7.5×10^{-6} cm for Pt, 4×10^{-6} cm for Cr, and less than 2×10^{-6} cm for the ferromagnetic metals Fe, Co, and Ni. The investigation was extended to measurements of the resistance of contacts between crossed wires of copper or gold plated with various thicknesses of tin. Simultaneous measurements of the (longitudinal) resistance of the tin-plated gold or copper wires showed that these thin films of tin do not become superconducting for thicknesses below certain minimum values. These latter findings are in agreement with previous measurements at Toronto. The measurements at Toronto usually were believed to be unreliable because films of tin evaporated onto quartz substrates can be superconducting at thicknesses as small as 1.6×10^{-6} cm. It is now believed that just as superconducting electrons can drift into an adjoining normal conducting layer and make it superconducting, normal electrons can drift into an adjoining superconducting layer and prevent superconductivity.

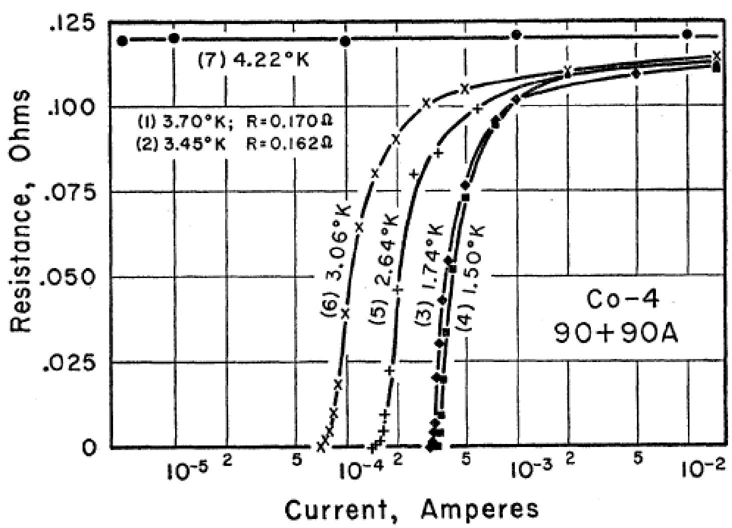


FIG. 1. Resistance *vs* current diagram of cobalt-plated contact Co 4, representative of diagrams type A.

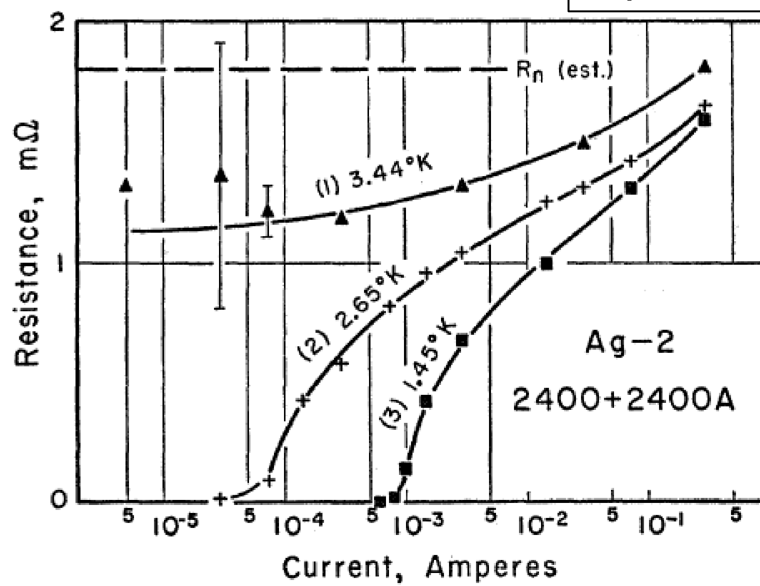
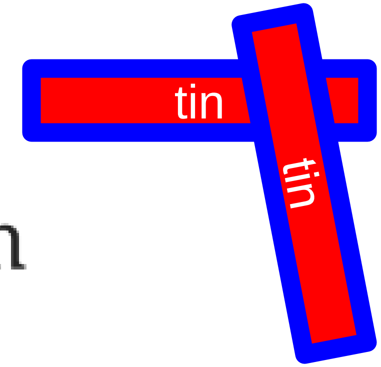


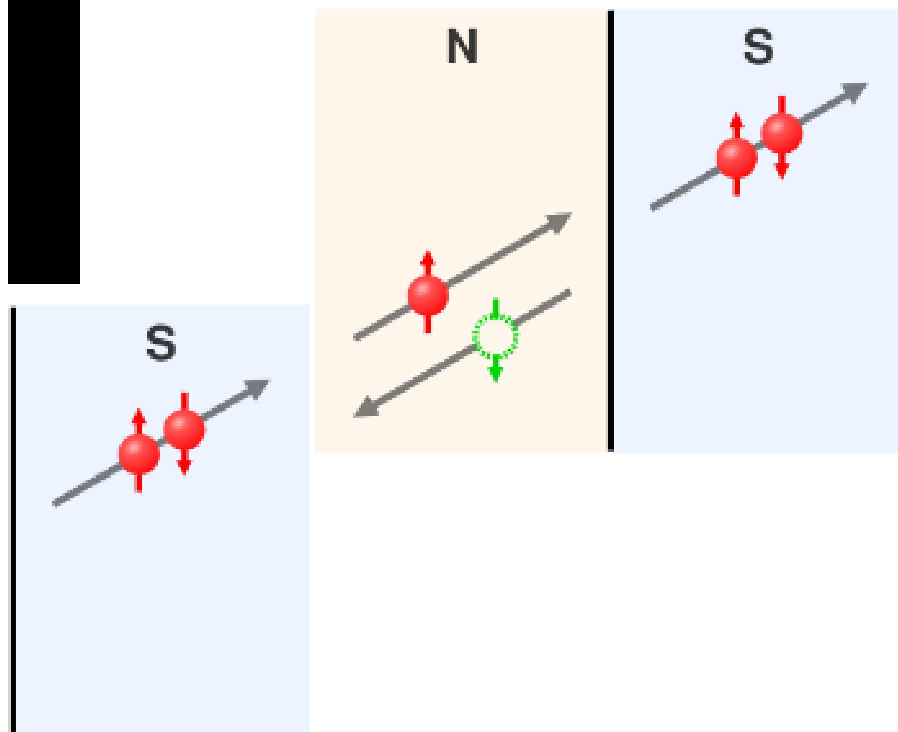
FIG. 2. Resistance *vs* current diagram of silver-plated contact Ag 2, representative of diagrams type B.

Explanation of the supercurrent in SNS junctions --- Andreev reflection



www.kapitza.ras.ru
www.kapitza.ras.ru/~andreev/afan...

Andreev reflection



A.F. Andreev, 1964

Non-Abelian Majorana Modes in Vortices

VOLUME 86, NUMBER 2 PHYSICAL REVIEW LETTERS 8 JANUARY 2001

Non-Abelian Statistics of Half-Quantum Vortices in *p*-Wave Superconductors

D. A. Ivanov
Institut für Theoretische Physik, ETH-Hönggerberg, CH-8093 Zürich, Switzerland
(Received 17 May 2000)

Excitation spectrum of a half-quantum vortex in a *p*-wave superconductor contains a zero-energy Majorana fermion. This results in a degeneracy of the ground state of the system of several vortices. From the properties of the solutions to Bogoliubov–de Gennes equations in the vortex core we derive the non-Abelian statistics of vortices identical to that for the Moore-Read (Pfaffian) quantum Hall state.

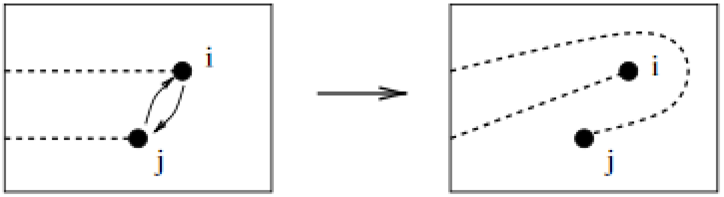


FIG. 3. Elementary braid interchange of two vortices.

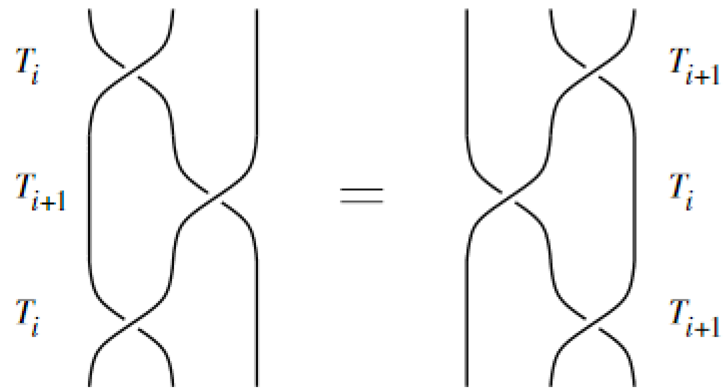
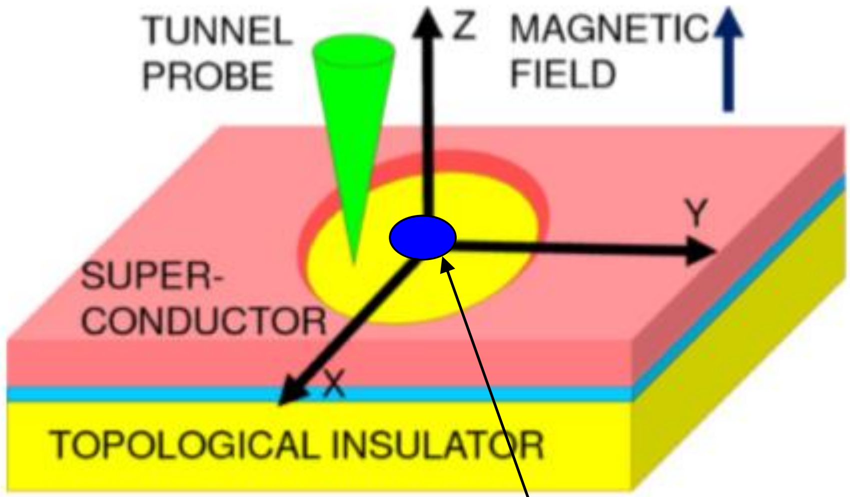


FIG. 2. Defining relation for the braid group: $T_i T_{i+1} T_i = T_{i+1} T_i T_{i+1}$.

Total number of quantum states
for N pairs of such Majorana vortices is:
2^N
Example N=100
$2^{100}=10^{30}$

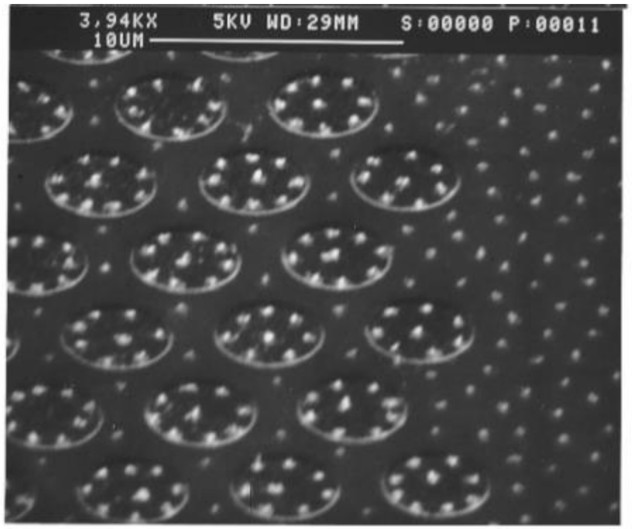
Majorana modes in a vortex



Theory: Vortex in the nano-hole contains Majorana states (gap is about Tc)

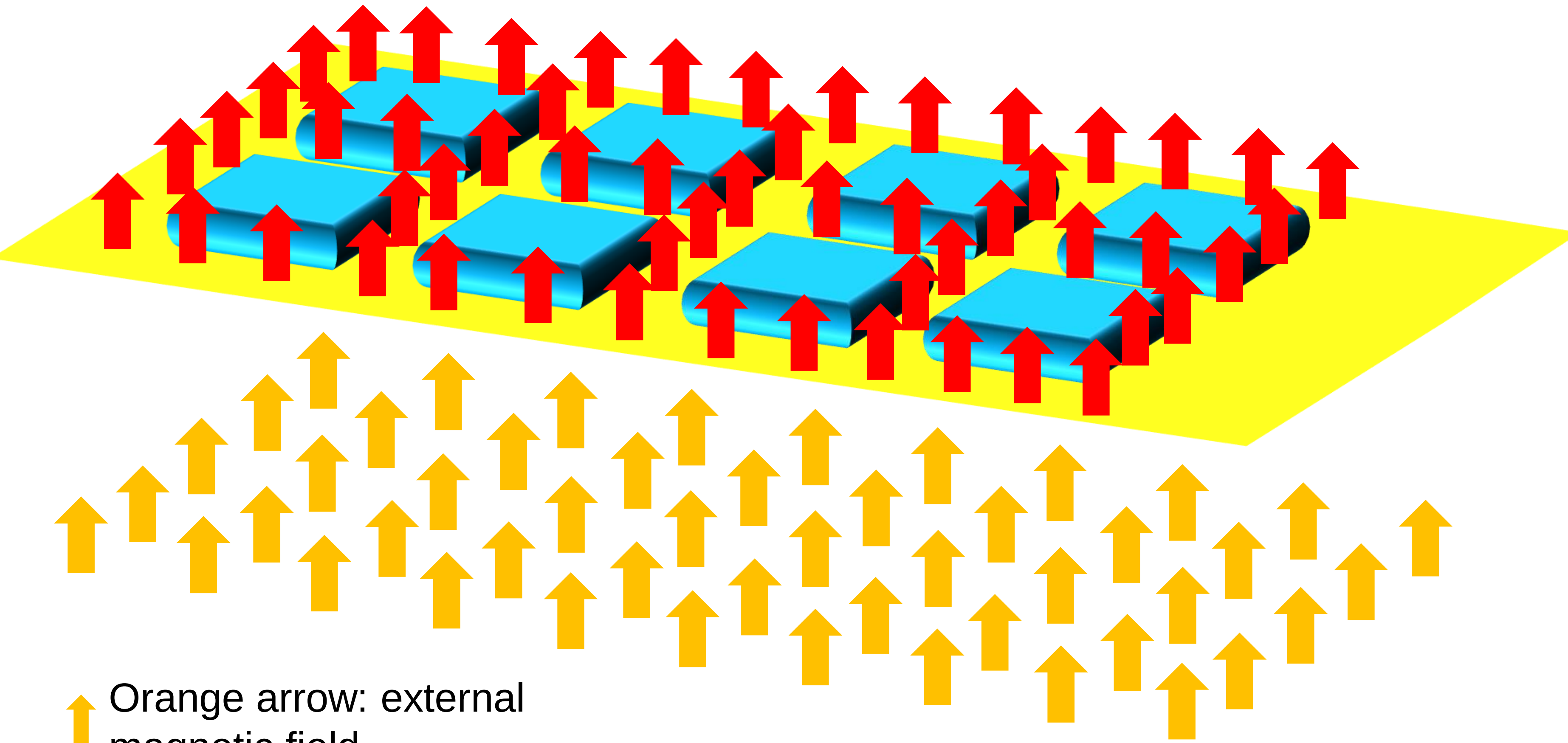
R.S. Akzyanov, A.V. Rozhkov, A.L.Rakhmanov, and F. Nori, PRB 89, 085409 (2014)

PHYSICAL REVIEW B **84**, 075141 (2011)



A. Bezryadin, Yu. Ovchinnikov, B. Pannetier, PRB 53, 8553 (1996)

Schematic of the array (22x22 islands)



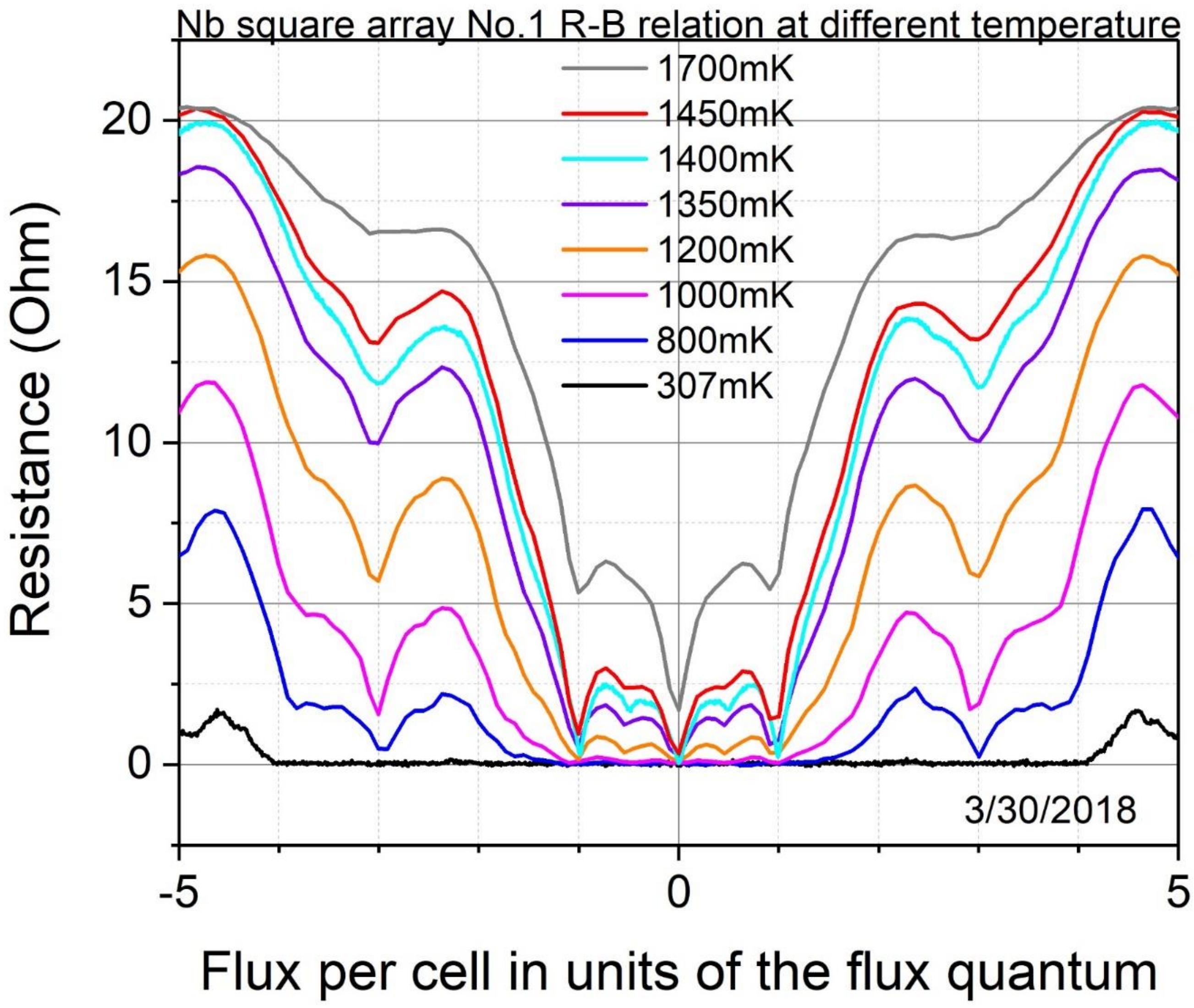
Orange arrow: external magnetic field

Red arrow: magnetic field that penetrates the array

Yellow film: Bi2Se3 topological insulator

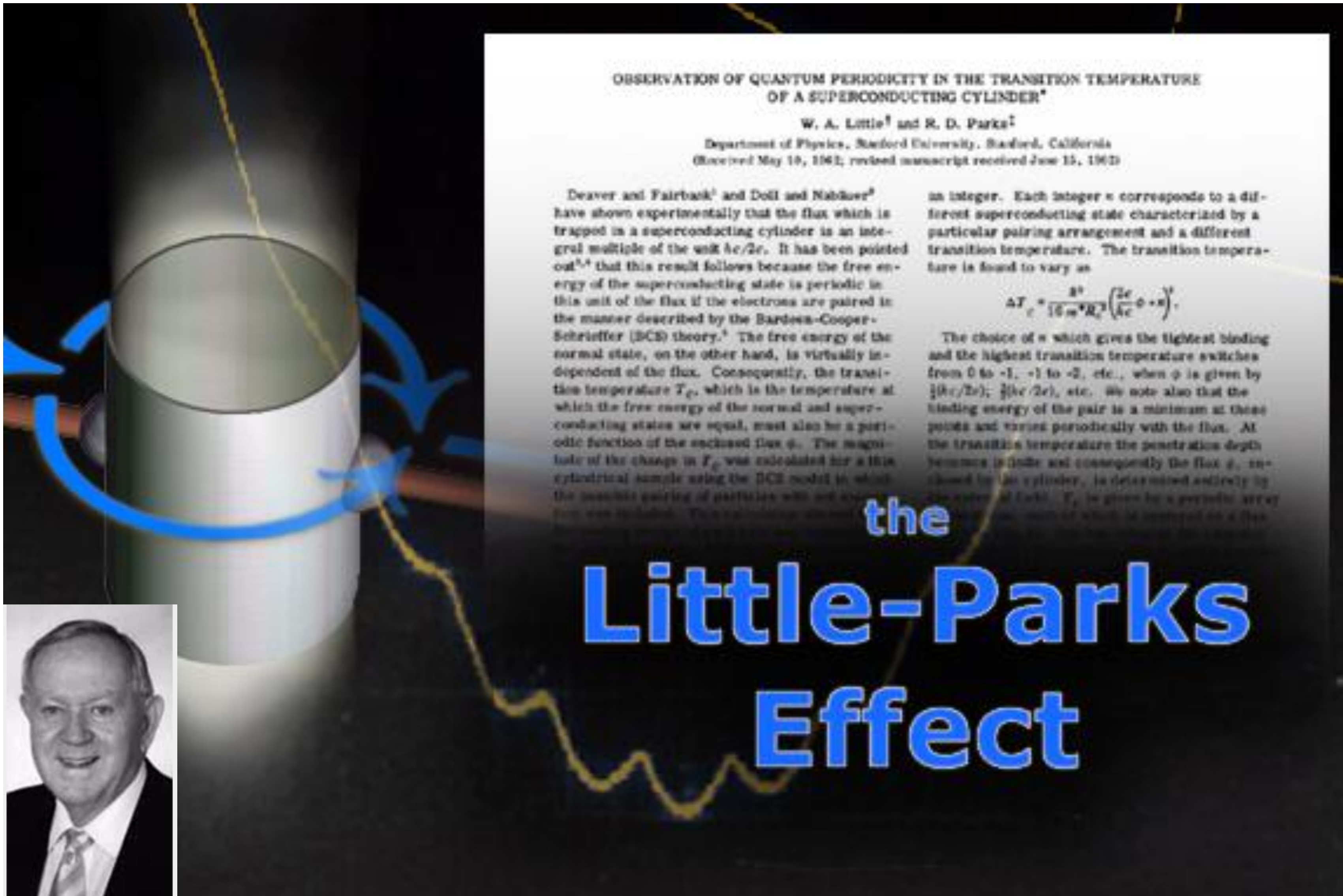
Blue block: Niobium square island

Resistance of the sample as a function of magnetic field at 300 mK (zoom into lower fields)

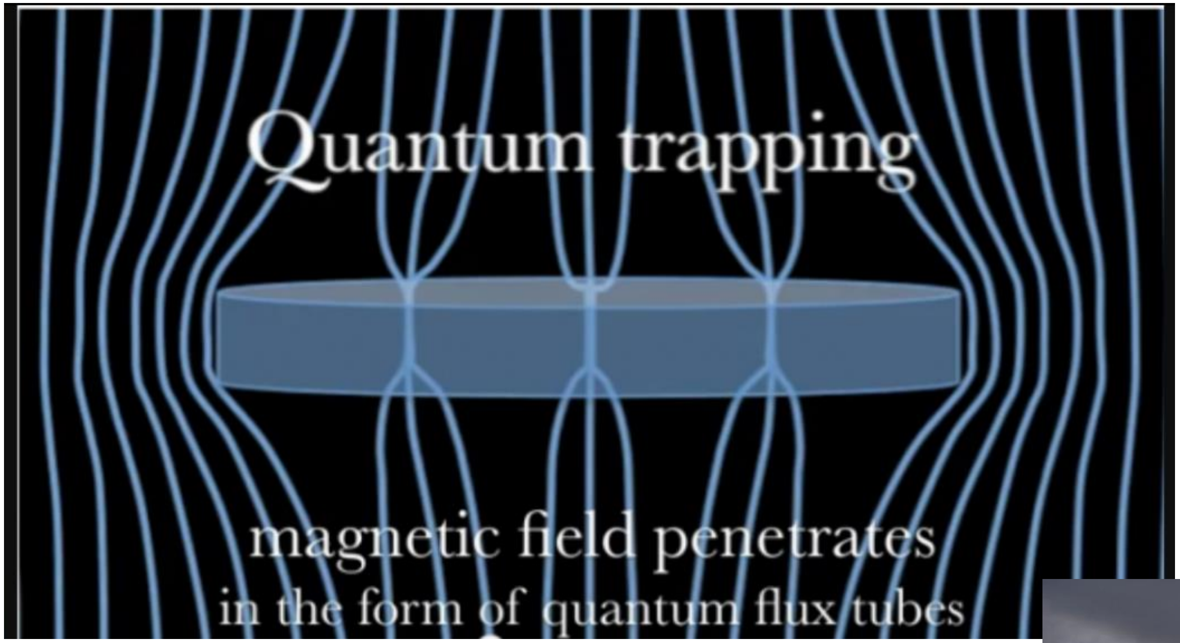


Searching for an explanation: Little-Parks effect ('62)

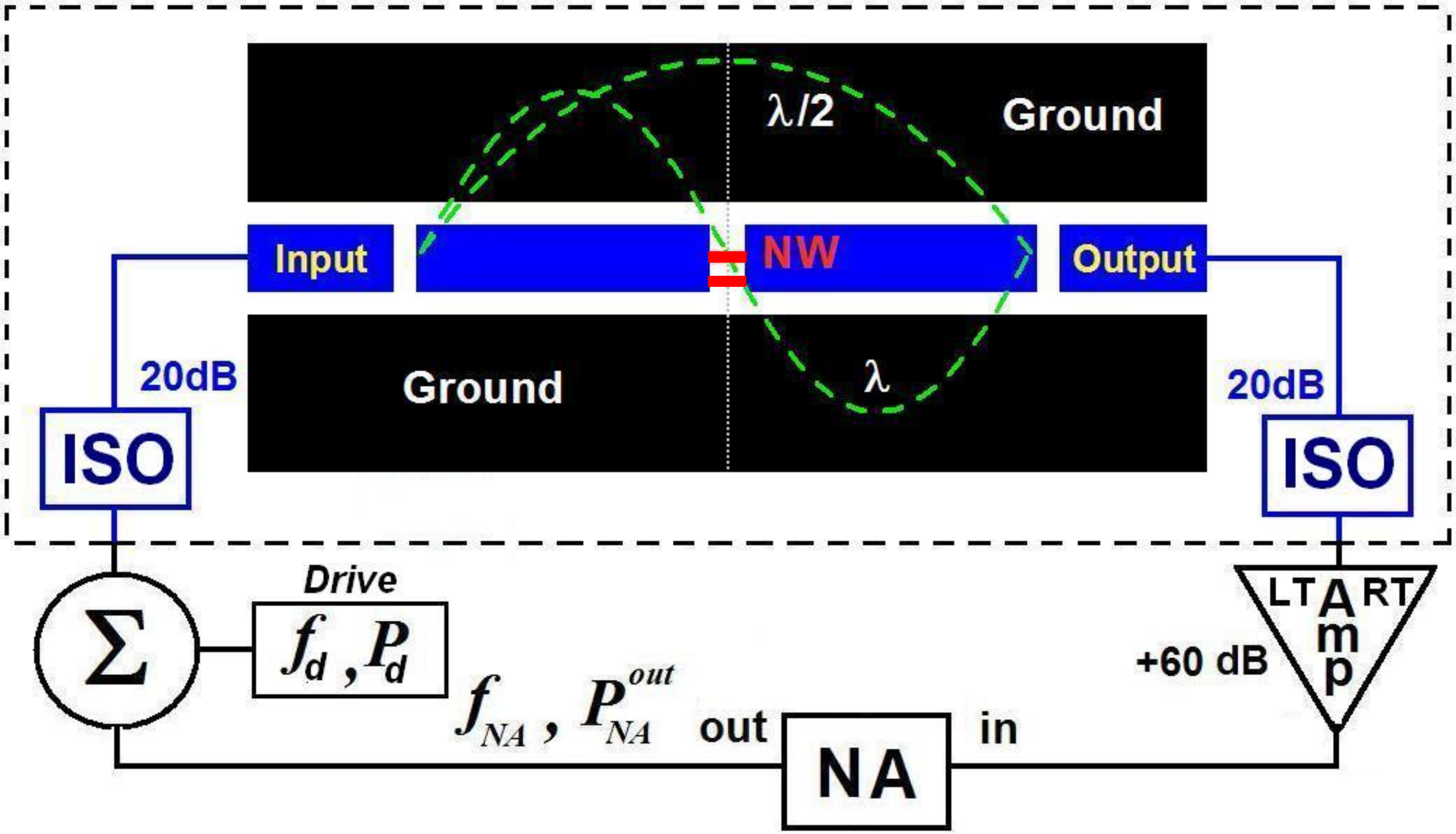
The basic idea: magnetic field induces non-zero vector-potential, which produces non-zero superfluid velocity, thus reducing the Tc.



Superconducting vortices produced by magnetic field

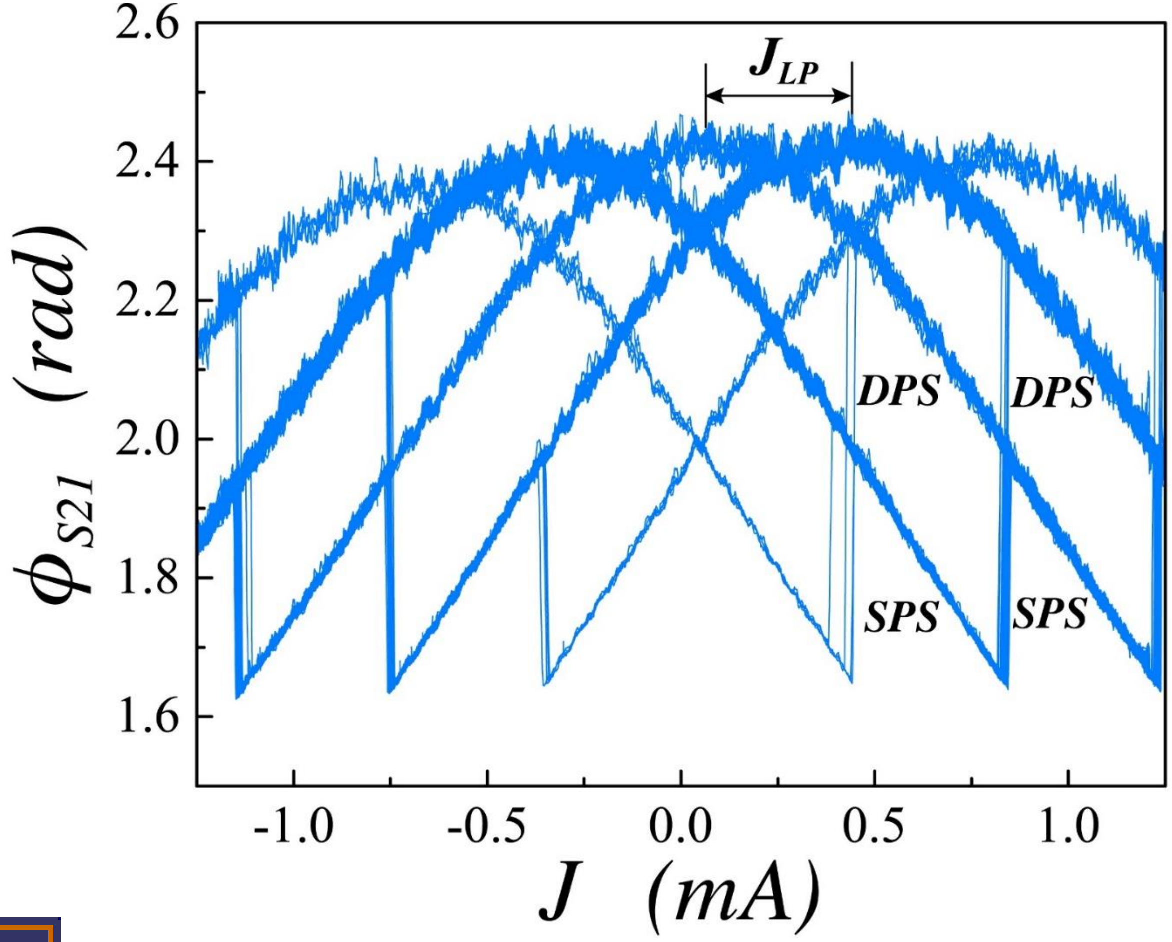


Measuring nanowires within GHz resonators. Detection of individual phase slips.



A. Belkin et al, *Appl. Phys. Lett.* **98**, 242504 (2011)

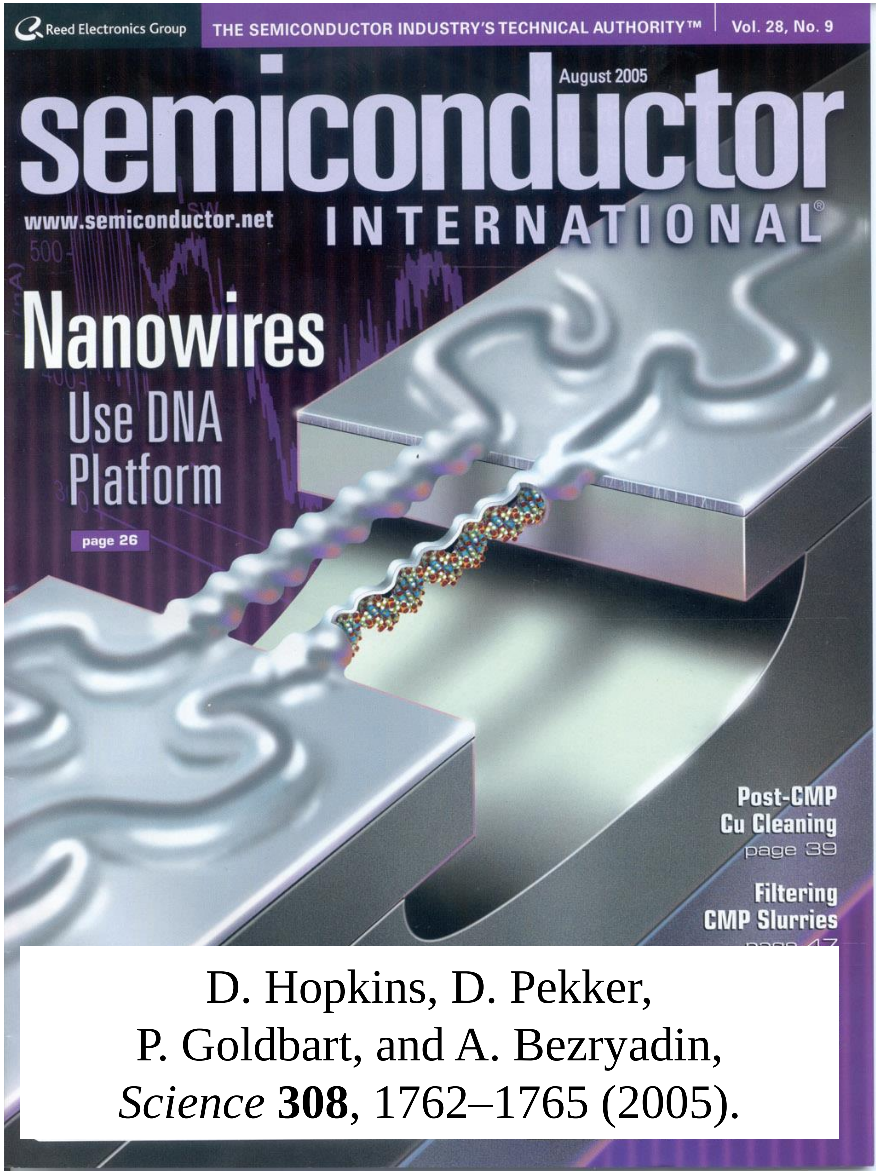
Resonators used to detect single phase slips (SPS) and double phase slips (DPS)



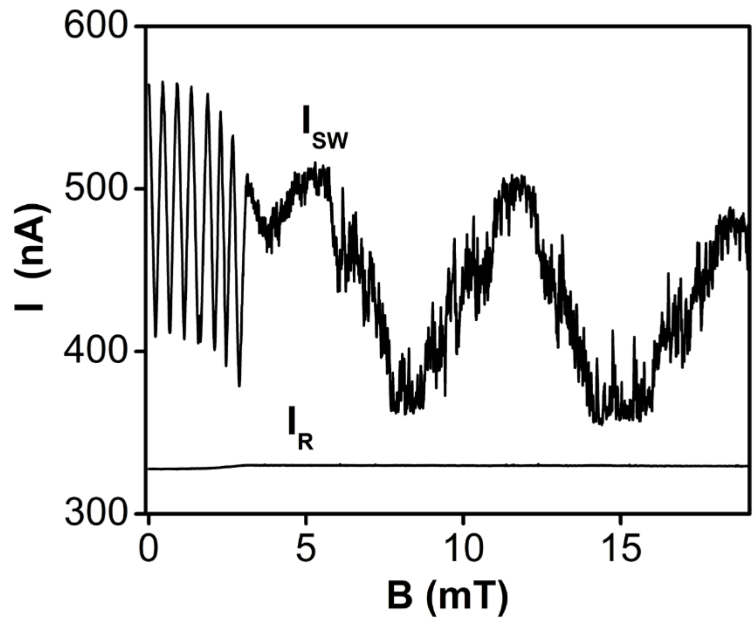
T = 360 mK
$f = f_0(H=0)$

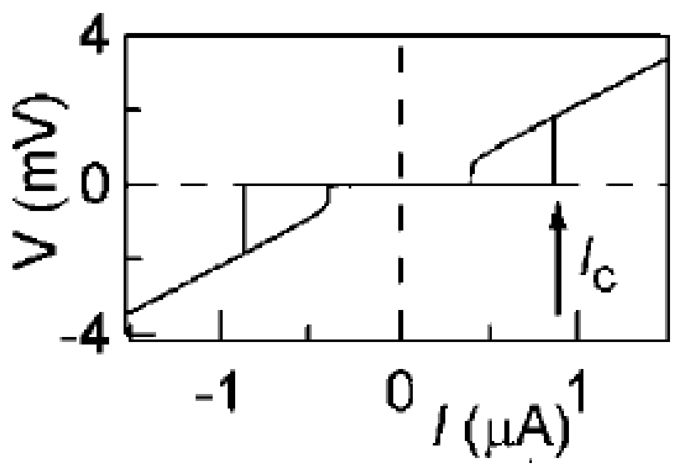
A. Belkin et al, PRX **5**, 021023 (2015)

Phase gradiometers templated by DNA



D. Hopkins, D. Pekker,
P. Goldbart, and A. Bezryadin,
Science **308**, 1762–1765 (2005).





SQUID – superconducting quantum interference device

SQUID helmet project at Los Alamos

Magnetic field scales:

Earth field: ~1G

Fields inside animals: ~0.01G-0.00001G

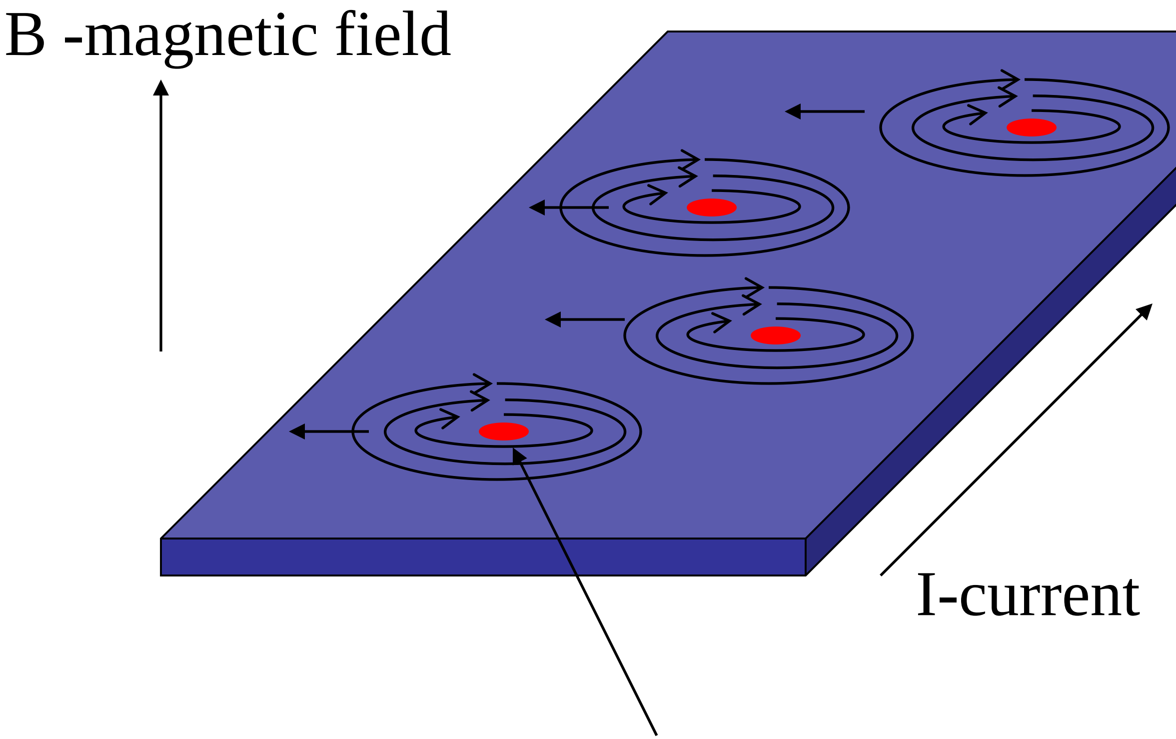
Fields on the **human brain**: ~0.3nG
This is less than a hundred-millionth of the Earth's magnetic field.

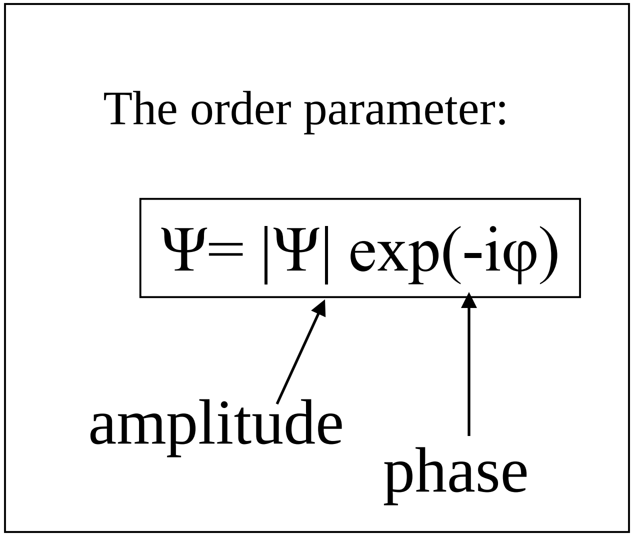
SQUIDs, or Superconducting Quantum Interference Devices, invented in 1964 by Robert Jaklevic, John Lambe, Arnold Silver, and James Mercereau of Ford Scientific Laboratories, are used to measure extremely small magnetic fields. They are currently the most sensitive magnetometers known, with the noise level as low as 3 fT•Hz−½. While, for example, the Earth magnet field is only about 0.0001 Tesla, some electrical processes in animals produce very small magnetic fields, typically between 0.000001 Tesla and 0.000000001 Tesla. SQUIDs are especially well suited for studying magnetic fields this small.
Measuring the brain's magnetic fields is even much more difficult because just above the skull the strength of the magnetic field is only about 0.3 picoTesla (0.0000000000003 Tesla). This is less than a hundred-millionth of Earth's magnetic field. In fact, brain fields can be measured only with the most sensitive magnetic-field sensor, i.e. with the superconducting quantum interference device, or SQUID.

Vortices introduce electrical resistence to otherwise superconducting materials

Magnetic field creates vortices--

Vortices cause dissipation (i.e. a non-zero electrical resistance)!

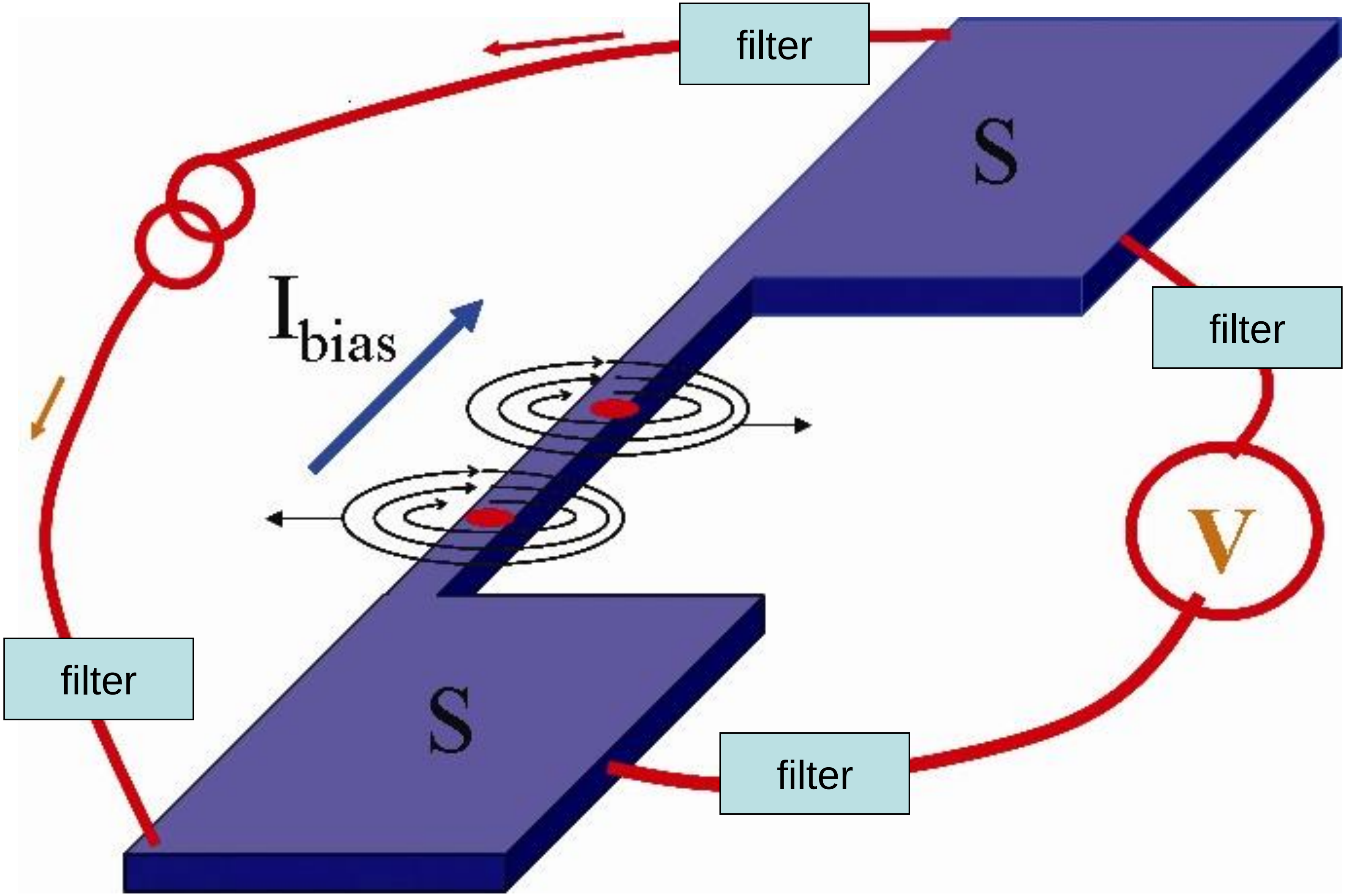




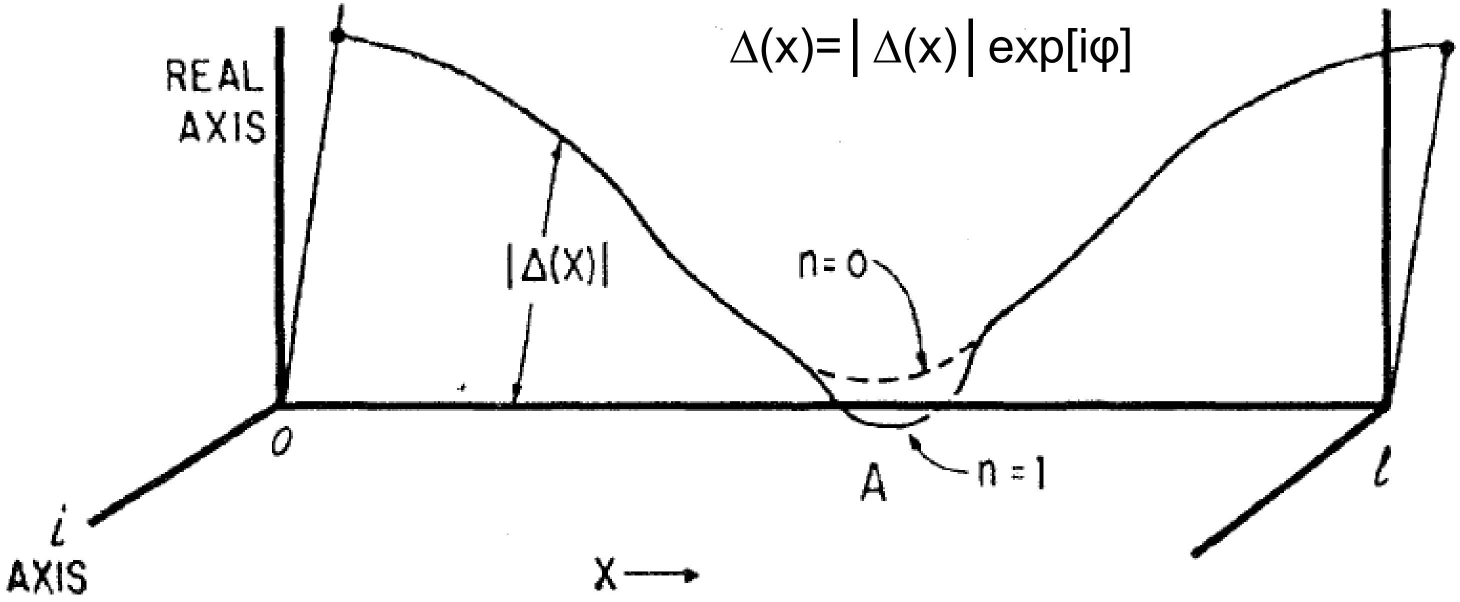
Vortex core: normal, not superconducting; diameter ξ~10 nm

DC transport measurement schematic

Phase slip events are shown as red dots



Transport properties: Little's Phase Slip

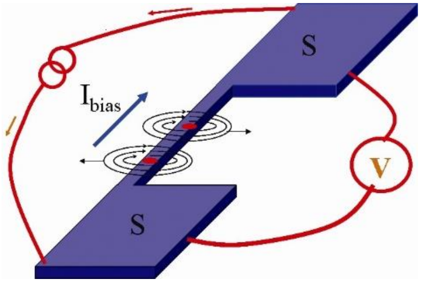


W. A. Little, "Decay of persistent currents in small superconductors", Physical Review, V.156, pp.396-403 (1967).

Two types of phase slips (PS) can be expected:

1. The usual, thermally activated PS (TAPS)
2. Quantum phase slip (QPS)

How to use voltage to determine the rate of phase slips?



Phase evolution equation: $d\varphi/dt = 2eV/\hbar$

Simplified derivation:

1. From Schrödinger equation: $i\hbar(d\Psi/dt)=E\,\Psi$
2. The solution is: $\Psi=\exp(-iEt/\hbar)$
3. The phase of the wavefunction is $\varphi=Et/\hbar$

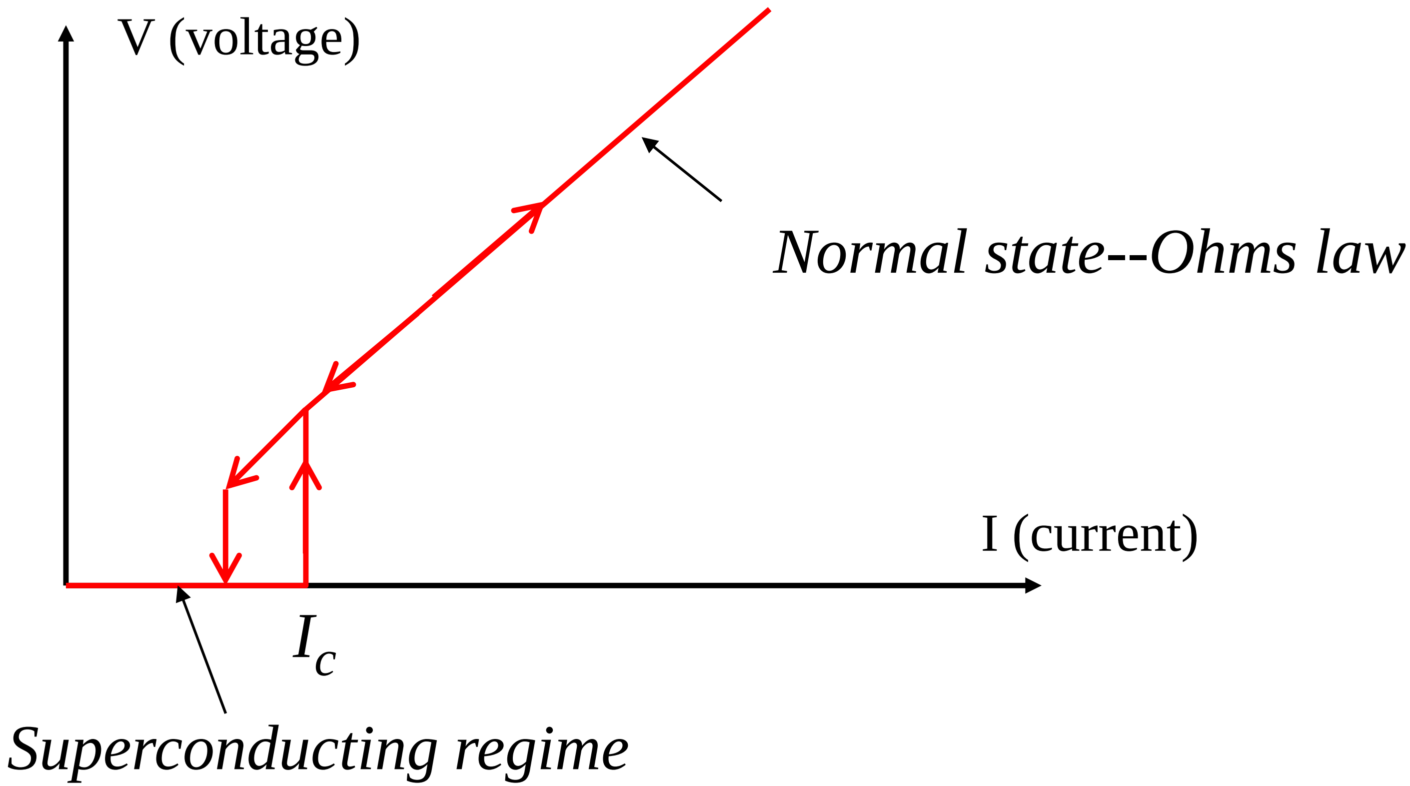
Superconducting electrons form pairs, so the energy is: $E=2eV$
(here V is the electric potential or voltage)

Thus the resulting equation is: $d\varphi/dt = 2eV/\hbar$

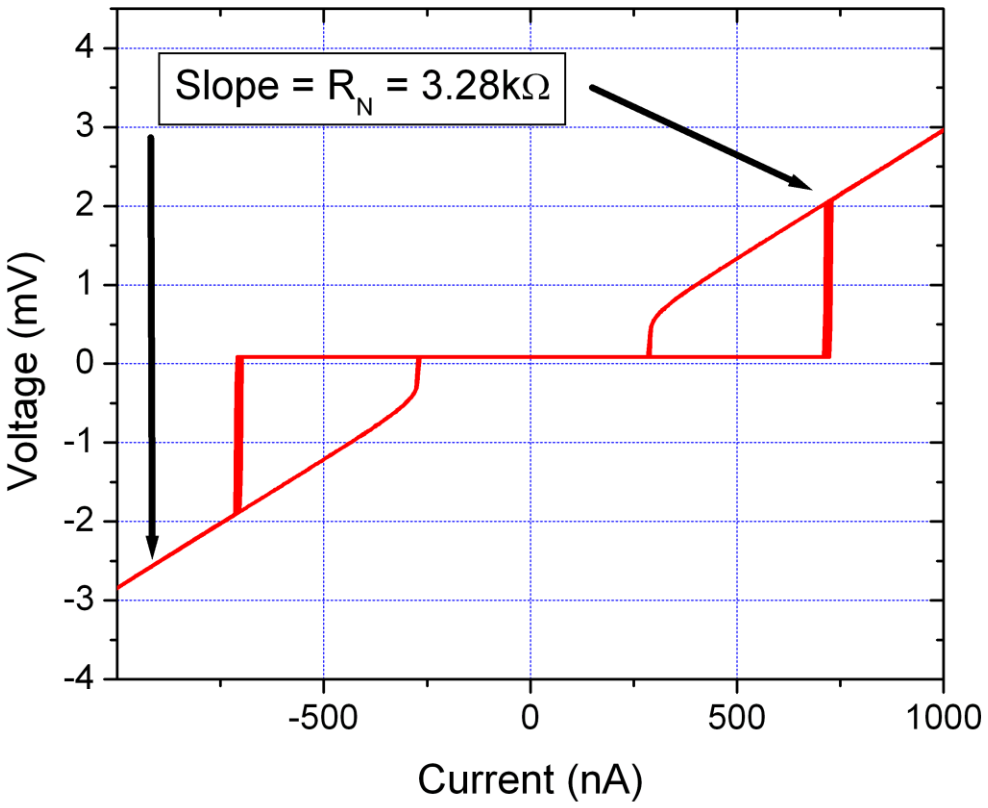
Gor'kov, L.P. (1958) *Exp. Theor. Phys. (USSR)*, **34**, 735;
(English transl.: (1958) *Sov. Phys. JETP*, **7**, 505.)

Superconductivity: very basic introduction

Electrical resistance is zero only if current is not too strong

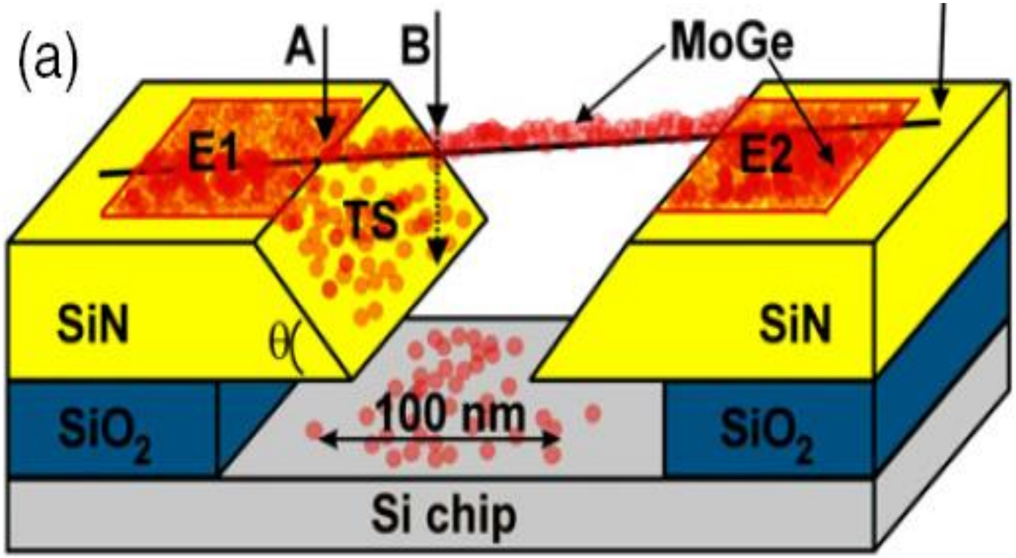


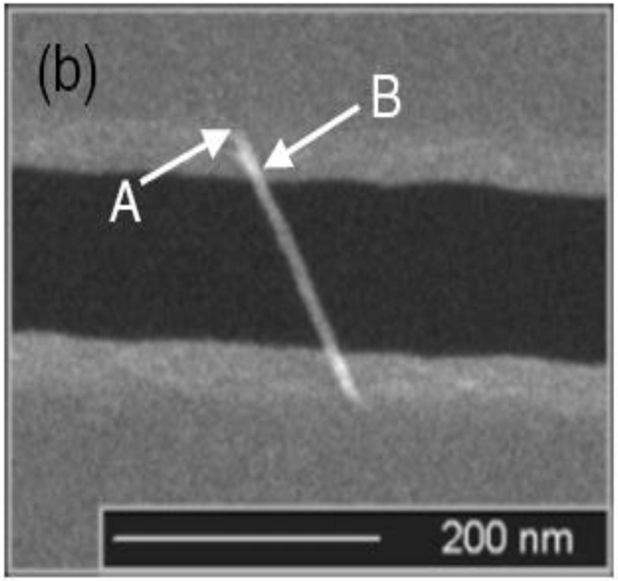
Search for QPS at high bias currents, by measuring the fluctuations of the switching current



Fabrication of nanowires

Method of Molecular Templating





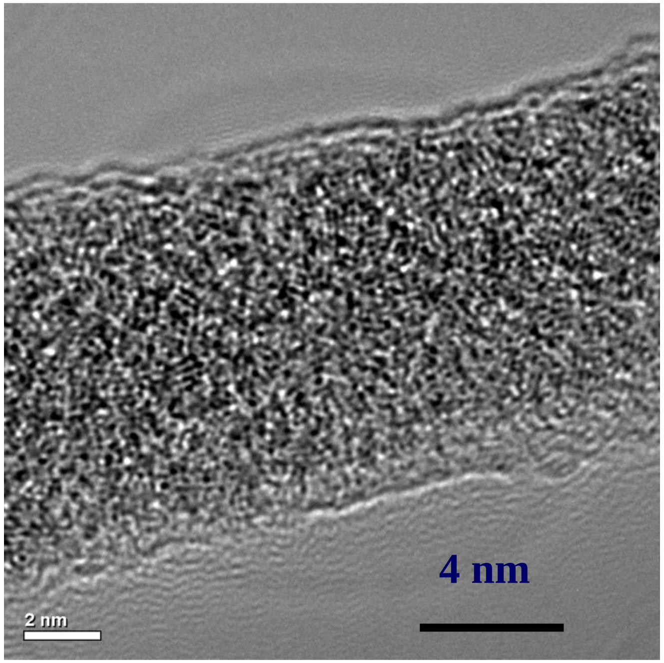
Si/ SiO_2/SiN substrate with undercut

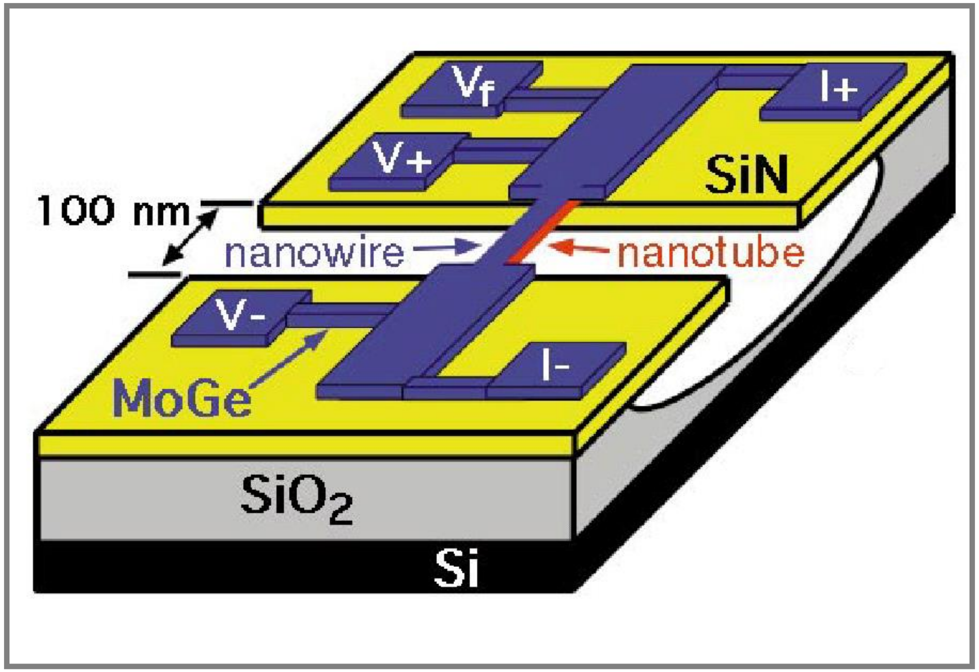
~ 0.5 mm Si wafer
500 nm SiO_2
60 nm SiN
Width of the trenches ~ 50 - 500 nm

HF wet etch for ~10 seconds to form undercut

Bezryadin, Lau, Tinkham, *Nature* **404**, 971 (2000)

Sample Fabrication

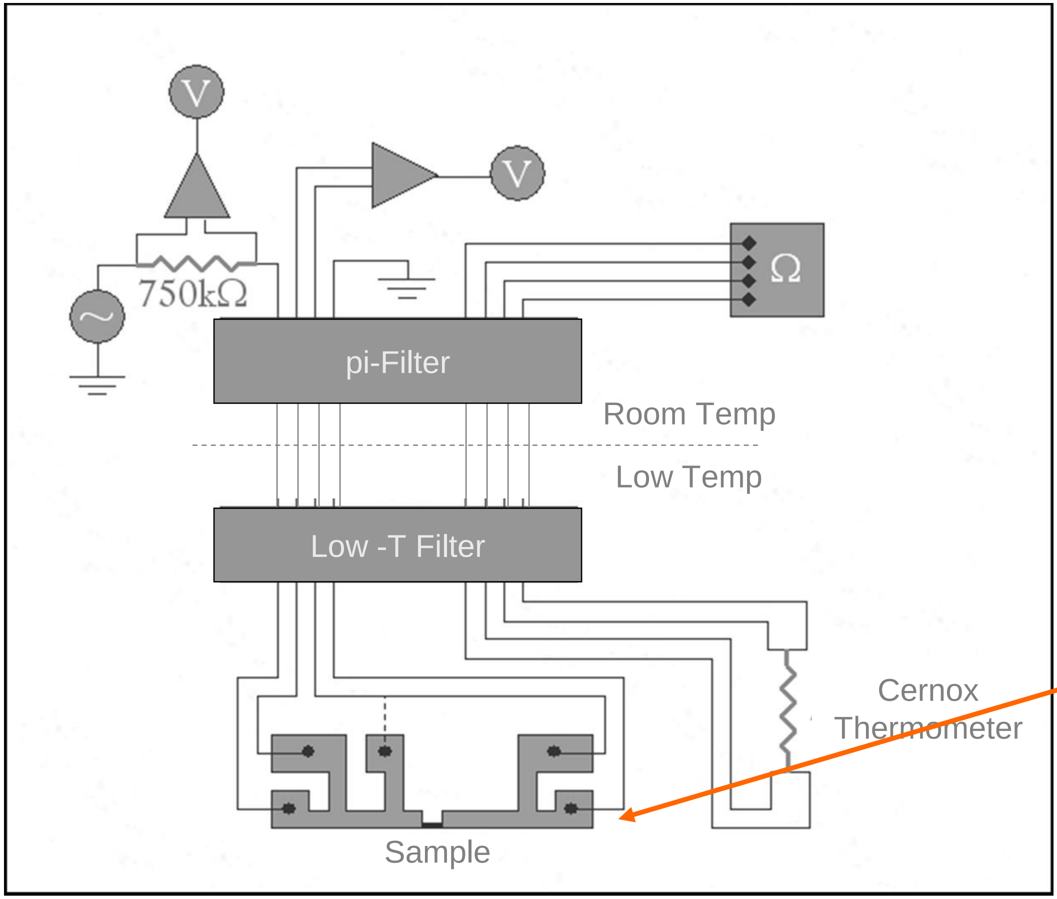


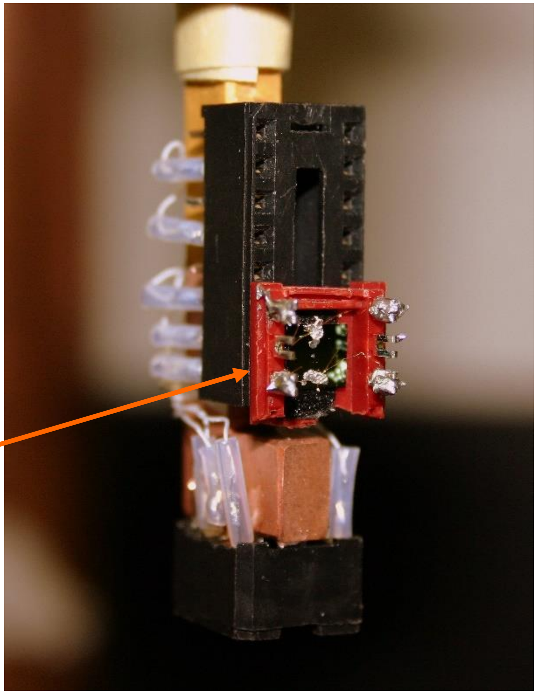


TEM image of a wire shows amorphous morphology.
Nominal MoGe thickness = 3 nm

Schematic picture of the pattern Nanowire + Film Electrodes used in transport measurements

Measurement Scheme





Circuit Diagram

Sample mounted on the ^{3}He insert.

Tony Bollinger's sample-mounting procedure in winter in Urbana

Procedure (~75% Success)

- Put on gloves

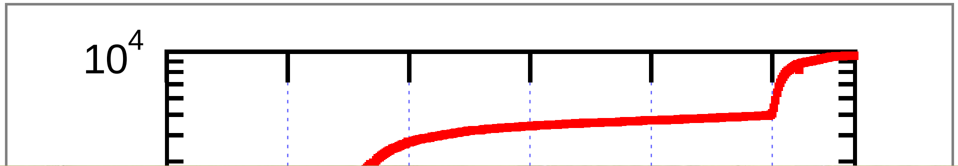
- Put grounded socket for mounting in vise with grounded indium dot tool connected
- Spray high-backed black chair all over and about 1 m square meter of ground with anti-static spray

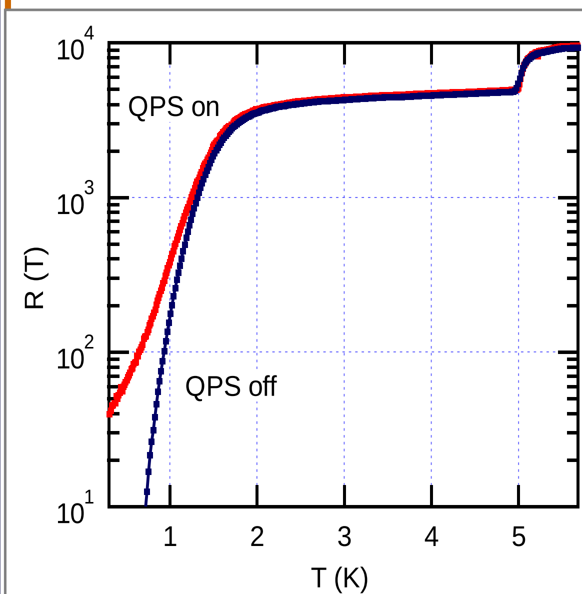
  - DO NOT use green chair
  - Not sure about short-backed black chairs

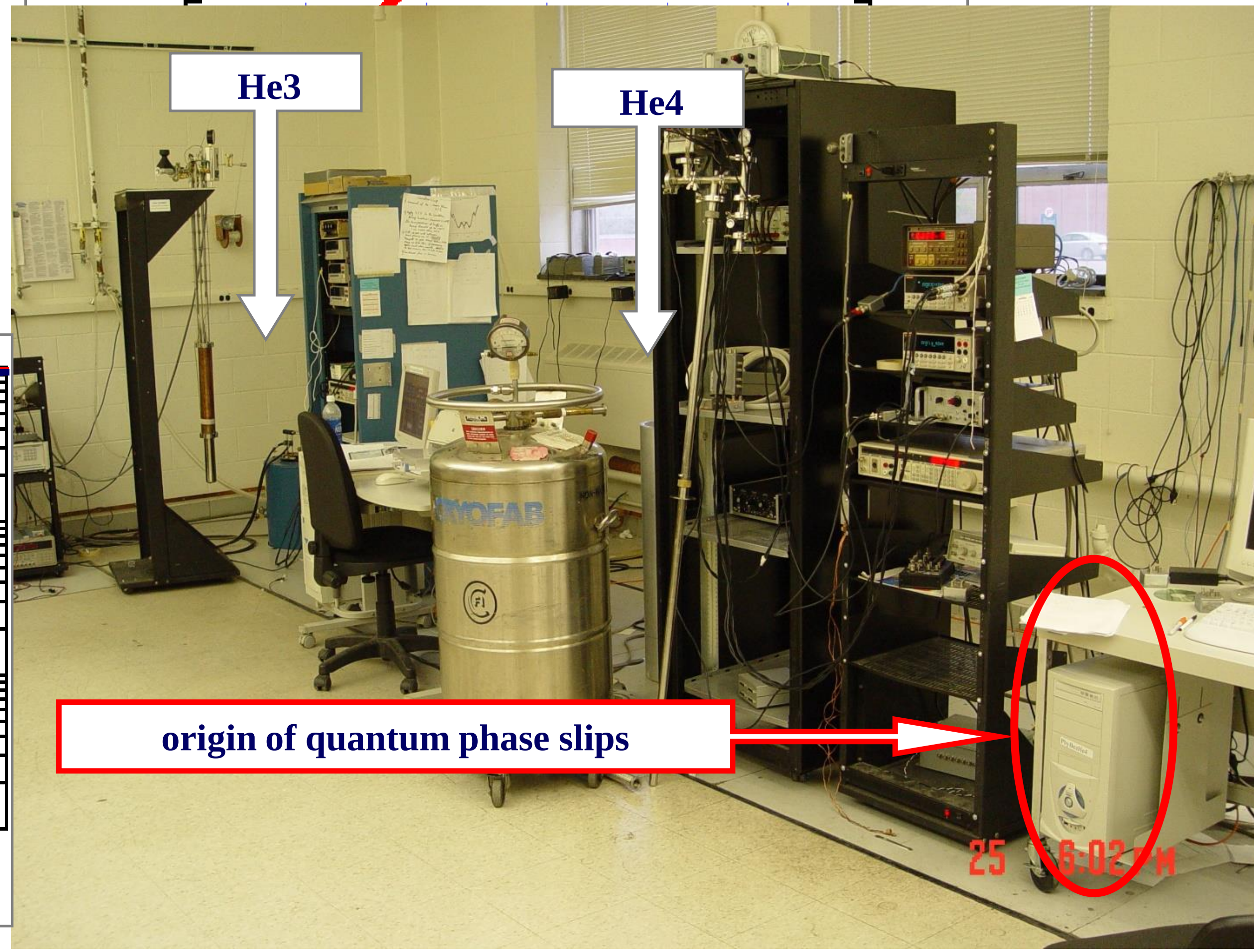
- Sit down
- Spray bottom of feet with anti-static spray

- Plant feet on the ground. ***Do not move your feet again for any reason until mounting is finished.***
- Mount sample
- Keep sample in grounded socket until last possible moment
- Test samples in dipstick at ~1 nA

Possible Origin of Quantum Phase Slips







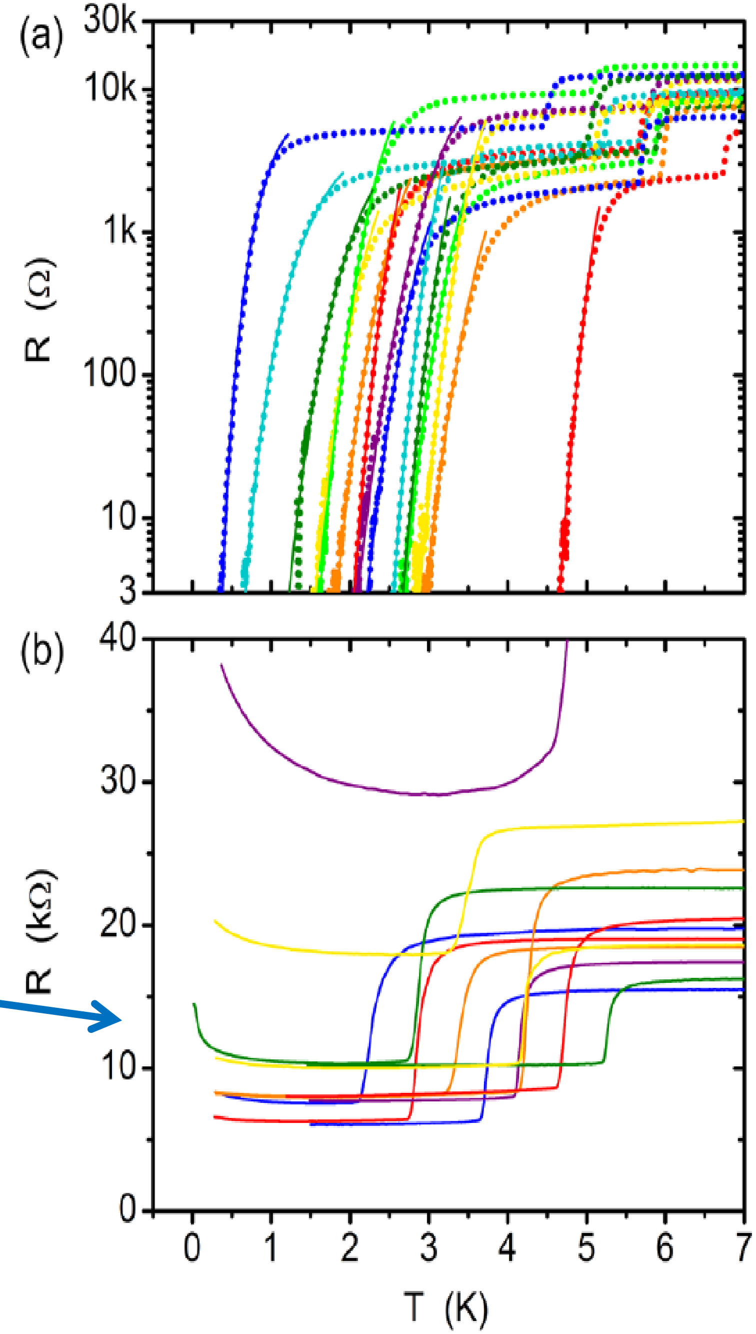
Dichotomy in nanowires: Evidence for superconductor-insulator transition (SIT)

R=V/I I~3 nA

The difference between samples is the amount of the deposited Mo79Ge21.

$$R_{\text{sheet}} = 100 - 400\,\Omega$$

Can the insulating behavior be due to Anderson localization of the BCS condensate?



Bollinger, Dinsmore, Rogachev, Bezryadin, Phys. Rev. Lett. **101**, 227003 (2008)

Useful Expression for the Free Energy of a Phase Slip

"Arrhenius-Little" formula for the wire resistance:

$$R_{AL} \approx R_N \, exp[-\Delta F(T)/k_B T]$$

$$\Delta F = (8\sqrt{2}/3)(H_c(T)^2/8\pi)(A\xi(T))$$

$$\frac{\Delta F(0)}{k_B T_c} = \sqrt{6}\,\frac{\hbar I_c(0)}{2ek_B T_c} = 0.83\,\frac{R_q L}{R_n \xi(0)} = 0.83\,\frac{R_q}{R_{\xi(0)}}$$

APPLIED PHYSICS LETTERS VOLUME 80, NUMBER 16 22 APRIL 2002

Quantum limit to phase coherence in thin superconducting wires

M. Tinkham[a] and C. N. Lau
Physics Department, Harvard University, Cambridge, Massachusetts 02138

Linearity of the Schrödinger's equation

Suppose Ψ_1 is a valid solution of the Schrödinger equation:

$$i\hbar\frac{\partial\psi_1}{\partial t}=\frac{\partial^2\psi_1}{\partial x^2}+U(x)\psi_1$$

And suppose that Ψ_2 is another valid solution of the Schrödinger equation:

$$i\hbar\frac{\partial\psi_2}{\partial t}=\frac{\partial^2\psi_2}{\partial x^2}+U(x)\psi_2$$

Then $(\Psi_1 + \Psi_2)/\sqrt{2}$ is also a valid solution, because:

$$\boldsymbol{i\hbar\frac{\partial(\psi_1+\psi_2)}{\partial t}=\frac{\partial^2(\psi_1+\psi_2)}{\partial x^2}+U(x)(\psi_1+\psi_2)}$$

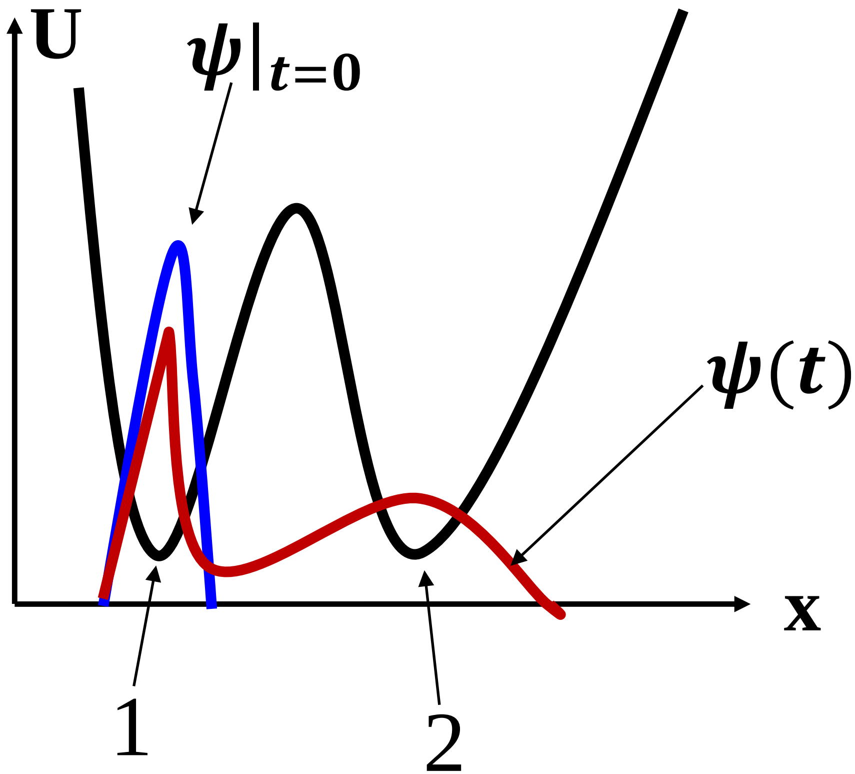
The state $(\Psi_1 + \Psi_2)/\sqrt{2}$ is a new combined state which is called "quantum superposition" of state (1) and (2)

TM

George Gamow

(He also developed Big Bang theory)

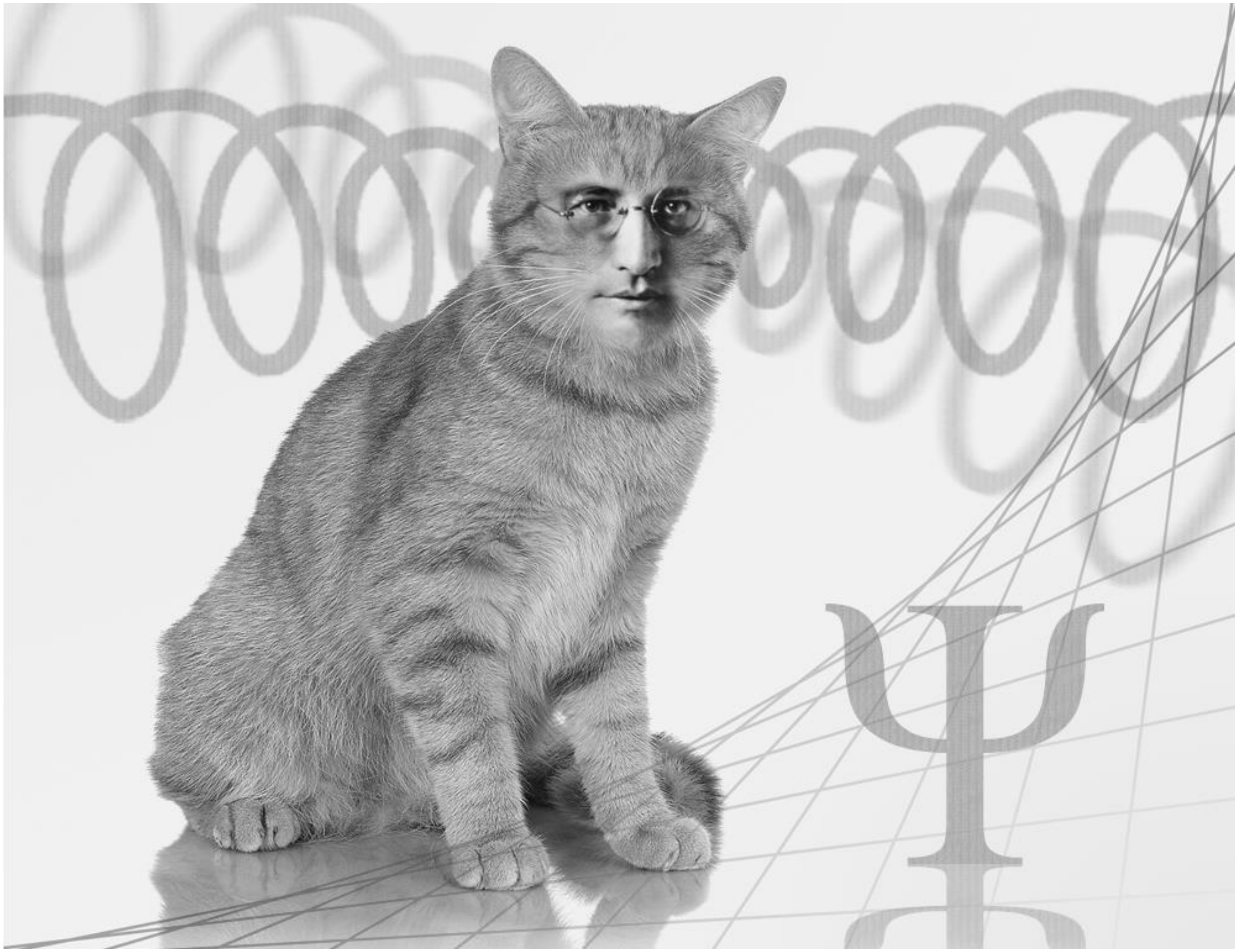
Quantum tunneling



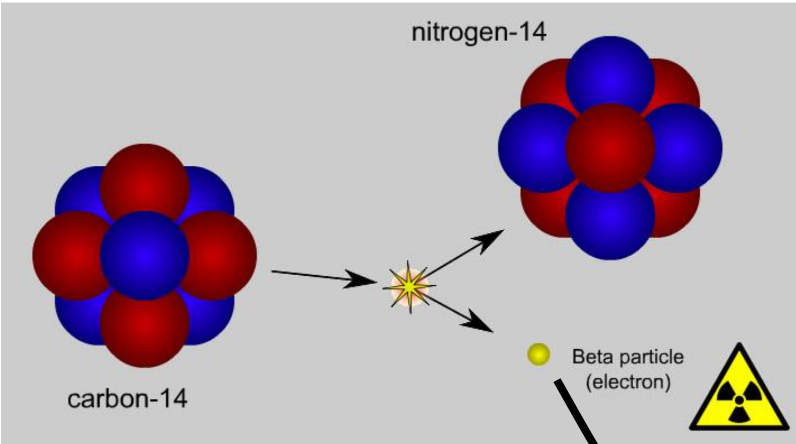
Quantum tunneling is possible since quantum superpositions of states are possible.

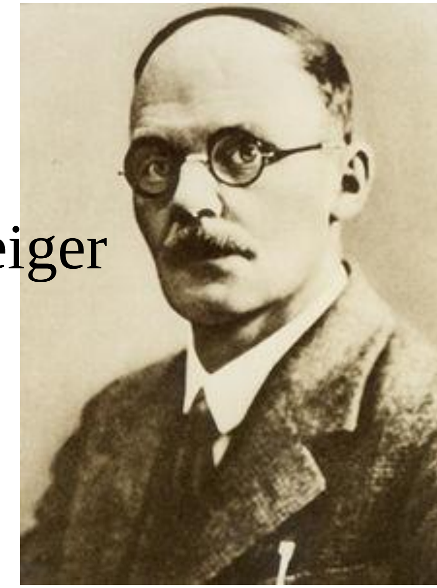
Schrödinger cat –
the ultimate macroscopic quantum phenomenon

E. Schrödinger, Naturwiss. **23** (1935), 807.



Schrödinger cat – thought experiment

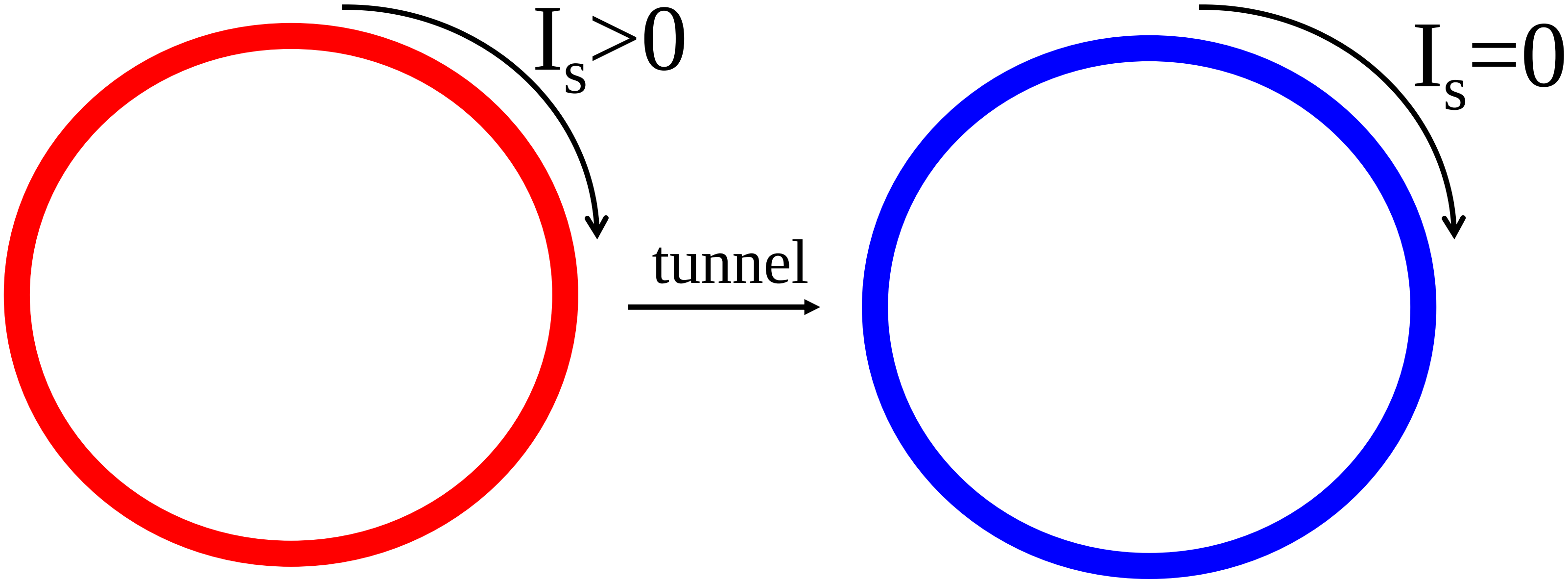




Hans Geiger

Geiger counter

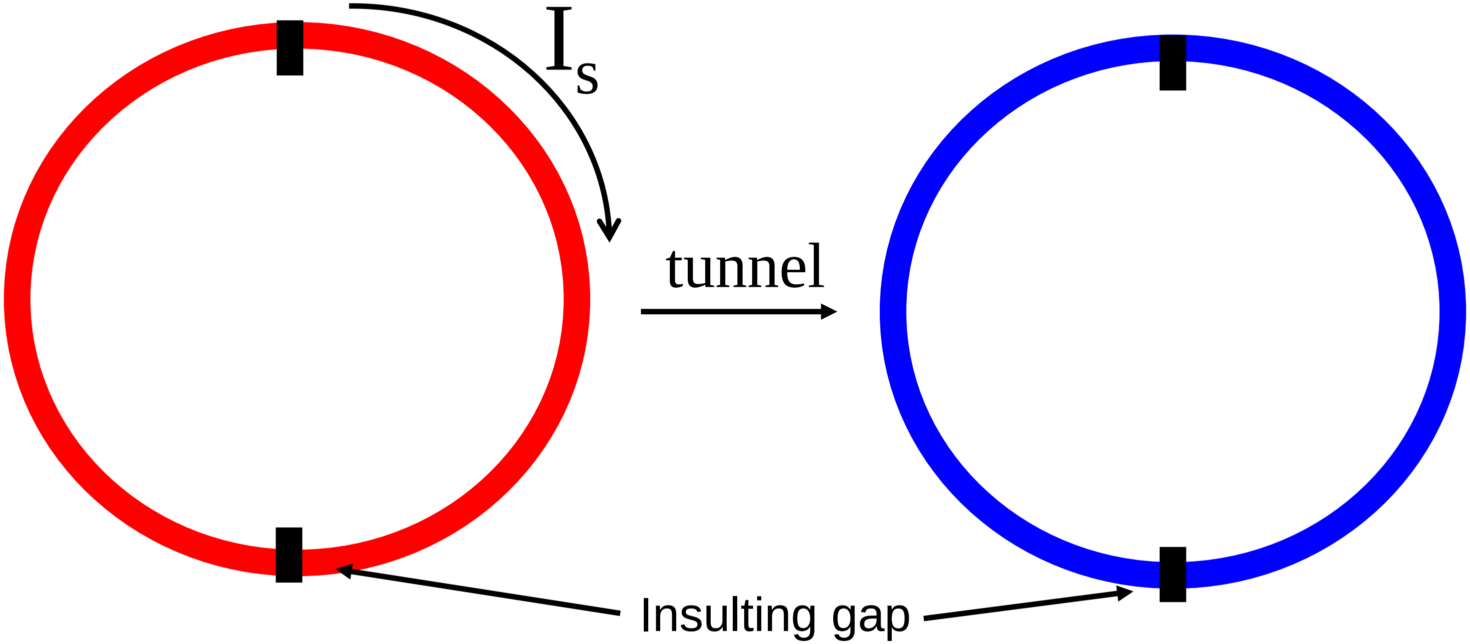
What sort of tunneling we will consider?



-Red color represents some strong current in the superconducting wire loop

-Blue color represents no current or a much smaller current in the loop

Previous results relate loops with insulating interruptions (SQUIDs)



-Red color represents some strong current in the superconducting loop

-Blue color represents no current or very little current
in the superconducting loop

Leggett's prediction for macroscopic quantum tunneling (MQT) in SQUIDs

80 Supplement of the Progress of Theoretical Physics, No. 69, 1980

Macroscopic Quantum Systems and the Quantum Theory of Measurement

A. J. LEGGETT

School of Mathematical and Physical Sciences
University of Sussex, Brighton BN1 9QH

(Received August 27, 1980)

It is this property which makes a SQUID the most promising candidate to date for observing macroscopic quantum tunnelling; if it should ever become possible to observe macroscopic quantum *coherence*, the low entropy and consequent lack of dissipation will be absolutely essential.[21)]

MQT report by Kurkijarvi and collaborators (1981)

VOLUME 47, NUMBER 9 PHYSICAL REVIEW LETTERS 31 AUGUST 1981

Decay of the Zero-Voltage State in Small-Area, High-Current-Density Josephson Junctions

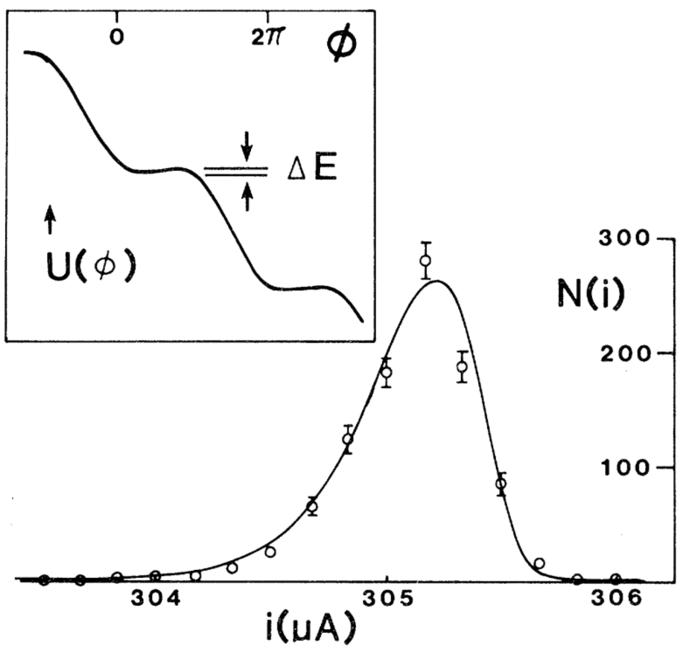


FIG. 1. Measured distribution for $T = 1.6$ K for small high-current-density junction. The solid line is a fit by the CL theory for $R = 20\ \Omega$, $C = 8$ fF, and $i_{cFF} = 310.5\ \mu$A. The inset is $U(\varphi)$ for $x = 0.8$ with barrier ΔE.

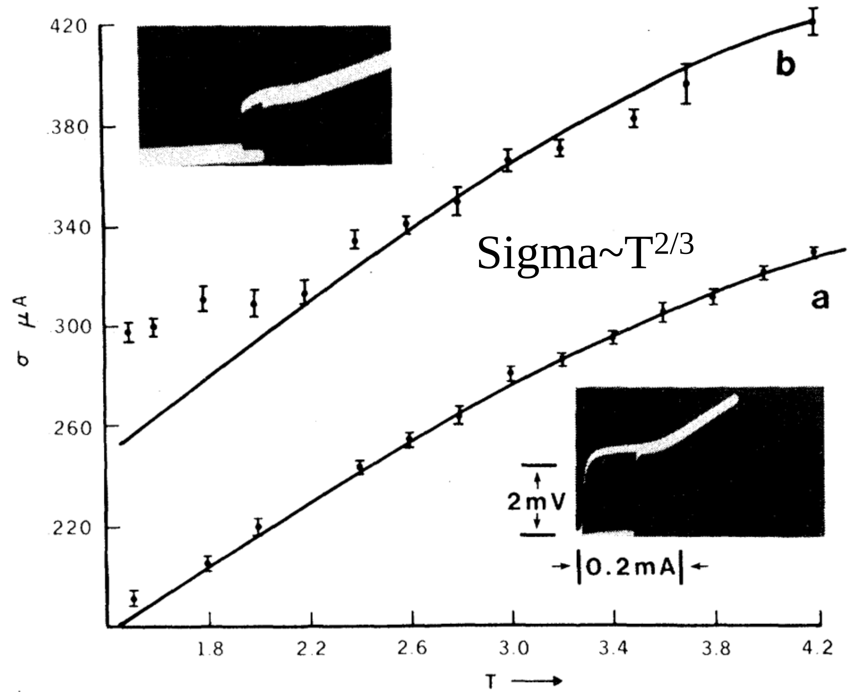
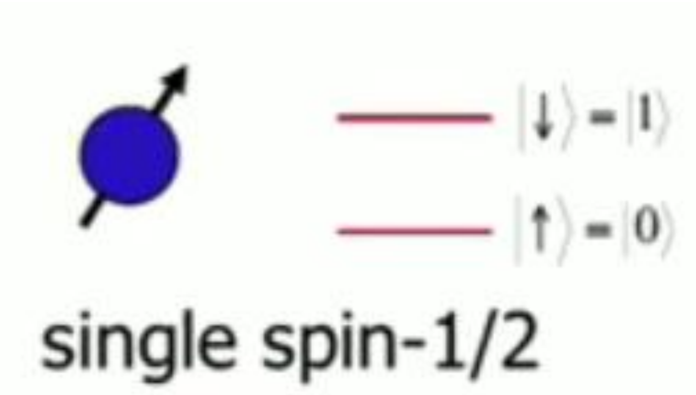
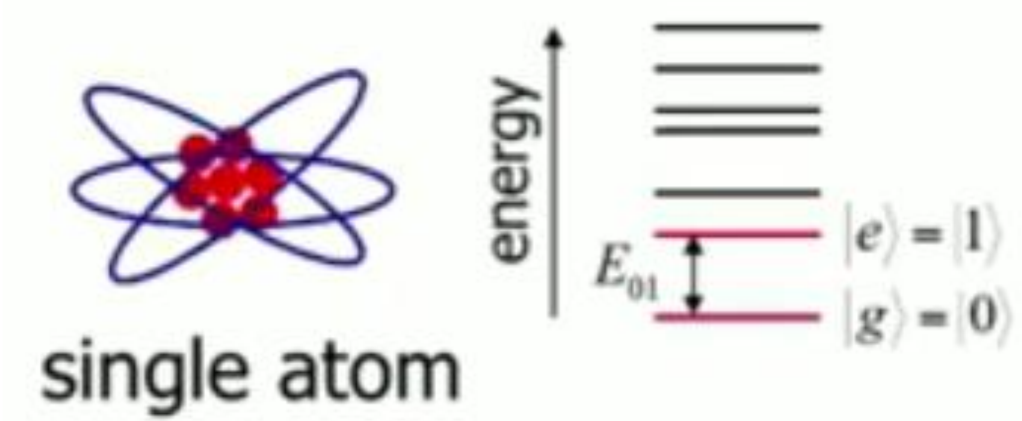


FIG. 2. Measured distribution widths σ vs T for two junctions with current sweep of ~400 μA/sec. Curve a is lower current density junction data and curve b is higher density junction data. The traces adjacent to the plots are the corresponding $I-V$ characteristics at 4.2 K. The scales are the same for both traces.

Types of Qubit



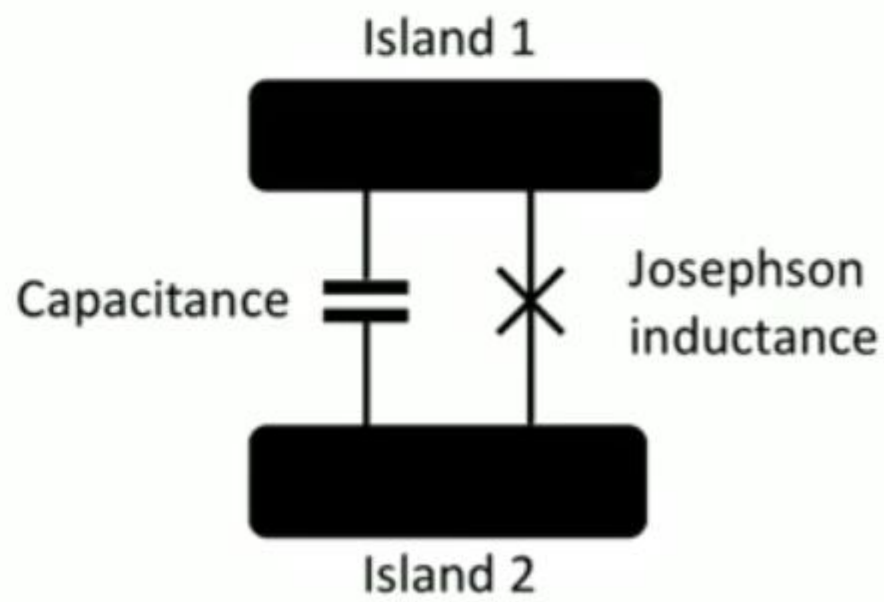


Quantum state:
$|\psi>=A*|0>+B*1>$

$A^2+B^2=1$

A and B are
complex numbers

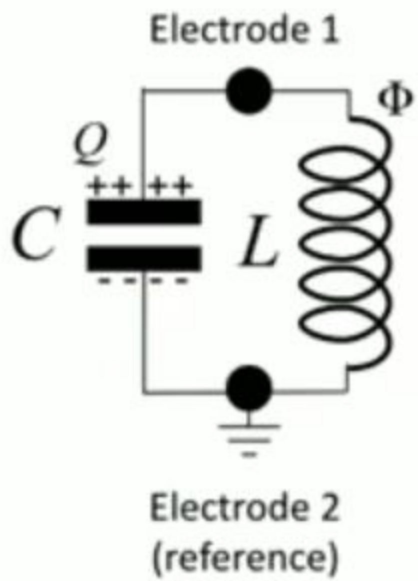
Transmon Qubit



Theory of transmons: J. Koch et al., Phys. Rev. A **76**, 042319 (2007).

Quantization of electrical circuits

The quantized *LC* oscillator



Hamiltonian:

$$\hat{H}_{LC} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L}$$

Capacitive term   Inductive term

Canonically conjugate variables:

$\hat{\Phi}$ = Flux through the inductor.

$\hat{Q}$ = Charge on capacitor plate.

$$\left[\hat{\Phi}, \hat{Q}\right] = i\hbar$$

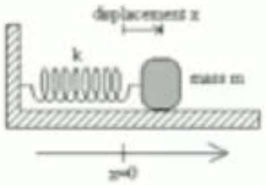
M. Devoret, Les Houches Session LXIII (1995)

Discrete energy spectrum of the LC-circuit

Correspondence with simple harmonic oscillator

$$\hat{H}_{LC} = \frac{\hat{\Phi}^2}{2L} + \frac{\hat{Q}^2}{2C} \qquad \left[\hat{\Phi}, \hat{Q}\right] = i\hbar$$

$$\hat{H}_{SHO} = \frac{k\hat{X}^2}{2} + \frac{\hat{P}^2}{2m} \qquad \left[\hat{X}, \hat{P}\right] = i\hbar$$



Correspondence: $\hat{\Phi} \leftrightarrow \hat{X}$, $\hat{Q} \leftrightarrow \hat{P}$, $L \leftrightarrow \frac{1}{k}$, $C \leftrightarrow m$, $\omega = \frac{1}{\sqrt{LC}} \leftrightarrow \sqrt{\frac{k}{m}}$

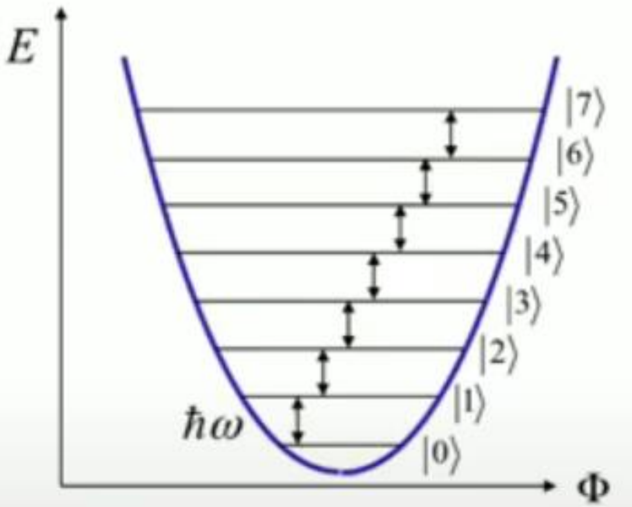
Solve using ladder operators:

$$\hat{a} = \left(\frac{\hat{Q}}{Q_{zpf}} - i\frac{\hat{\Phi}}{\Phi_{zpf}}\right) \qquad \Phi_{zpf} = \sqrt{2\hbar Z}$$

$$\hat{a}^\dagger = \left(\frac{\hat{Q}}{Q_{zpf}} + i\frac{\hat{\Phi}}{\Phi_{zpf}}\right) \qquad Q_{zpf} = \sqrt{2\hbar / Z}$$

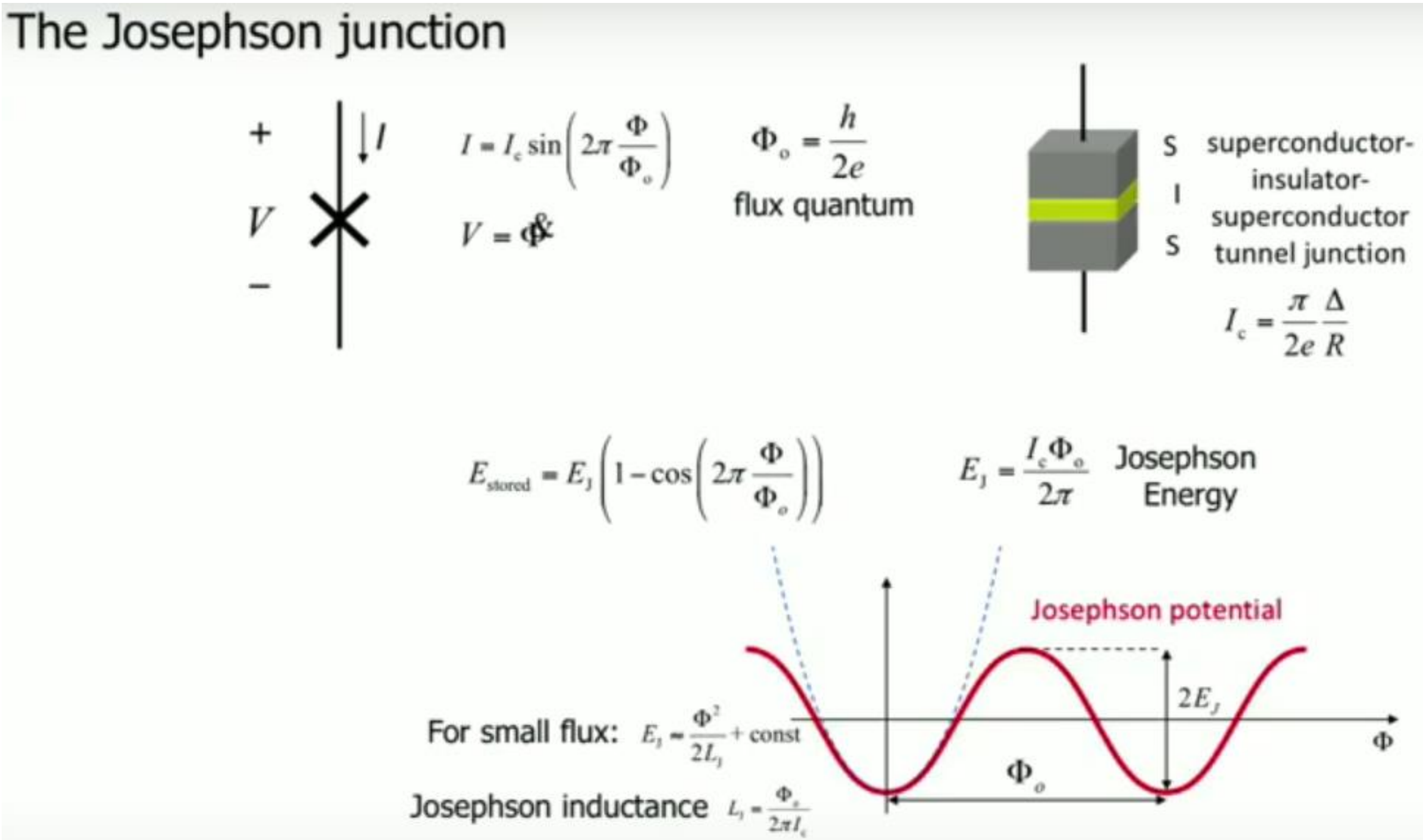
$$Z = \omega L = \frac{1}{\omega C} = \sqrt{\frac{L}{C}}$$

$$\hat{H}_{LC} = \hbar\omega\left(\hat{a}^\dagger \hat{a} + \frac{1}{2}\right) \qquad \left[\hat{a}_r, \hat{a}_r^\dagger\right] = 1$$



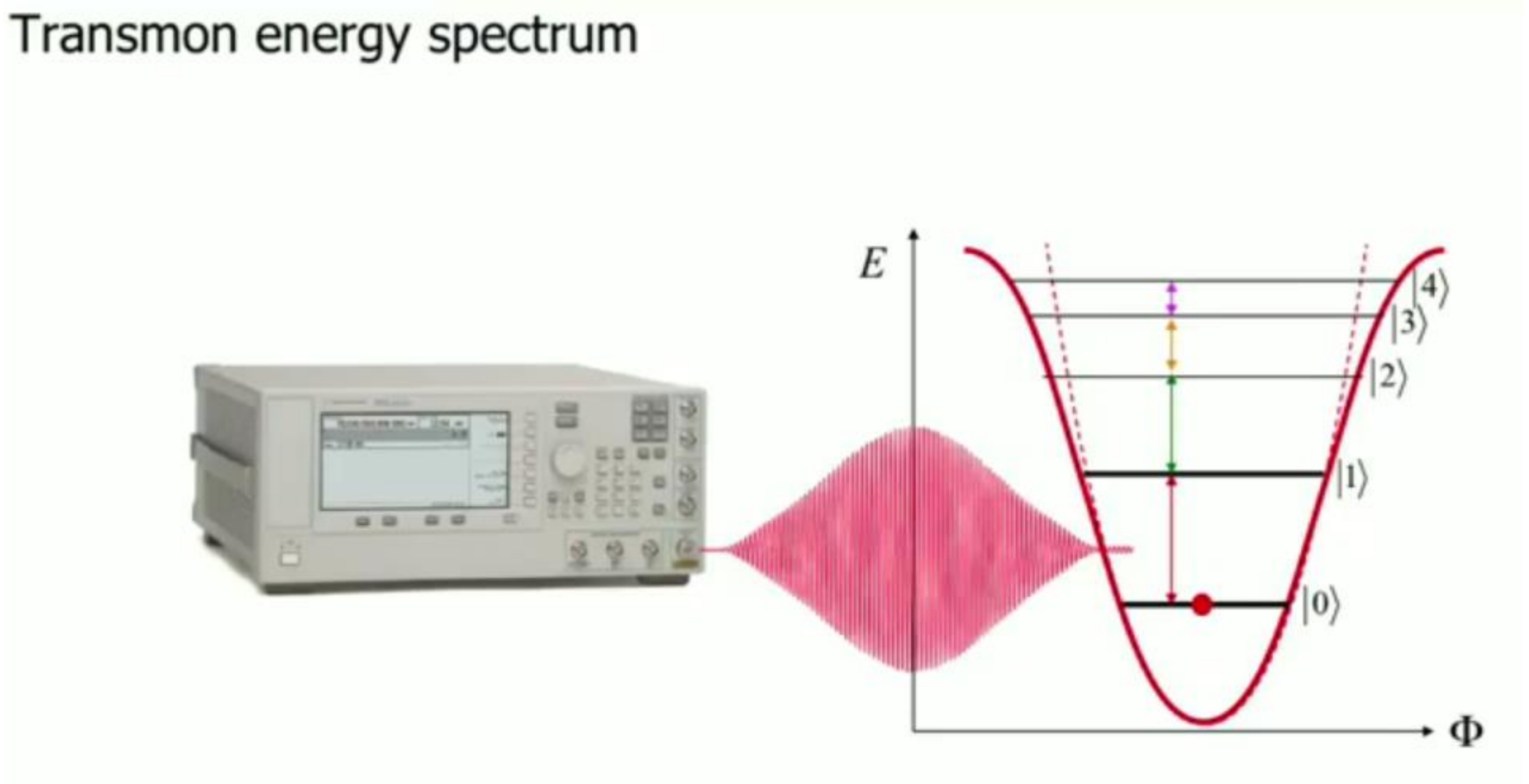
M. Devoret, Les Houches Session LXIII (1995)

Non-harmonicity is the key factor

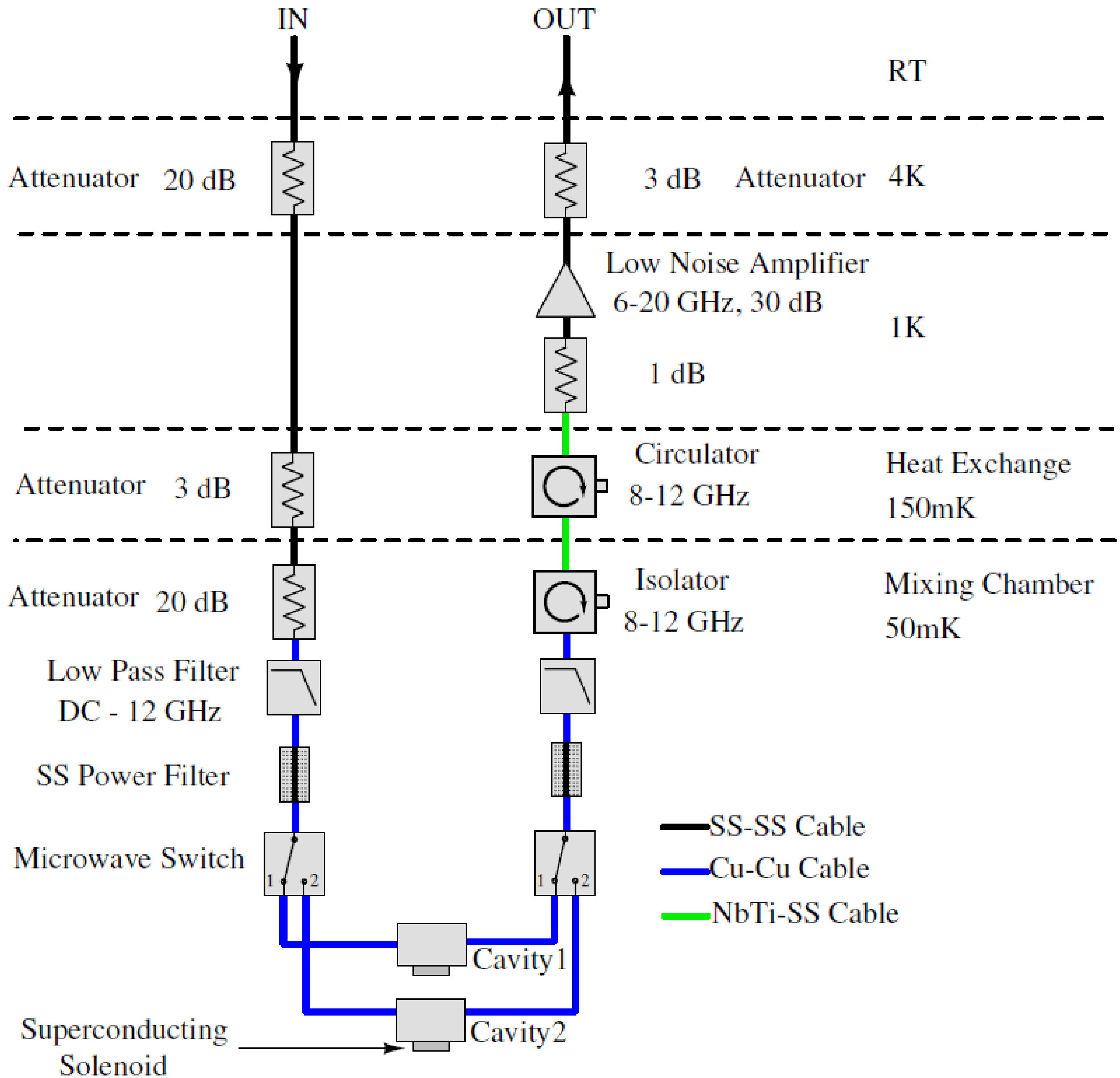


M. Devoret, Les Houches Session LXIII (1995)

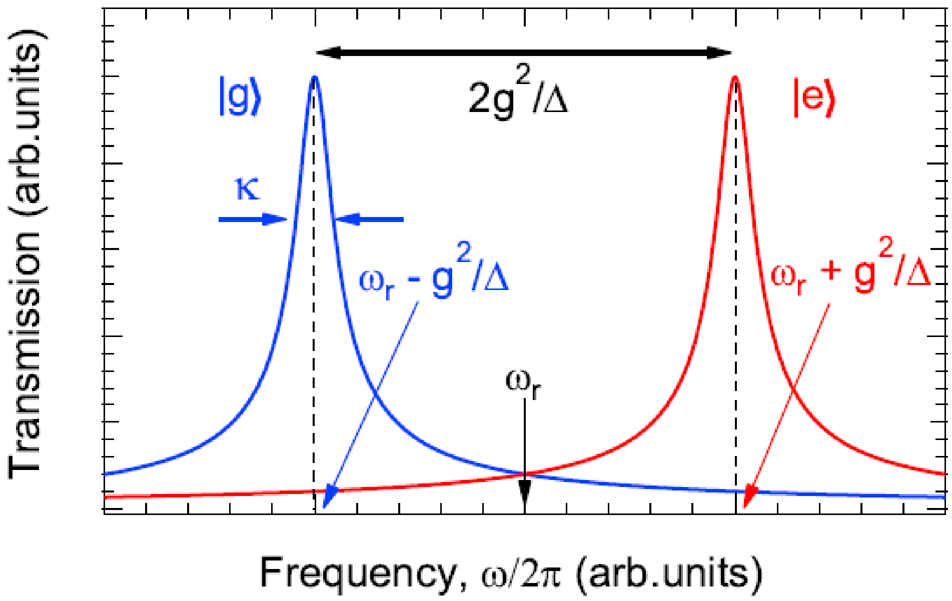
Non-harmonicity is the key factor



How transmons are measured

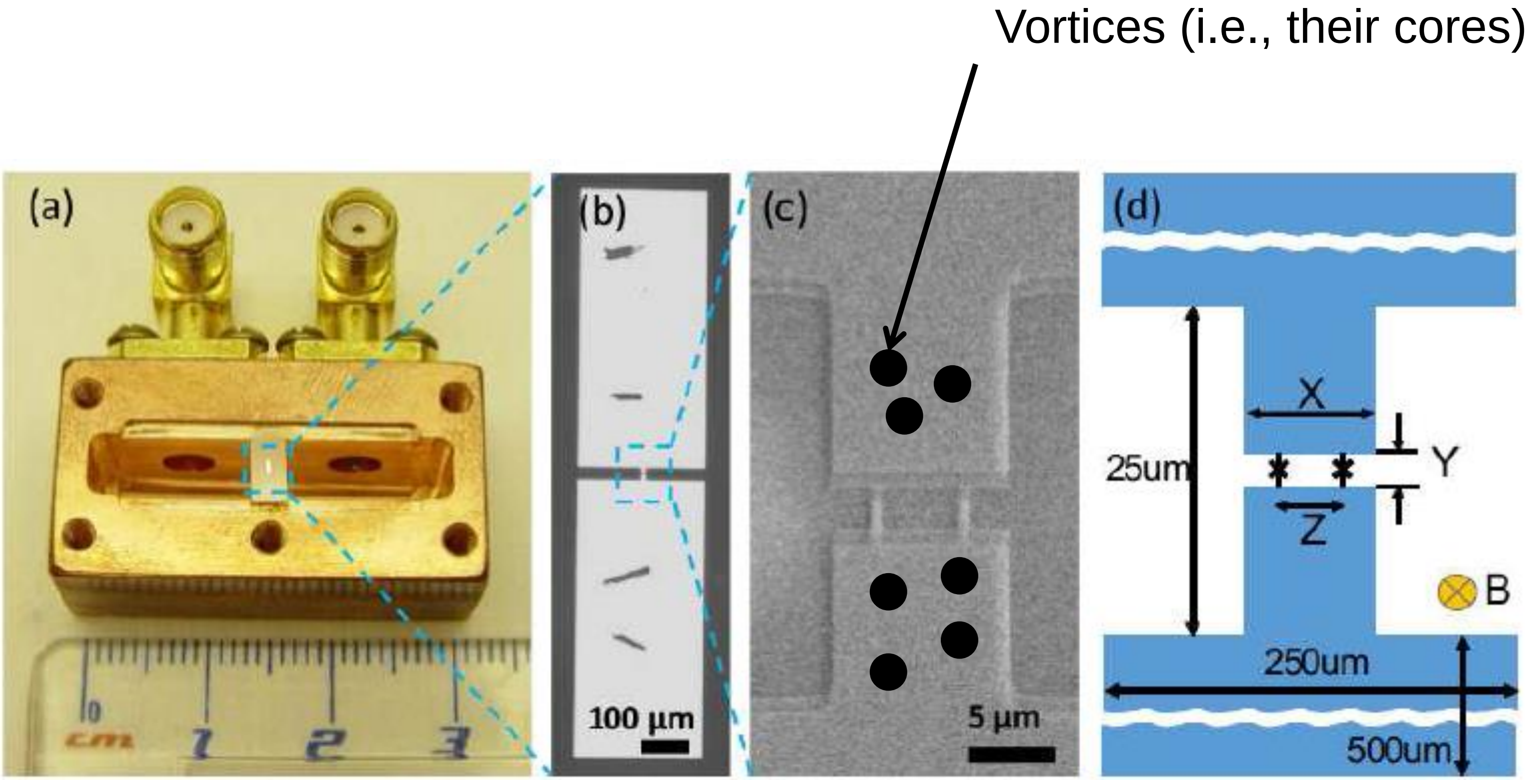


How transmons are measured



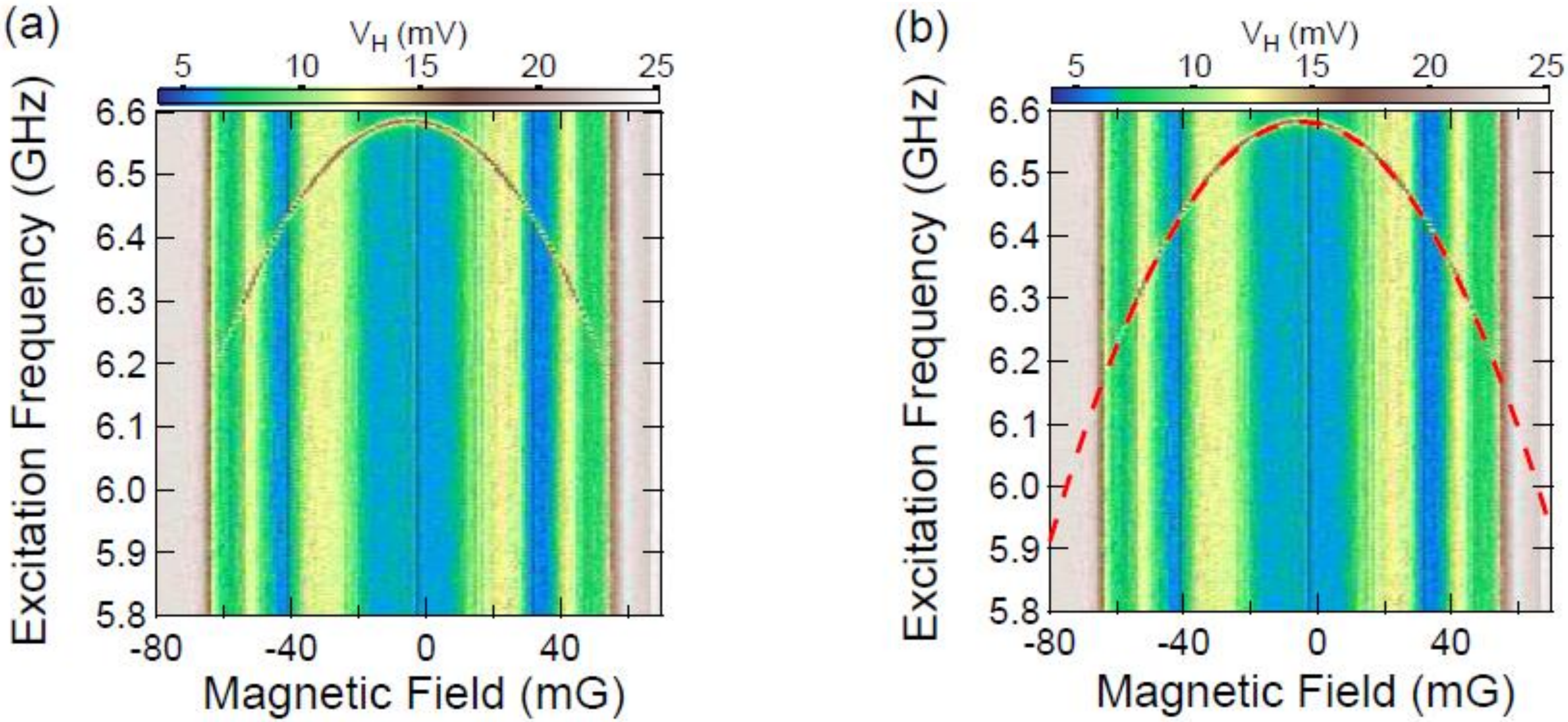
Transmission versus frequency for dispersive measurement. ω_r denotes the resonant frequency without the dispersive shift. Depending on the qubit state, the cavity frequency is pulled by $\pm g^2/\Delta$.

Transmon Meissner Qubit in Cu 3D cavity

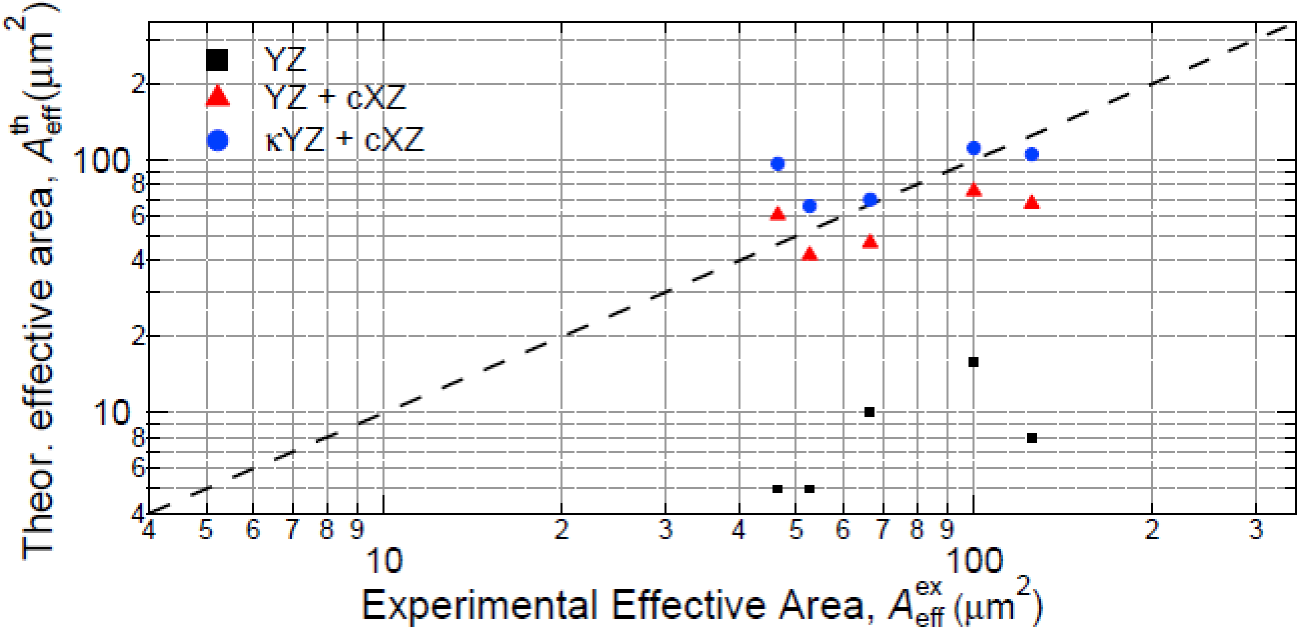


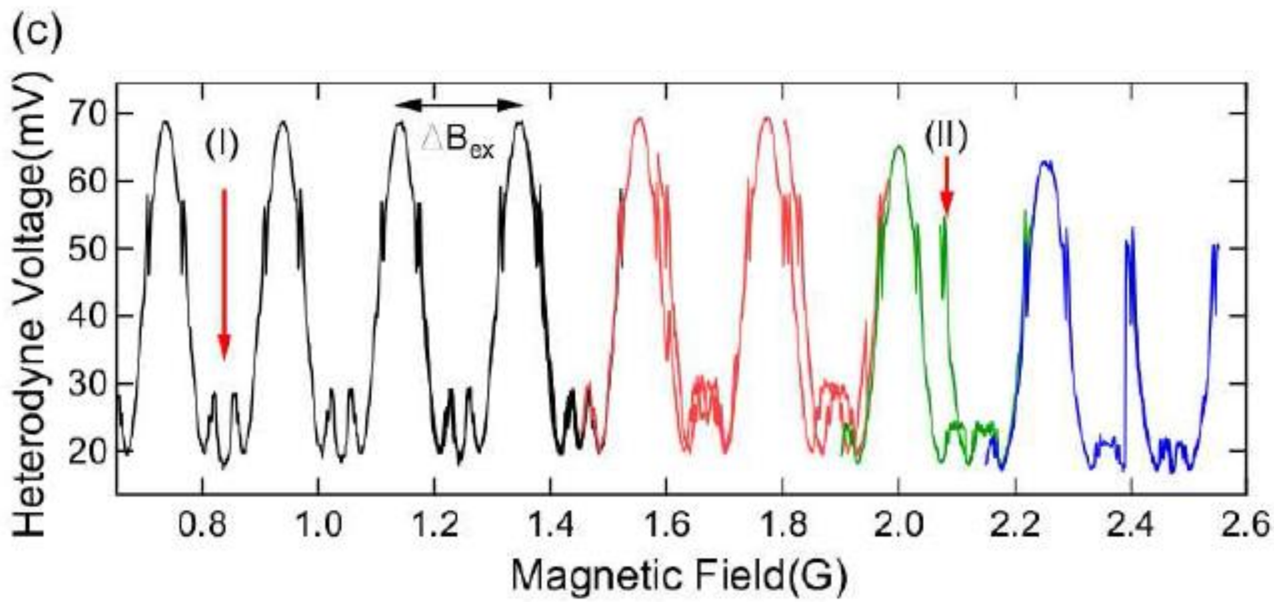
J. Ku, Z. Yoscovits, A. Levchenko, J. Eckstein, and A. Bezryadin,
Decoherence and radiation-free relaxation in Meissner transmon qubit coupled to Abrikosov vortices,
Physical Review B **94**, 165128(1-14) (2016).

Magnetic field effects

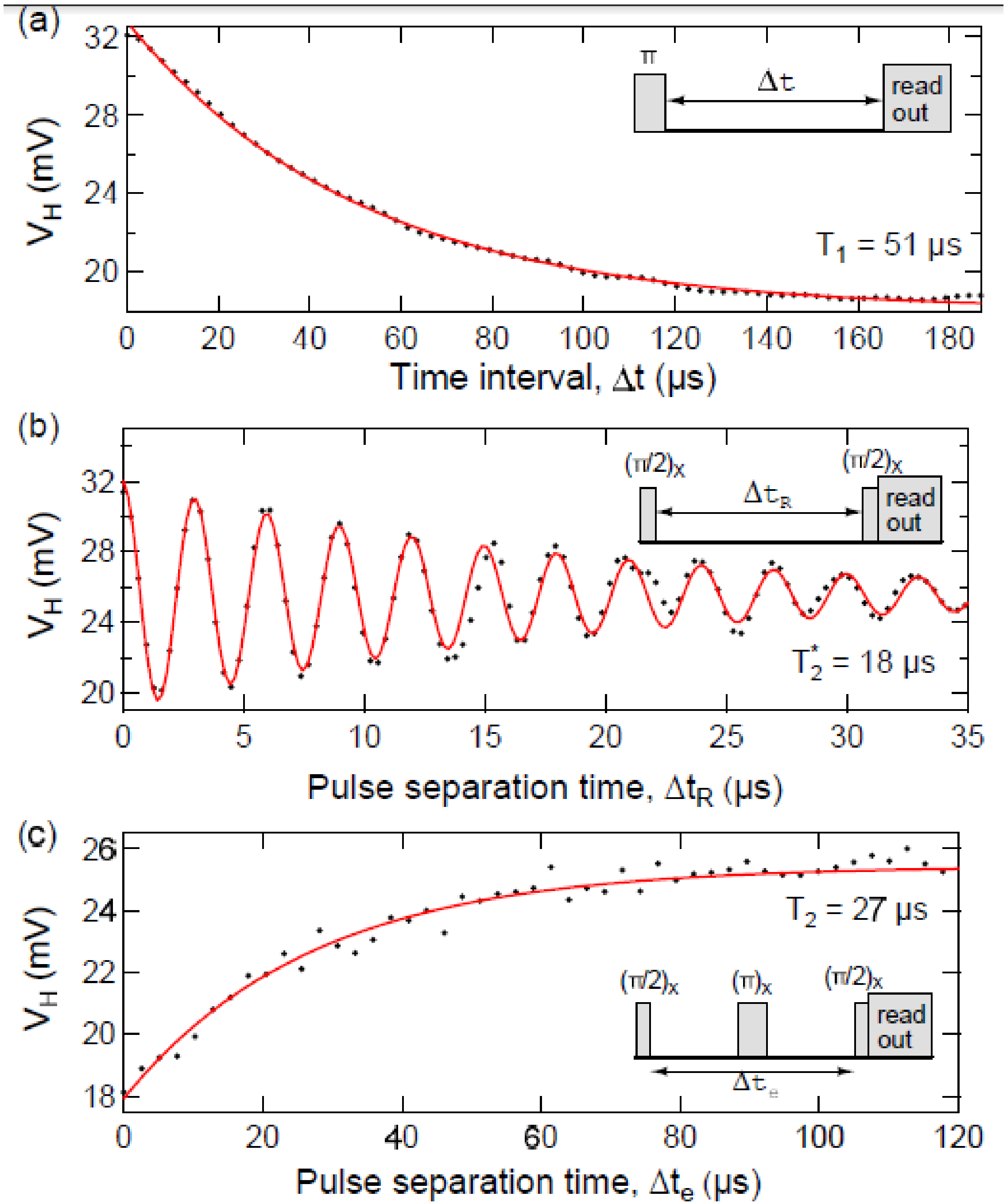


Magnetic field effects





Examples quantum time-domain oscillation: Ramsey fringe



Conclusions

- Superconductivity allows us to test fundamental quantum phenomena, for example the macroscopic quantum tunneling and macroscopic quantum coherence

- Superconductivity has been used to design many types of useful devices. The examples considered are the SQUIDS and the qubits for quantum computers