

# Superconductivity in pictures

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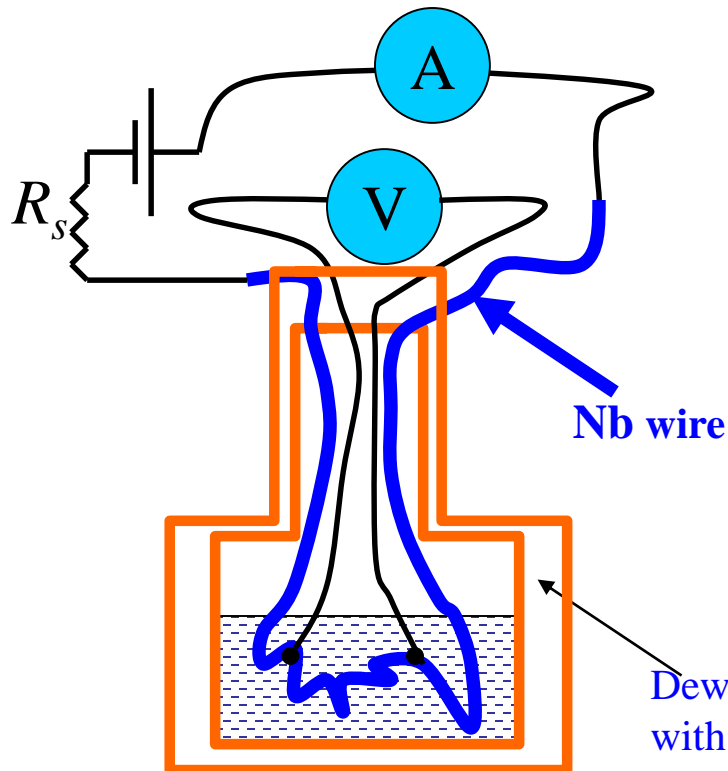


# How one can measure superconductivity?

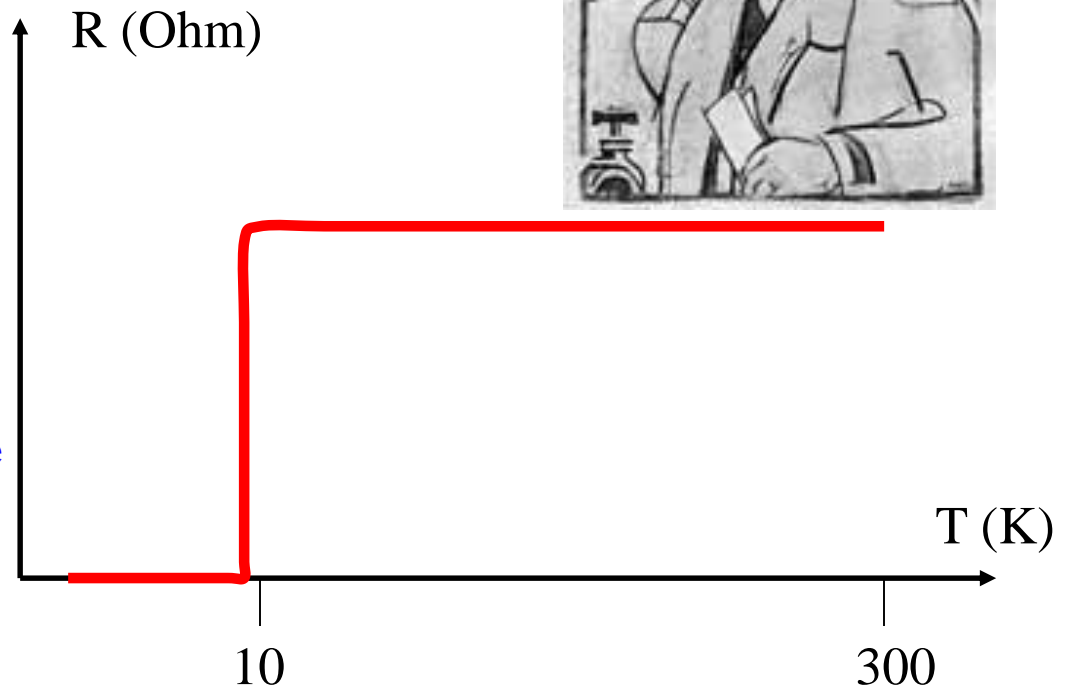
Electrical resistance of some metals drops to zero below a certain temperature which is called "critical temperature" (H. K. O. 1911)

## How to observe superconductivity

- Take Nb wire
- Connect to a voltmeter and a current source
- Put into helium Dewar
- Measure electrical resistance



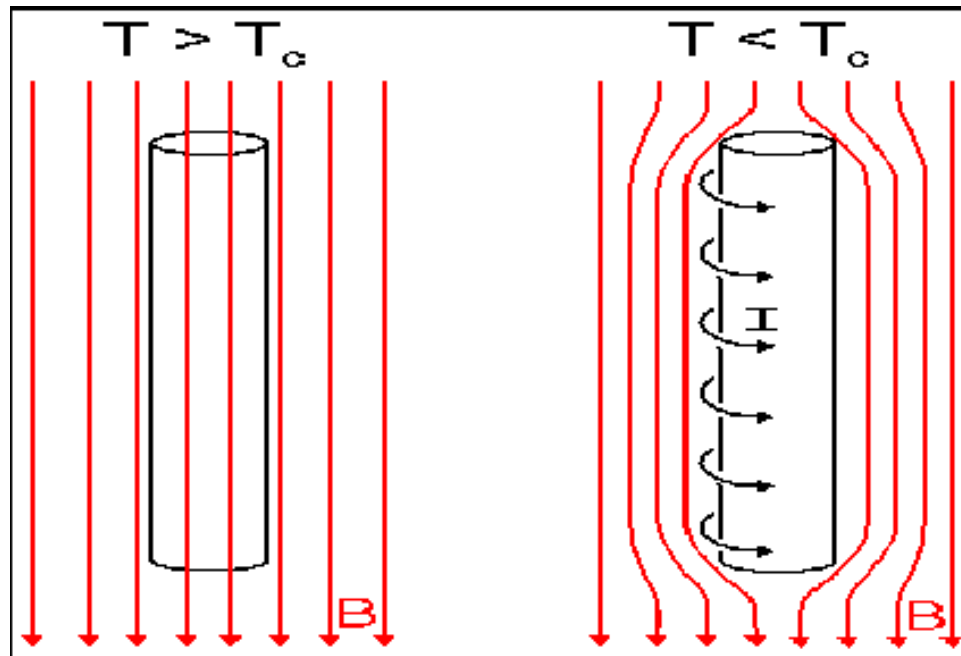
## Heike Kamerling Onnes



Dewar with liquid Helium (4.2K)



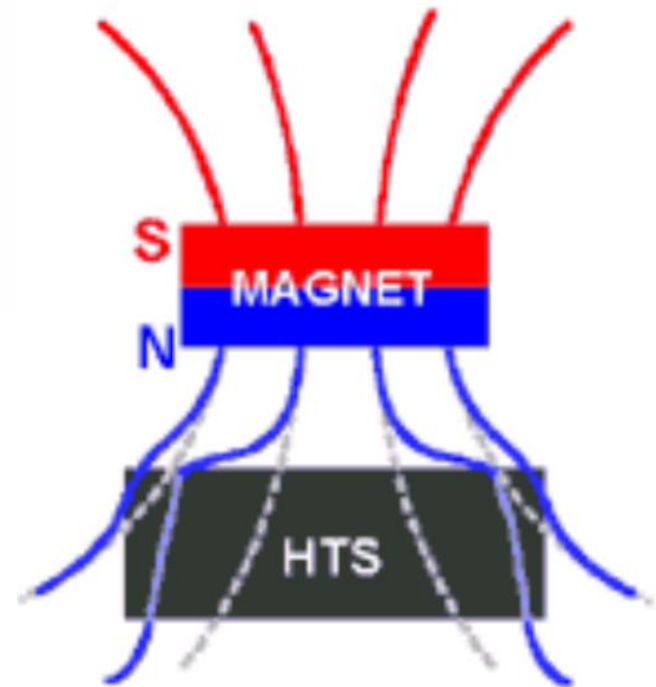
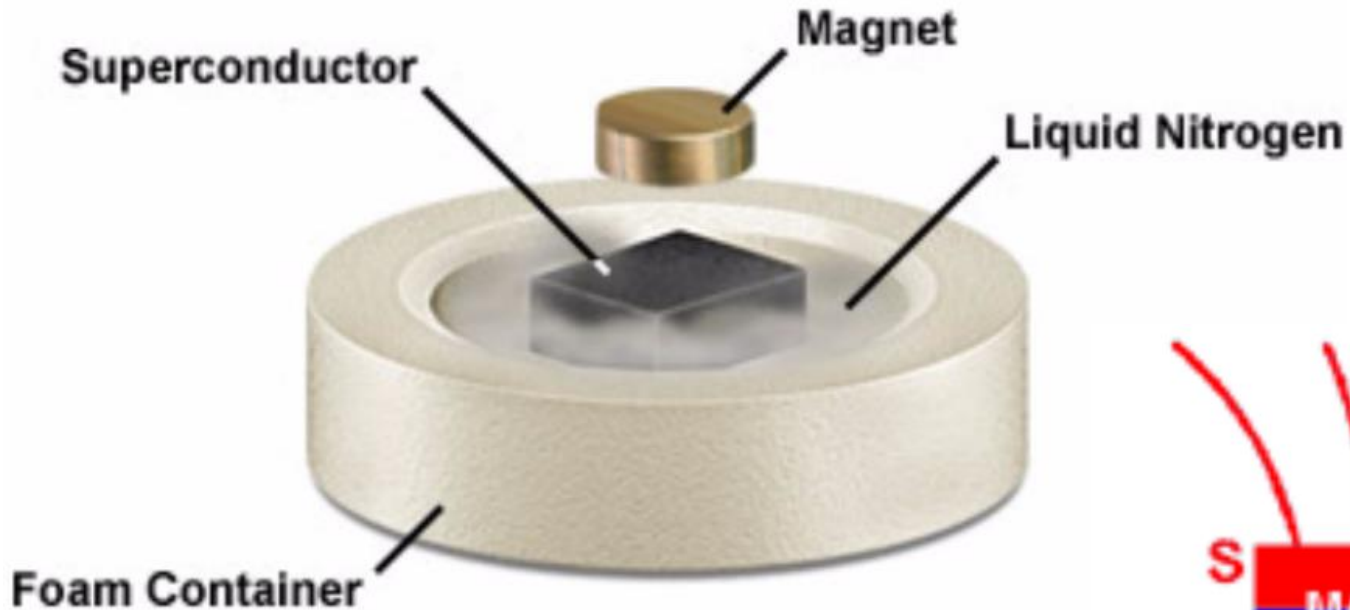
# Meissner effect – the key signature of superconductivity



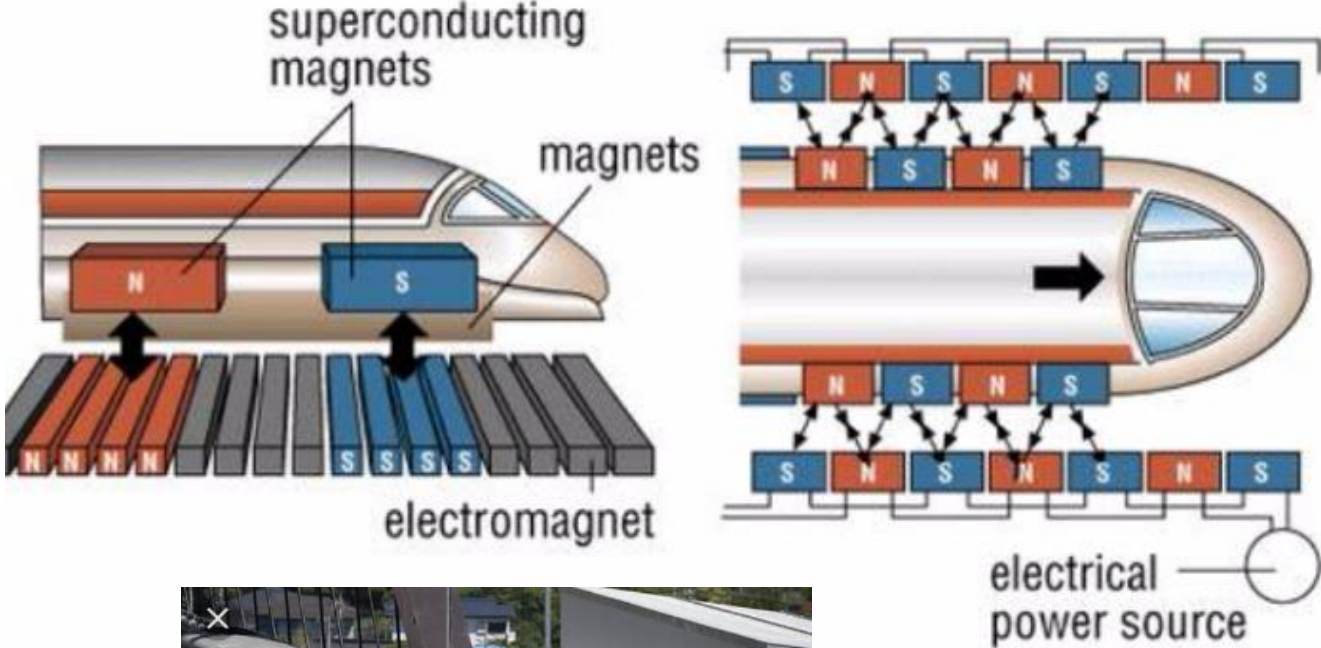
Formula	$T_c$ (K)	$H_c$ (T)	Type	BCS
<b>Elements</b>				
Al	1.20	0.01	I	yes
Cd	0.52	0.0028	I	yes
Diamond:B	11.4	4	II	yes
Ga	1.083	0.0058	I	yes
Hf	0.165		I	yes
$\alpha$ -Hg	4.15	0.04	I	yes
$\beta$ -Hg	3.95	0.04	I	yes
In	3.4	0.03	I	yes
Ir	0.14	0.0016 <sup>[7]</sup>	I	yes
$\alpha$ -La	4.9		I	yes
$\beta$ -La	6.3		I	yes
Mo	0.92	0.0096	I	yes
Nb	9.26	0.82	II	yes
Os	0.65	0.007	I	yes

# Magnetic levitation

## The Meissner Effect



# Magnetic levitation train



blogs.wsj.com

U.S. Transportation Secretary Foxx Rides on Japan's Maglev Train - Jap...





Discovery of the supercurrent, known now as proximity effect, in SNS junctions

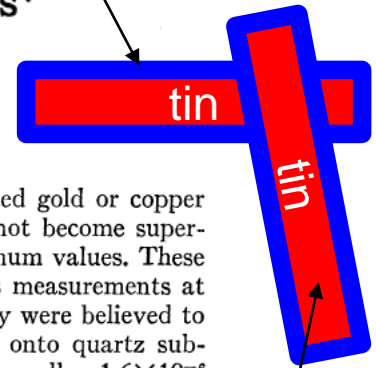
# Superconductivity of Contacts with Interposed Barriers\*

HANS MEISSNER†

*Department of Physics, The Johns Hopkins University, Baltimore, Maryland*

(Received August 25, 1959)

Non-superconductor (normal metal, i.e., Ag)



Resistance vs current diagrams and "Diagrams of State" have been obtained for 63 contacts between crossed wires of tin. The wires were plated with various thicknesses of the following metals: copper, silver, gold, chromium, iron, cobalt, nickel, and platinum. The contacts became superconducting, or showed a noticeable decrease of their resistance at lower temperatures if the plated films were not too thick. The limiting thicknesses were about  $35 \times 10^{-6}$  cm for Cu, Ag, and Au;  $7.5 \times 10^{-6}$  cm for Pt,  $4 \times 10^{-6}$  cm for Cr, and less than  $2 \times 10^{-6}$  cm for the ferromagnetic metals Fe, Co, and Ni. The investigation was extended to measurements of the resistance of contacts between crossed wires of copper or gold plated with various thicknesses of tin. Simultaneous measurements

of the (longitudinal) resistance of the tin-plated gold or copper wires showed that these thin films of tin do not become superconducting for thicknesses below certain minimum values. These latter findings are in agreement with previous measurements at Toronto. The measurements at Toronto usually were believed to be unreliable because films of tin evaporated onto quartz substrates can be superconducting at thicknesses as small as  $1.6 \times 10^{-6}$  cm. It is now believed that just as superconducting electrons can drift into an adjoining normal conducting layer and make it superconducting, normal electrons can drift into an adjoining superconducting layer and prevent superconductivity.

Superconductor (tin)

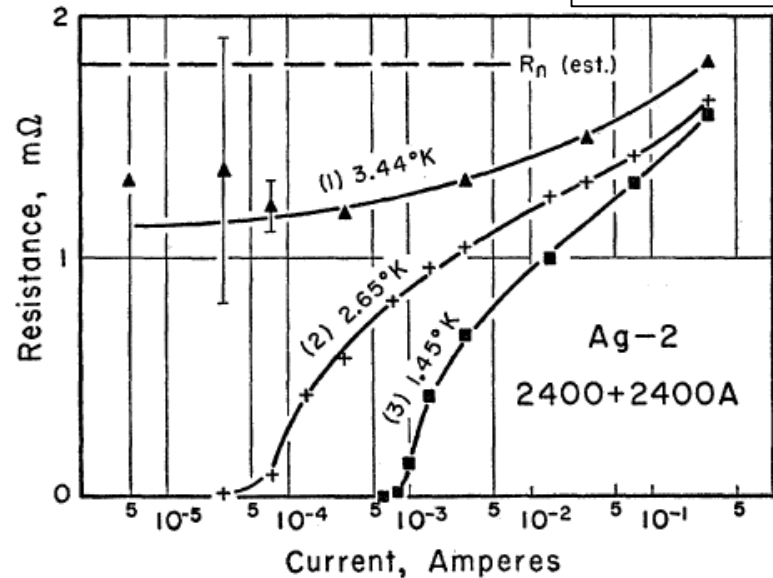
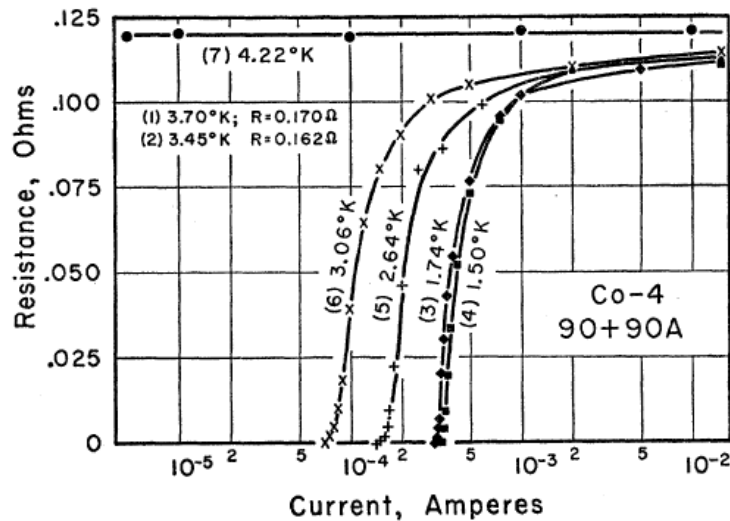
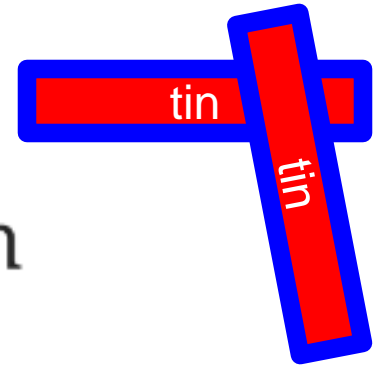


FIG. 1. Resistance vs current diagram of cobalt-plated contact Co 4, representative of diagrams type A.

FIG. 2. Resistance vs current diagram of silver-plated contact Ag 2, representative of diagrams type B.



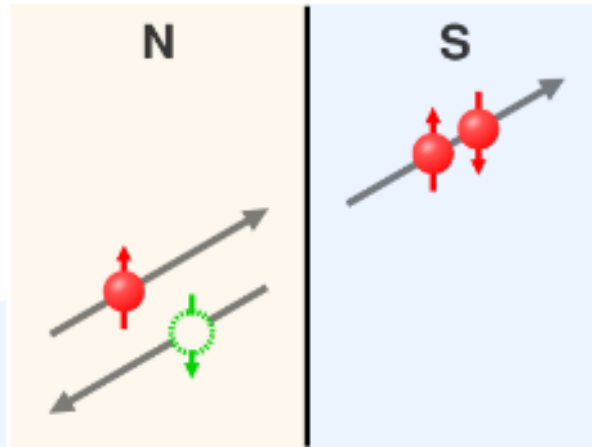
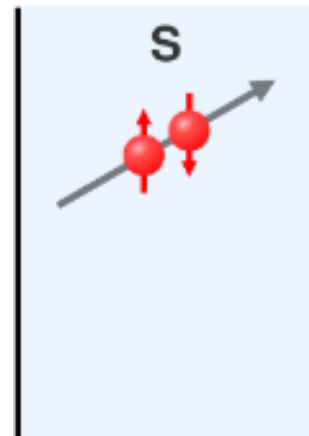
# Explanation of the supercurrent in SNS junctions --- Andreev reflection



[www.kapitza.ras.ru](http://www.kapitza.ras.ru)  
[www.kapitza.ras.ru/~andreev/afan...](http://www.kapitza.ras.ru/~andreev/afan...)



## Andreev reflection



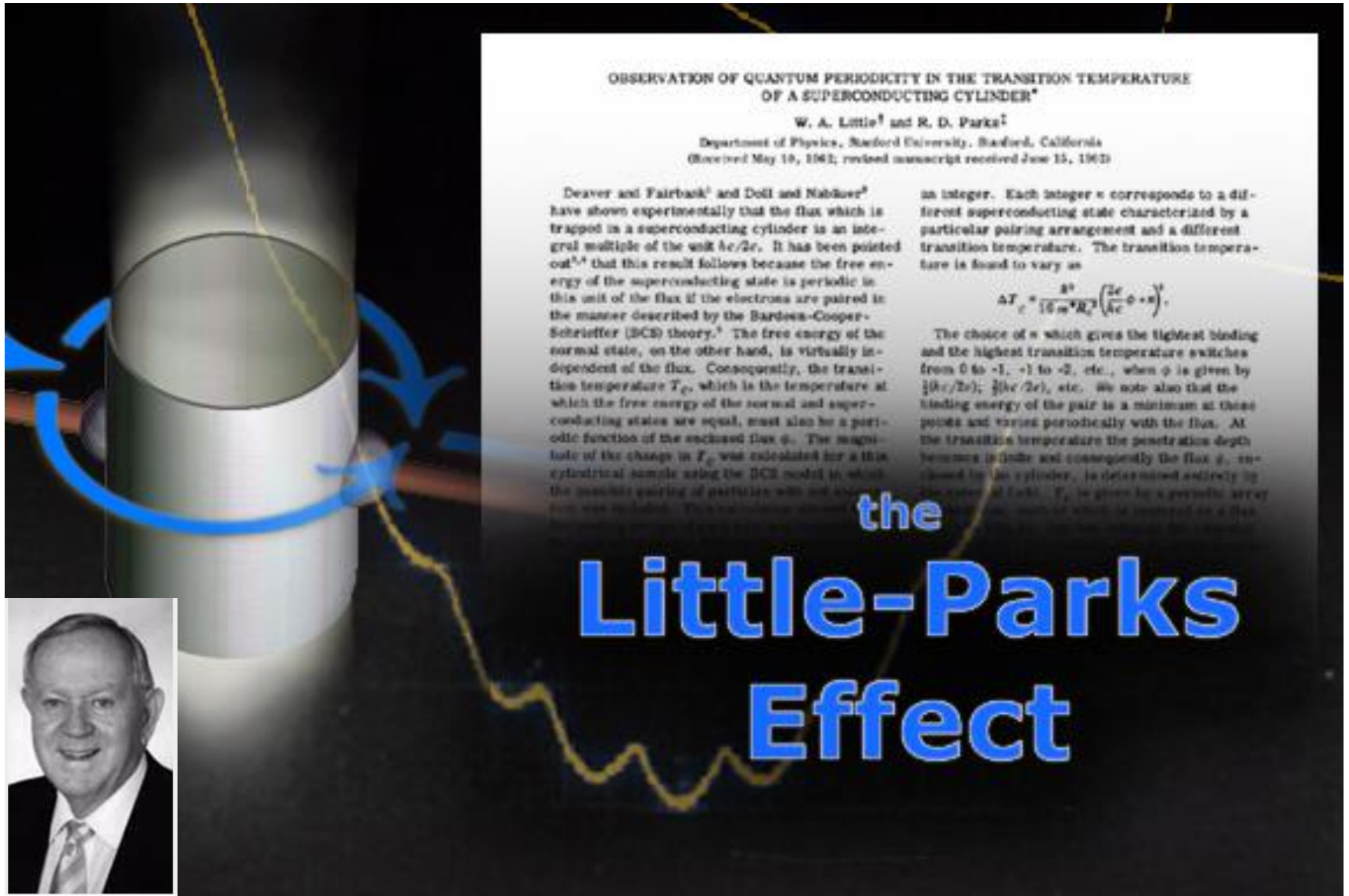
A.F. Andreev, 1964



TM

# Searching for an explanation: Little-Parks effect ('62)

The basic idea: magnetic field induces non-zero vector-potential, which produces non-zero superfluid velocity, thus reducing the  $T_c$ .



OBSERVATION OF QUANTUM PERIODICITY IN THE TRANSITION TEMPERATURE OF A SUPERCONDUCTING CYLINDER\*

W. A. Little<sup>†</sup> and R. D. Parks<sup>‡</sup>


Department of Physics, Stanford University, Stanford, California  
(Received May 19, 1962; revised manuscript received June 15, 1962)

Deaver and Fairbank<sup>1</sup> and Doll and Nabauer<sup>2</sup> have shown experimentally that the flux which is trapped in a superconducting cylinder is an integral multiple of the unit  $hc/2e$ . It has been pointed out<sup>3,4</sup> that this result follows because the free energy of the superconducting state is periodic in this unit of the flux if the electrons are paired in the manner described by the Bardeen-Cooper-Schrieffer (BCS) theory.<sup>5</sup> The free energy of the normal state, on the other hand, is virtually independent of the flux. Consequently, the transition temperature  $T_c$ , which is the temperature at which the free energy of the normal and superconducting states are equal, must also be a periodic function of the enclosed flux  $\phi$ . The magnitude of the change in  $T_c$  was calculated for a thin cylindrical sample using the BCS model in which the possible pairing of particles with net spin zero was included. The calculation showed that  $T_c$  is an integer. Each integer  $n$  corresponds to a different superconducting state characterized by a particular pairing arrangement and a different transition temperature. The transition temperature is found to vary as

$$\Delta T_c = \frac{8^3}{15\pi^2 R^2} \left( \frac{2e}{hc} \phi + n \right)^2.$$

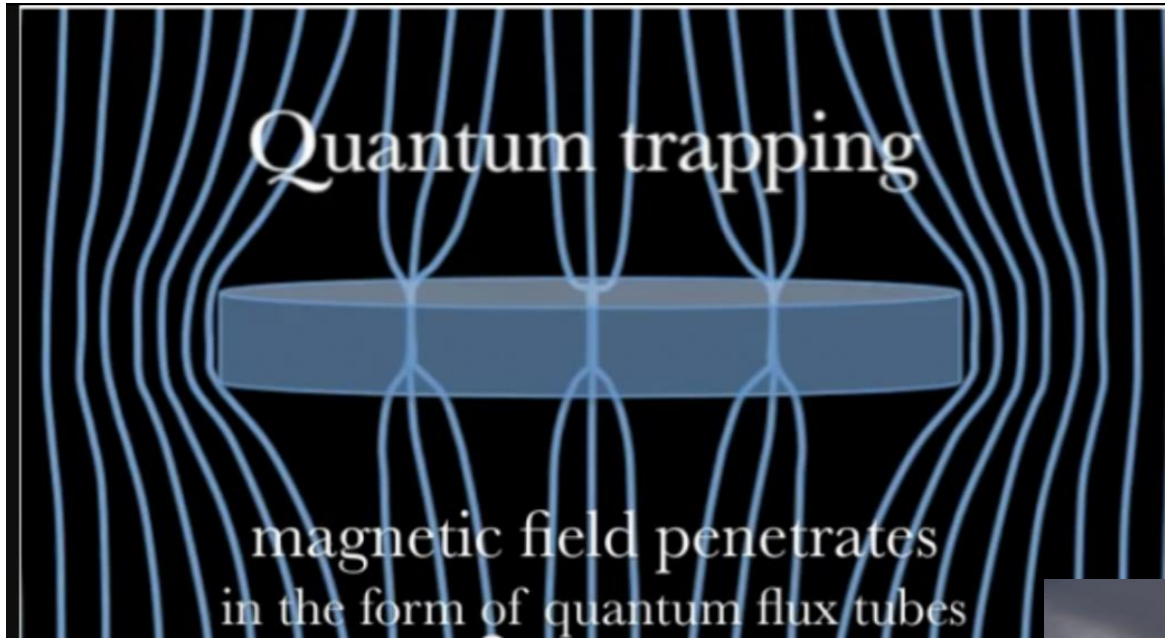
The choice of  $n$  which gives the tightest binding and the highest transition temperature switches from 0 to -1, -1 to -2, etc., when  $\phi$  is given by  $\frac{1}{2}(hc/2e)$ ;  $\frac{3}{2}(hc/2e)$ , etc. We note also that the binding energy of the pair is a minimum at these points and varies periodically with the flux. At the transition temperature the penetration depth becomes infinite and consequently the flux  $\phi$ , enclosed by the cylinder, is determined entirely by the external field.  $T_c$  is given by a periodic array of peaks and valleys, the period of which is constant at a flux

the  
**Little-Parks  
Effect**





# Superconducting vortices produced by magnetic field

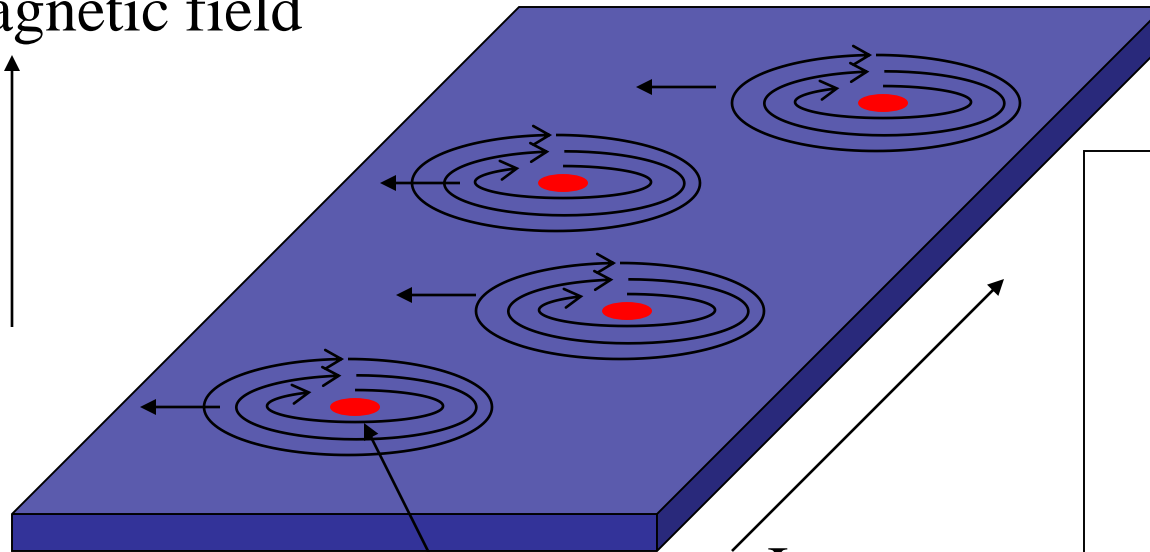


# Vortices introduce electrical resistance to otherwise superconducting materials

Magnetic field creates vortices--

Vortices cause dissipation (i.e. a non-zero electrical resistance)!

B -magnetic field



I-current

The order parameter:

$$\Psi = |\Psi| \exp(-i\phi)$$

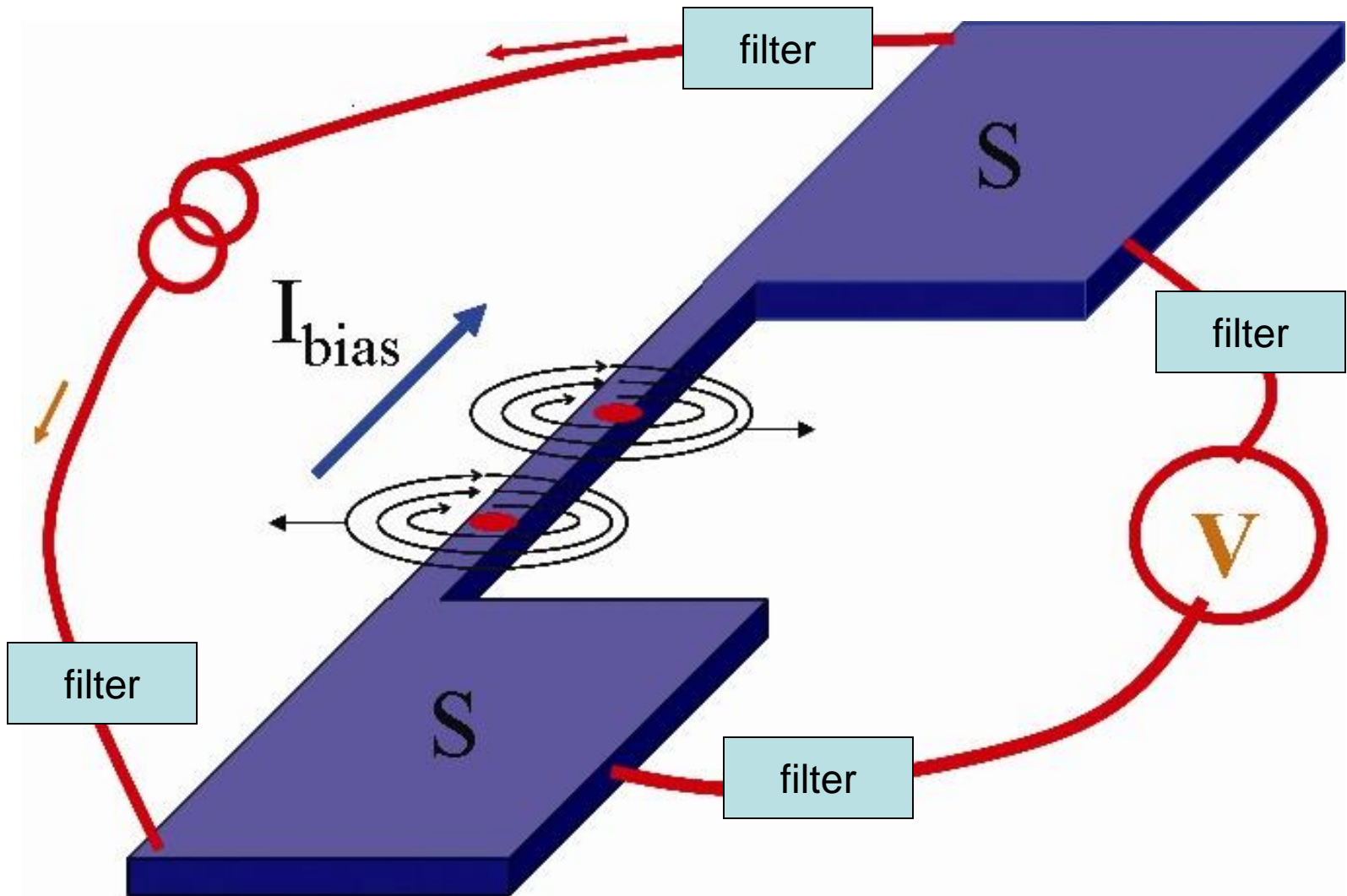
amplitude

phase

Vortex core: normal, not superconducting; diameter  $\xi \sim 10$  nm

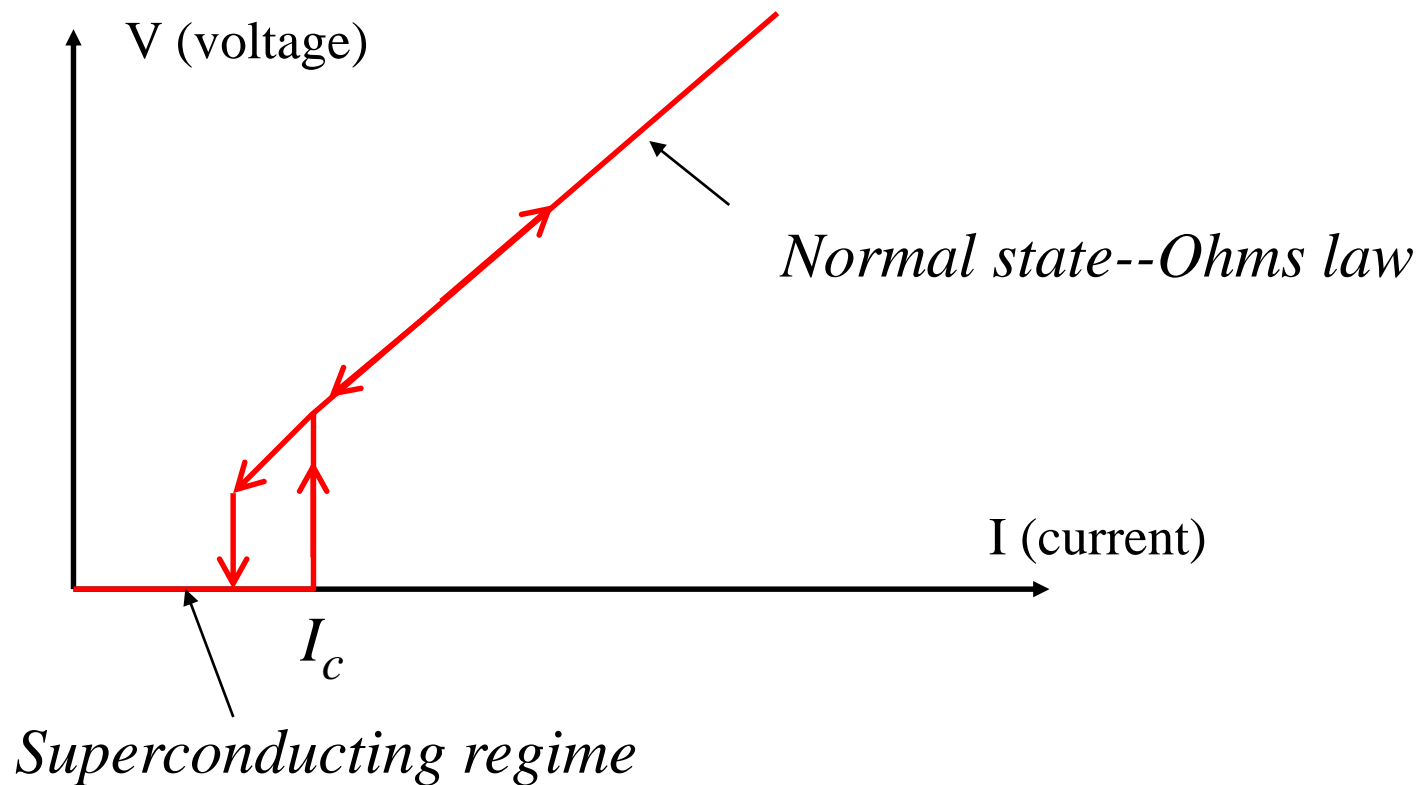
# DC transport measurement schematic

Phase slip events are shown as red dots

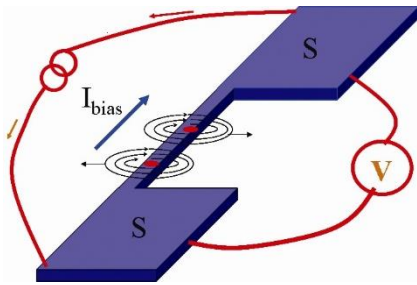


# Superconductivity: very basic introduction

Electrical resistance is zero only if current is not too strong



# How to use voltage to figure out the rate of phase slips?



$$2eV = \hbar \, d\phi/dt$$

According to Shrodinger,  $\phi = Et / \hbar$

Remember  $i\hbar(d\Psi/dt) = E \Psi$

But for BCS pairs  $E = 2eV$ ,

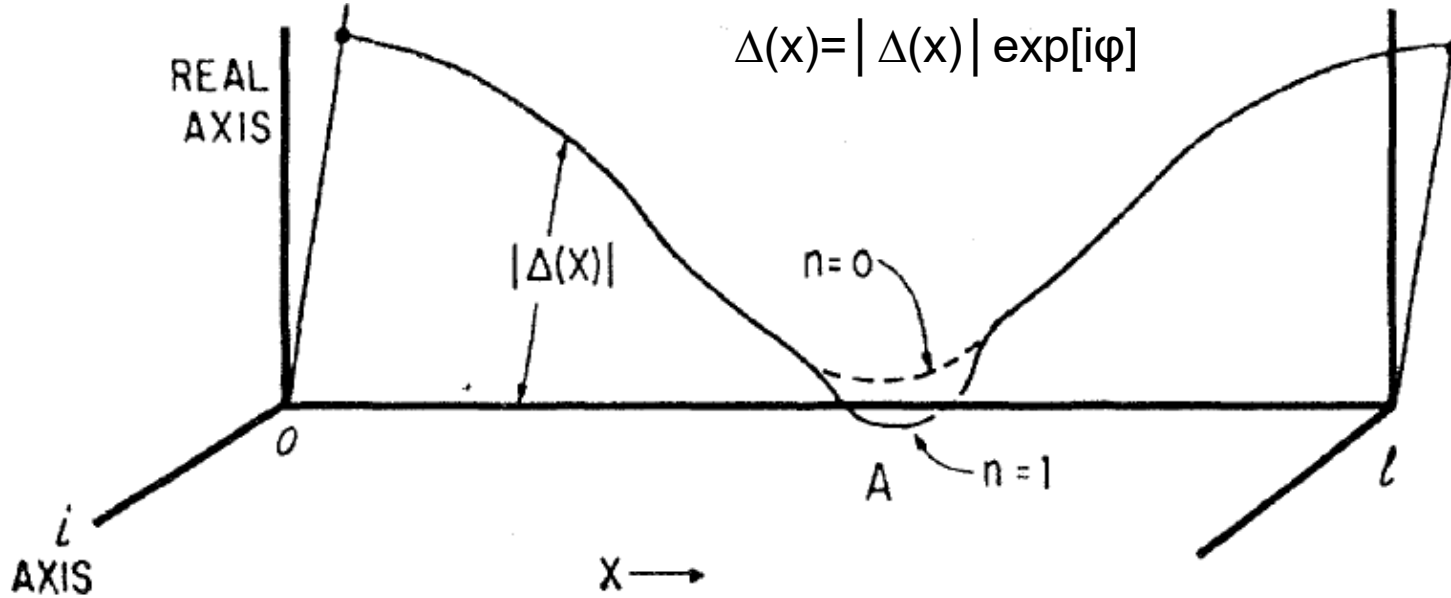
where  $V$  is the electric potential.

Thus the equation above can be obtained:

$$2eV = \hbar \, d\phi/dt$$



# Transport properties: Little's Phase Slip



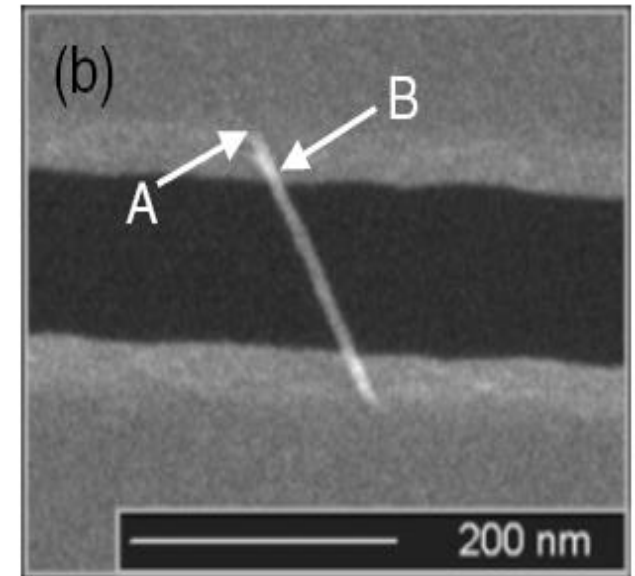
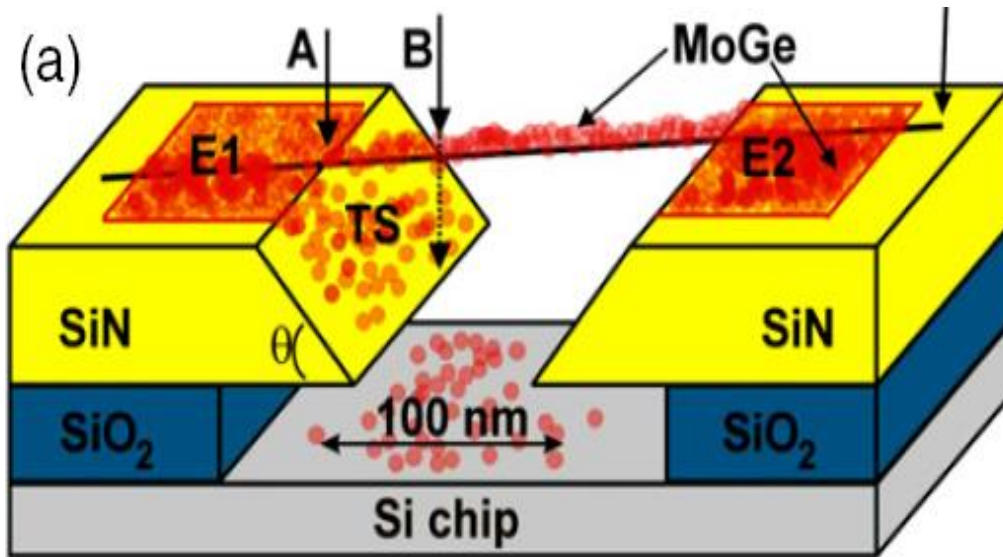
W. A. Little, "Decay of persistent currents in small superconductors", *Physical Review*, V.156, pp.396-403 (1967).

Two types of phase slips (PS) can be expected:

1. The usual, thermally activated PS (TAPS)
2. Quantum phase slip (QPS)

# Fabrication of nanowires

## *Method of Molecular Templating*



**Si/ SiO<sub>2</sub>/SiN substrate with undercut**

~ 0.5 mm Si wafer

500 nm SiO<sub>2</sub>

60 nm SiN

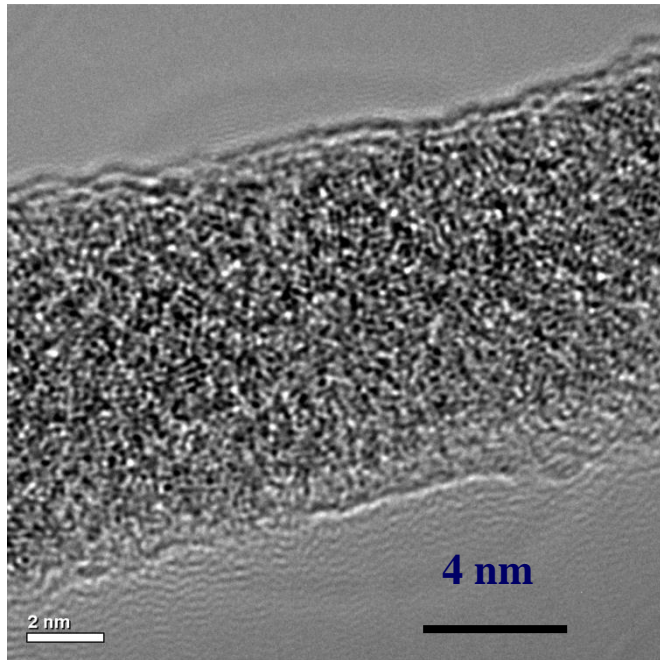
Width of the trenches ~ 50 - 500 nm

**HF wet etch for ~10 seconds  
to form undercut**

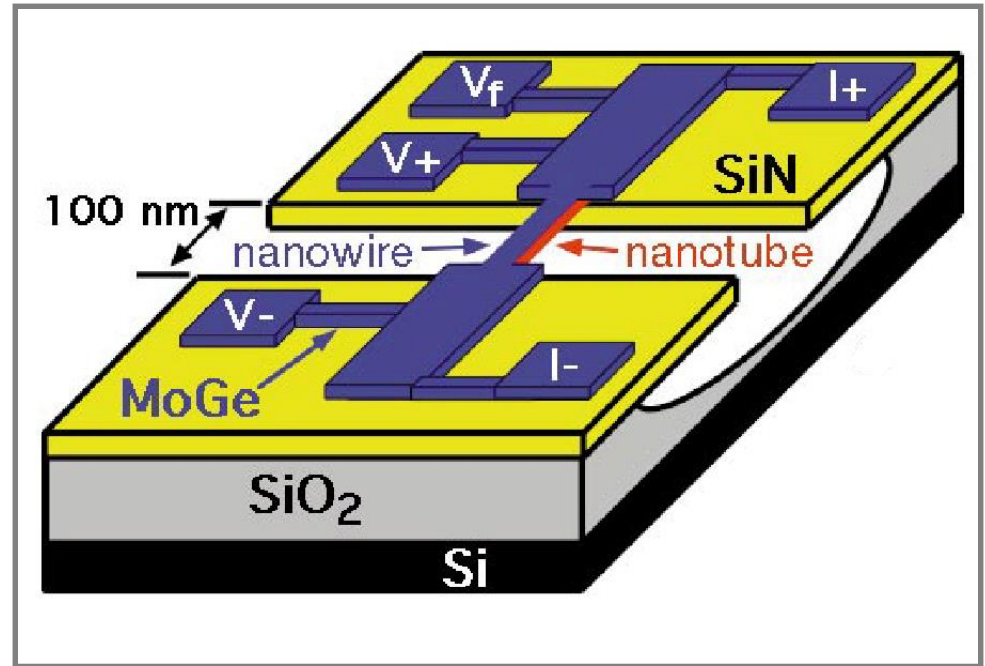
Bezryadin, Lau, Tinkham, *Nature* **404**, 971 (2000)



# Sample Fabrication



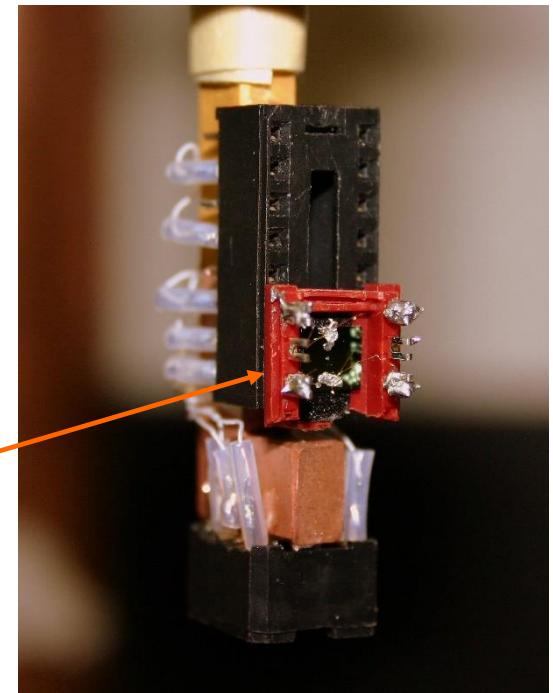
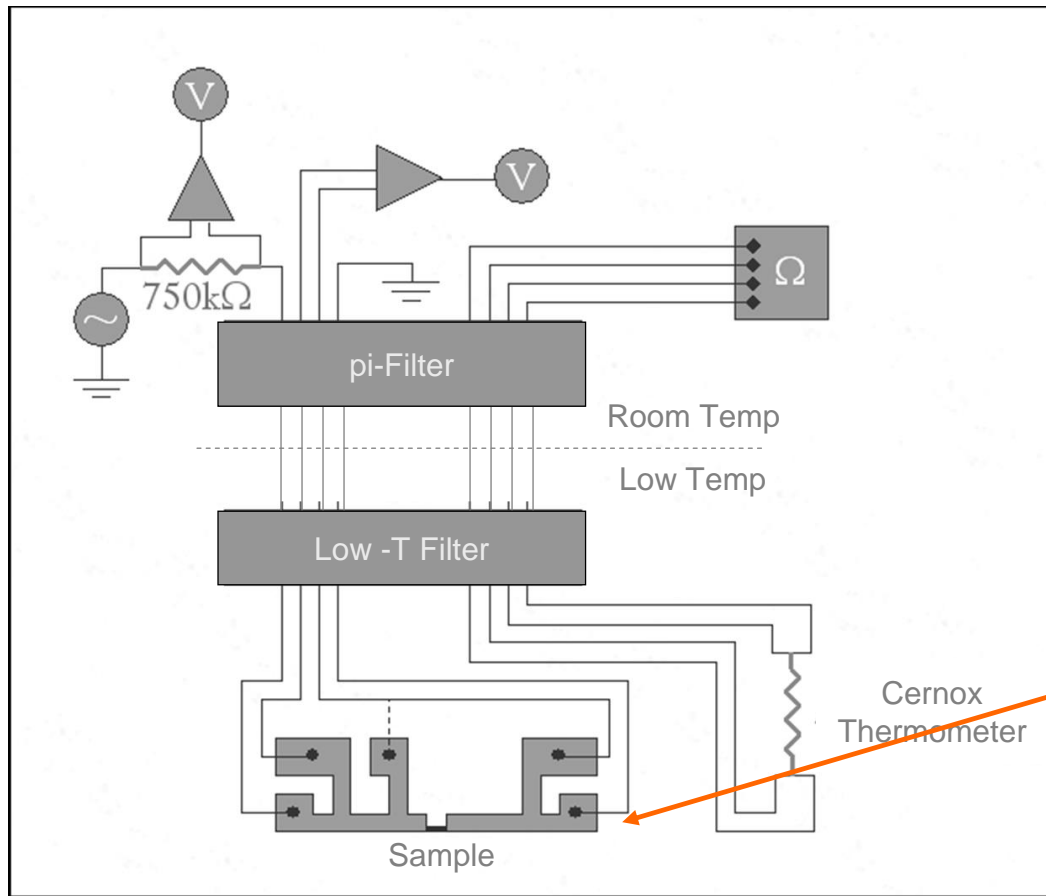
**TEM image of a wire shows amorphous morphology.  
Nominal MoGe thickness = 3 nm**



**Schematic picture of the pattern  
Nanowire + Film Electrodes used in  
transport measurements**



## Measurement Scheme



Circuit Diagram

Sample mounted on the  $^3\text{He}$  insert.



# Tony Bollinger's sample-mounting procedure in winter in Urbana

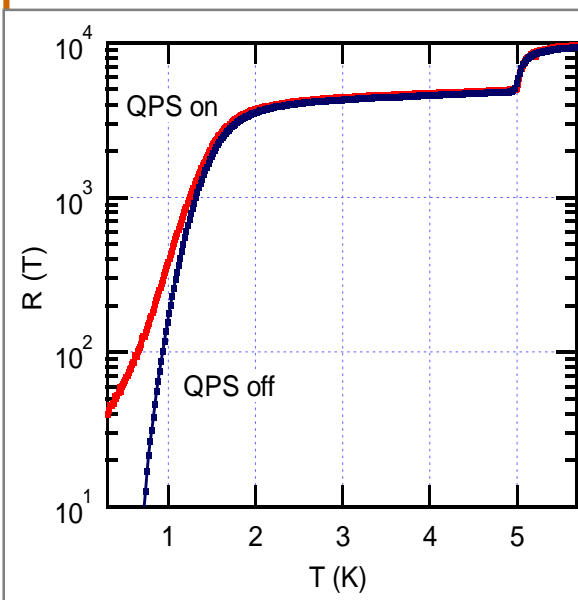
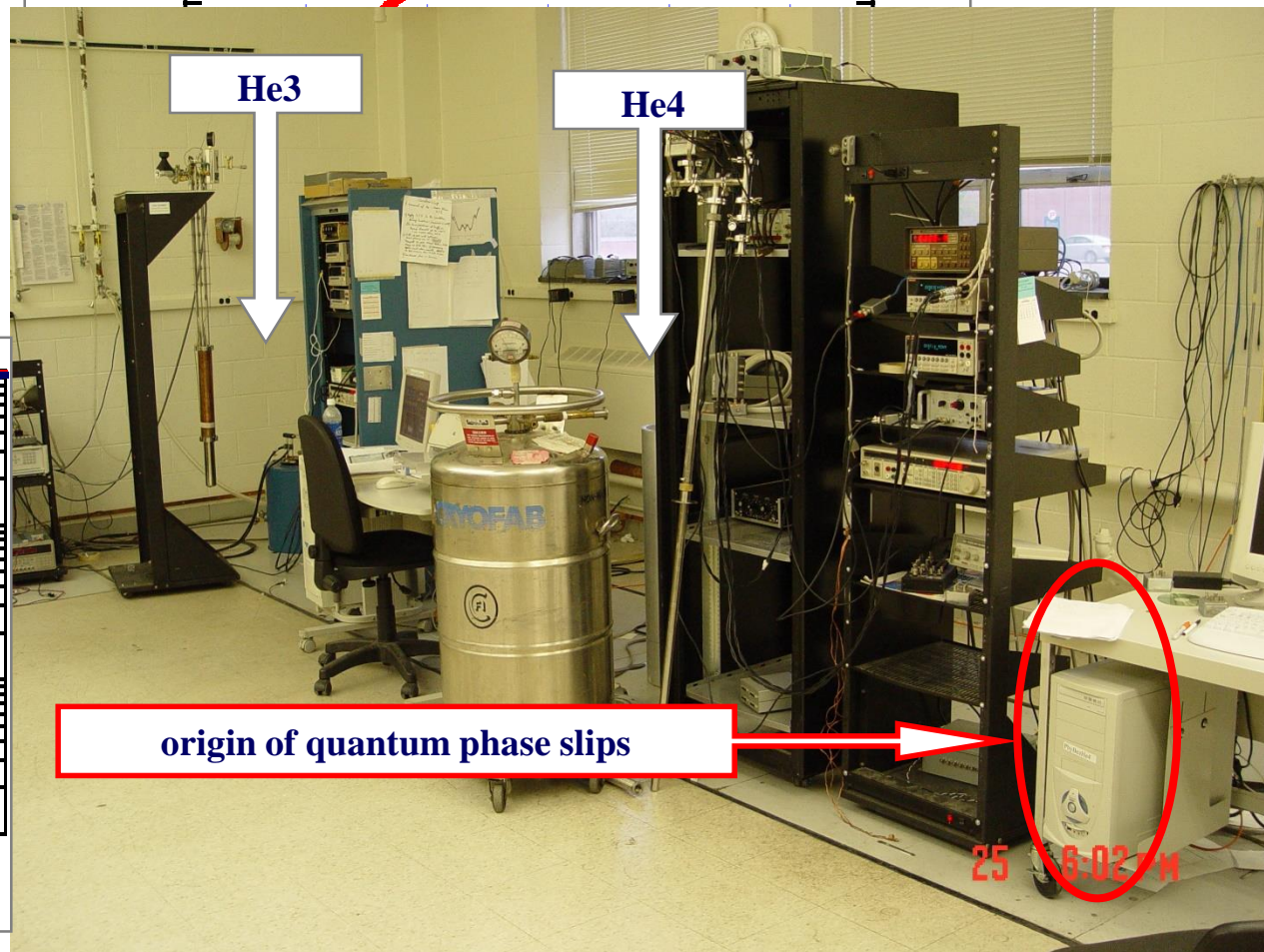
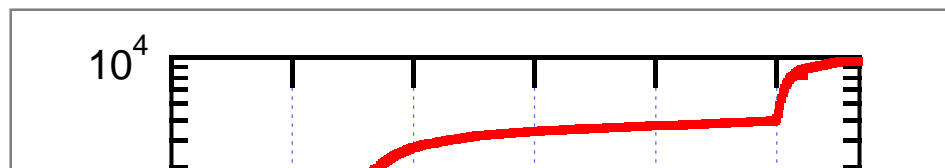
## Procedure (~75% Success)

- Put on gloves
- Put grounded socket for mounting in vise with grounded indium dot tool connected
- Spray high-backed black chair all over and about 1 m square meter of ground with anti-static spray
  - DO NOT use green chair
  - Not sure about short-backed black chairs
- Sit down
- Spray bottom of feet with anti-static spray
- Plant feet on the ground. ***Do not move your feet again for any reason until mounting is finished.***
- Mount sample
- Keep sample in grounded socket until last possible moment
- Test samples in dipstick at  $\sim 1$  nA

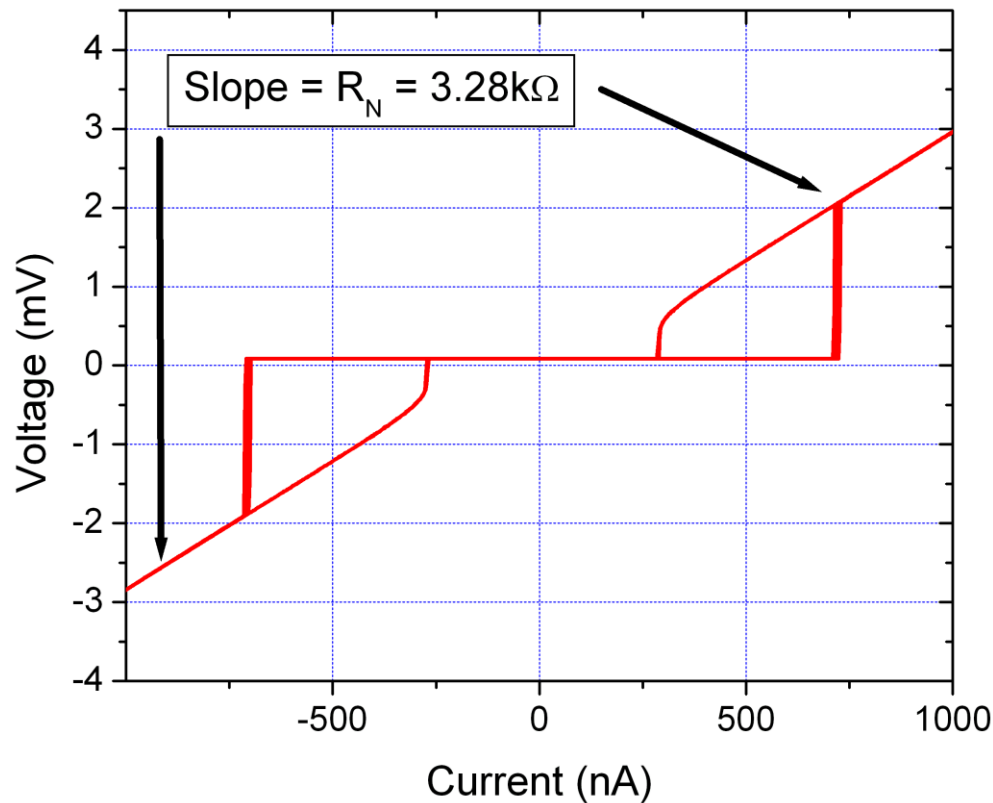




# Possible Origin of Quantum Phase Slips



# Search for QPS at high bias currents, by measuring the fluctuations of the switching current



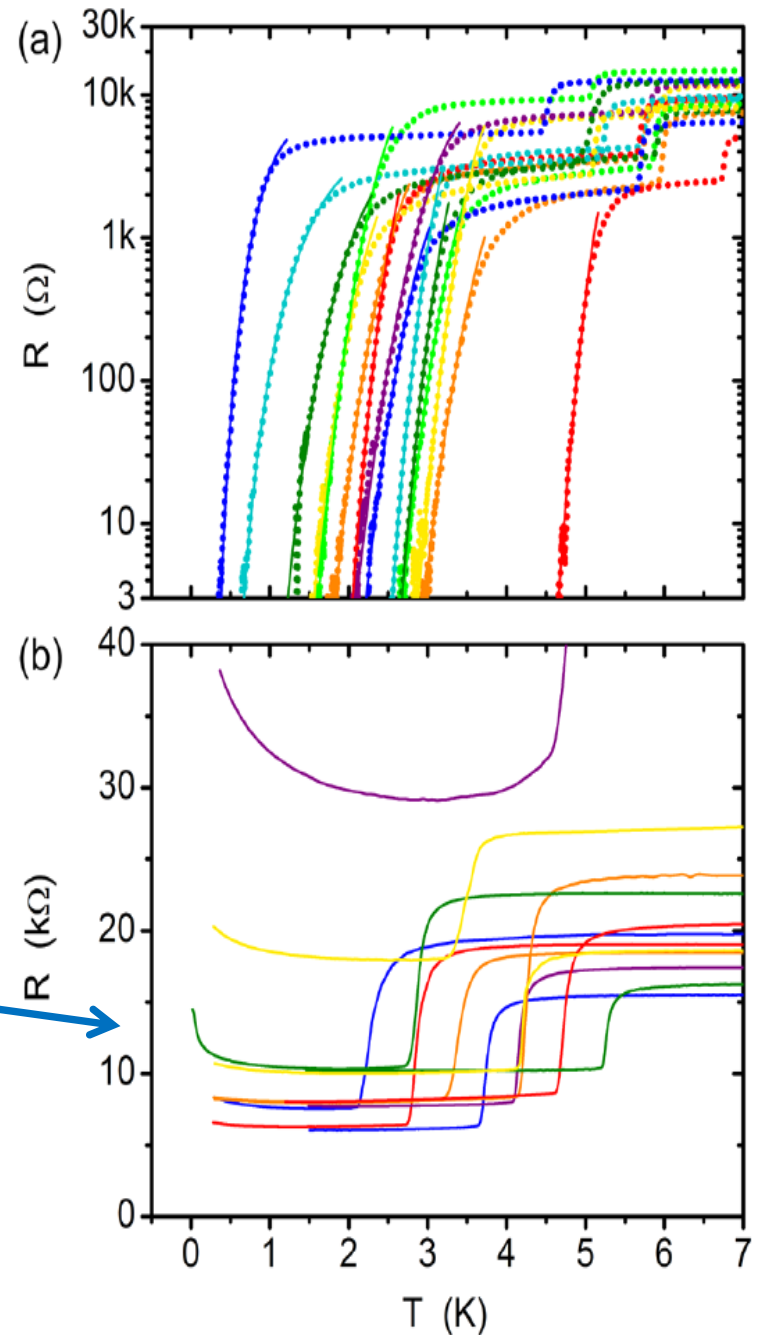
# Dichotomy in nanowires: Evidence for superconductor- insulator transition (SIT)

$$R=V/I \quad I \sim 3 \text{ nA}$$

The difference between samples is the amount of the deposited Mo<sub>79</sub>Ge<sub>21</sub>.

$$R_{\text{sheet}} = 100 - 400 \ \Omega$$

Can the insulating behavior be due to Anderson localization of the BCS condensate?



Bollinger, Dinsmore, Rogachev, Bezryadin,  
Phys. Rev. Lett. **101**, 227003 (2008)



# Useful Expression for the Free Energy of a Phase Slip

“Arrhenius-Little” formula for the wire resistance:

$$R_{AL} \approx R_N \exp[-\Delta F(T)/k_B T]$$

$$\Delta F = (8\sqrt{2}/3)(H_c(T)^2/8\pi)(A\xi(T))$$

$$\frac{\Delta F(0)}{k_B T_c} = \sqrt{6} \frac{\hbar I_c(0)}{2ek_B T_c} = 0.83 \frac{R_q L}{R_n \xi(0)} = 0.83 \frac{R_q}{R_{\xi(0)}}$$

## Quantum limit to phase coherence in thin superconducting wires

M. Tinkham<sup>a)</sup> and C. N. Lau

*Physics Department, Harvard University, Cambridge, Massachusetts 02138*



# Linearity of the Schrödinger's equation



Suppose  $\Psi_1$  is a valid solution of the Schrödinger equation:

$$i\hbar \frac{\partial \psi_1}{\partial t} = \frac{\partial^2 \psi_1}{\partial x^2} + U(x)\psi_1$$

And suppose that  $\Psi_2$  is another valid solution of the Schrödinger equation:

$$i\hbar \frac{\partial \psi_2}{\partial t} = \frac{\partial^2 \psi_2}{\partial x^2} + U(x)\psi_2$$

Then  $(\Psi_1 + \Psi_2)/\sqrt{2}$  is also a valid solution, because:

$$i\hbar \frac{\partial (\psi_1 + \psi_2)}{\partial t} = \frac{\partial^2 (\psi_1 + \psi_2)}{\partial x^2} + U(x)(\psi_1 + \psi_2)$$

The state  $(\Psi_1 + \Psi_2)/\sqrt{2}$  is a new combined state which is called “quantum superposition” of state (1) and (2)



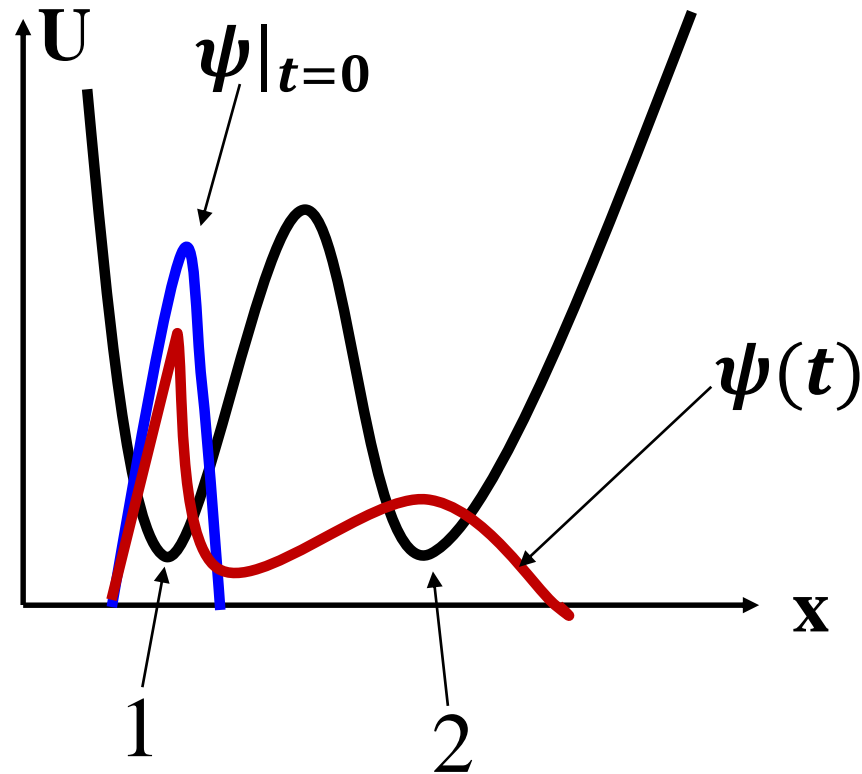


# Quantum tunneling



George Gamow

(He also developed  
Big Bang theory)

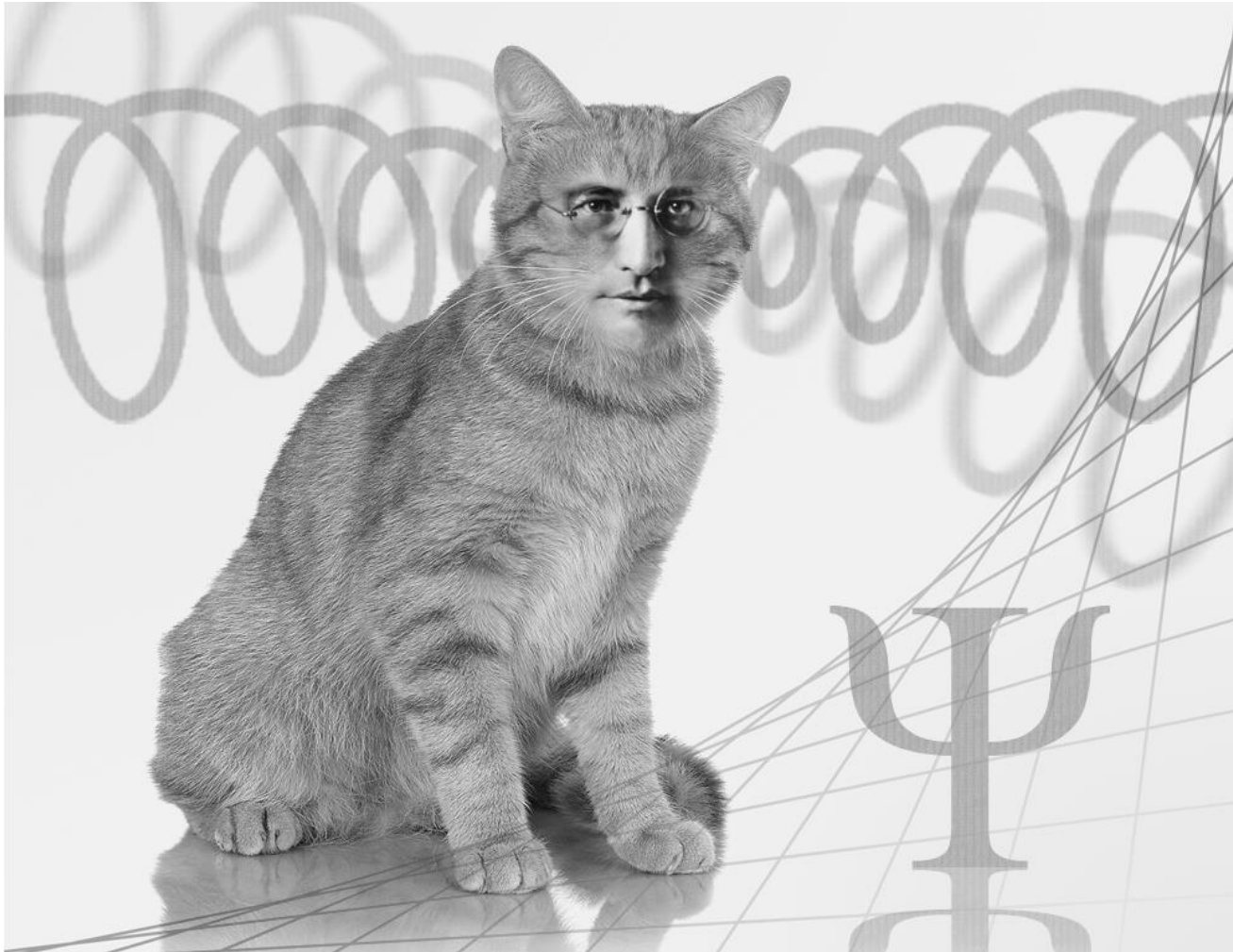


**Quantum tunneling is possible  
since quantum superpositions of  
states are possible.**

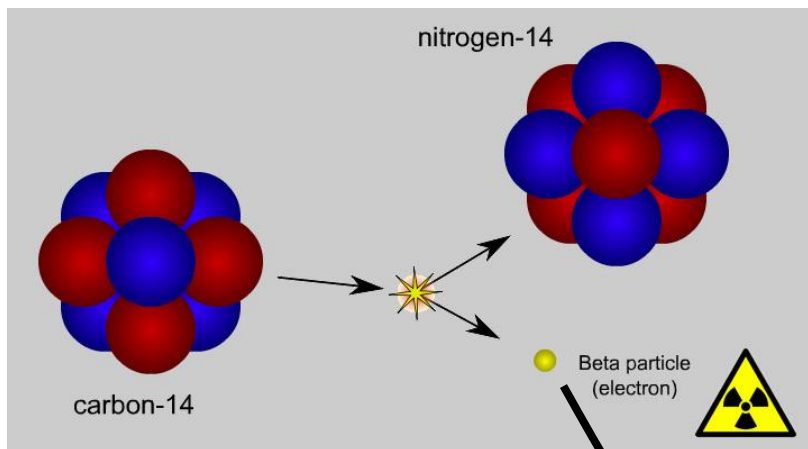


# Schrödinger cat – the ultimate macroscopic quantum phenomenon

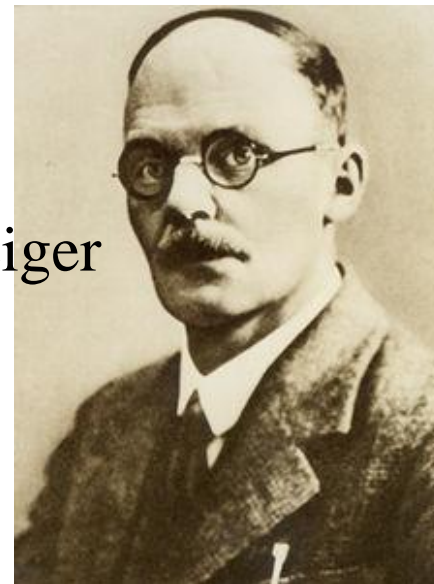
E. Schrödinger, Naturwiss. **23** (1935), 807.



# Schrödinger cat – thought experiment



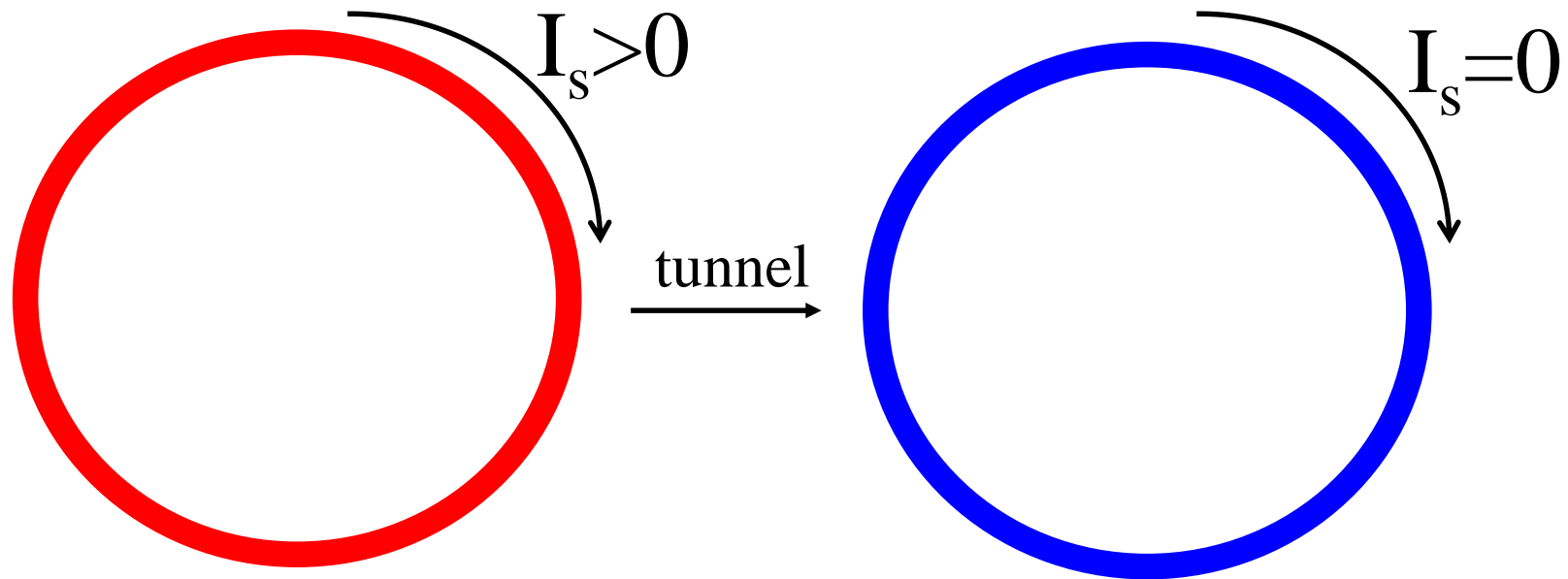
Hans Geiger



Geiger counter

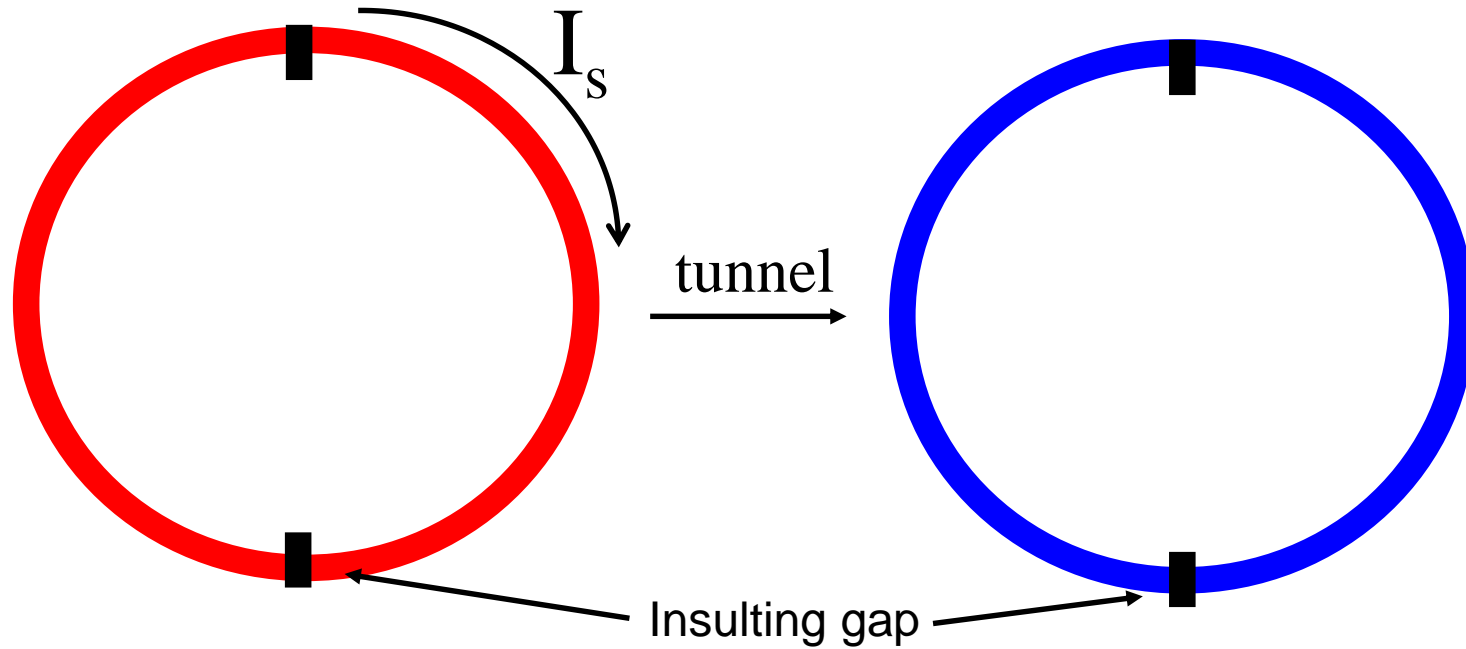


# What sort of tunneling we will consider?



- Red color represents some strong current in the superconducting wire loop
- Blue color represents no current or a much smaller current in the loop

# Previous results relate loops with insulating interruptions (SQUIDS)



-Red color represents some strong current in the superconducting loop

-Blue color represents no current or very little current in the superconducting loop



# Leggett's prediction for macroscopic quantum tunneling (MQT) in SQUIDs

80

Supplement of the Progress of Theoretical Physics, No. 69, 1980

## Macroscopic Quantum Systems and the Quantum Theory of Measurement

A. J. LEGGETT

*School of Mathematical and Physical Sciences  
University of Sussex, Brighton BN1 9QH*

(Received August 27, 1980)

It is this property which makes a SQUID the most promising candidate to date for observing macroscopic quantum tunnelling; if it should ever become possible to observe macroscopic quantum *coherence*, the low entropy and consequent lack of dissipation will be absolutely essential.<sup>21)</sup>



# MQT report by Kurkijarvi and collaborators (1981)

VOLUME 47, NUMBER 9

PHYSICAL REVIEW LETTERS

31 AUGUST 1981

## Decay of the Zero-Voltage State in Small-Area, High-Current-Density Josephson Junctions

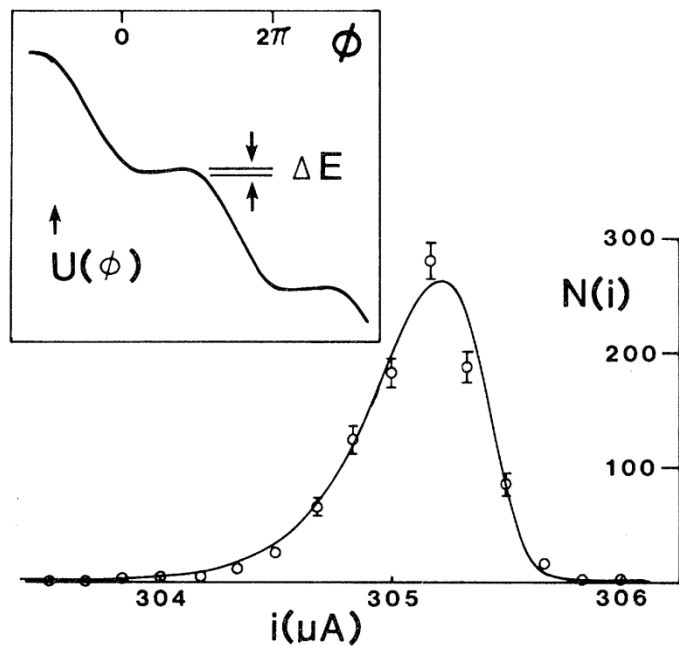


FIG. 1. Measured distribution for  $T = 1.6$  K for small high-current-density junction. The solid line is a fit by the CL theory for  $R = 20 \Omega$ ,  $C = 8$  fF, and  $i_{\text{CFF}} = 310.5 \mu\text{A}$ . The inset is  $U(\phi)$  for  $x = 0.8$  with barrier  $\Delta E$ .

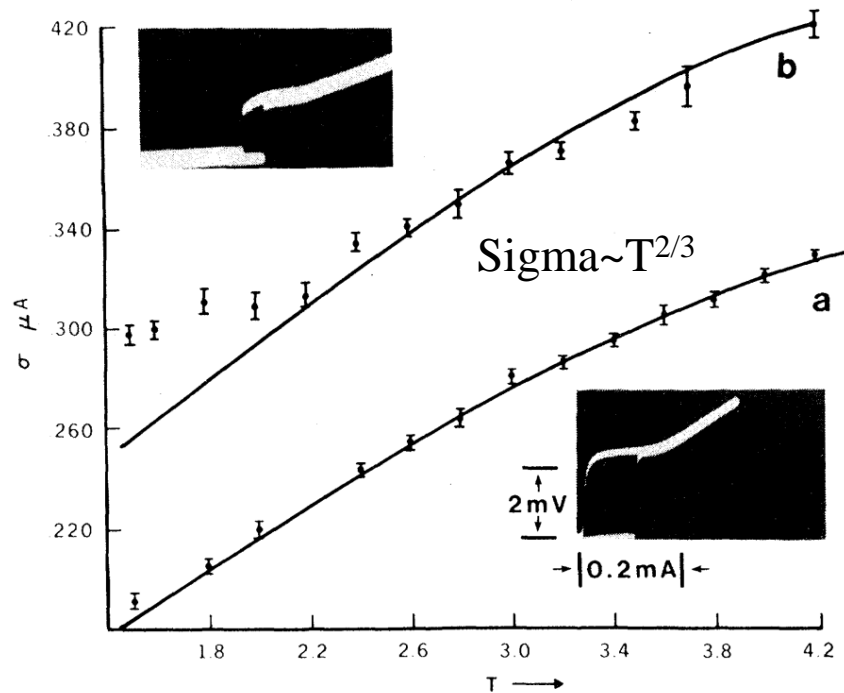


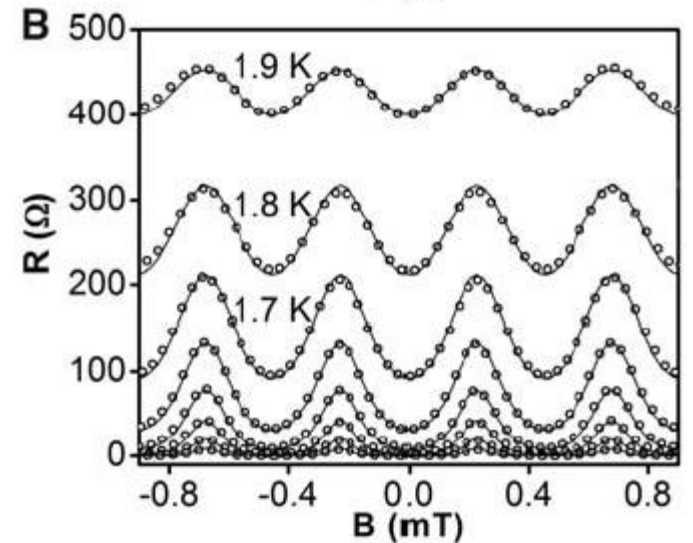
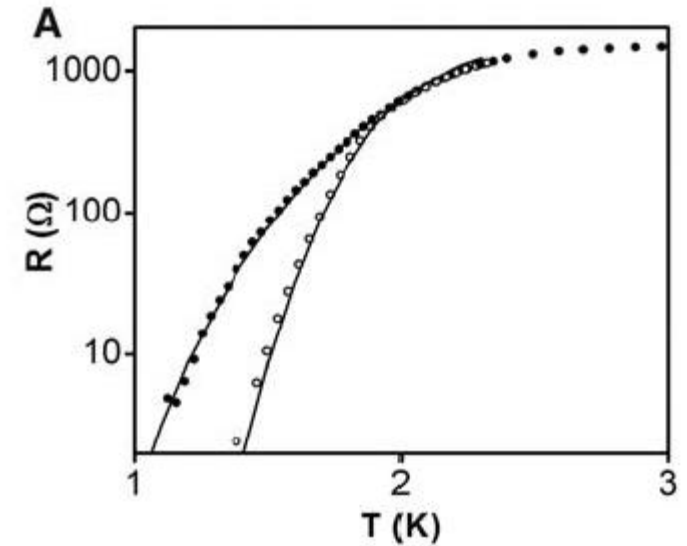
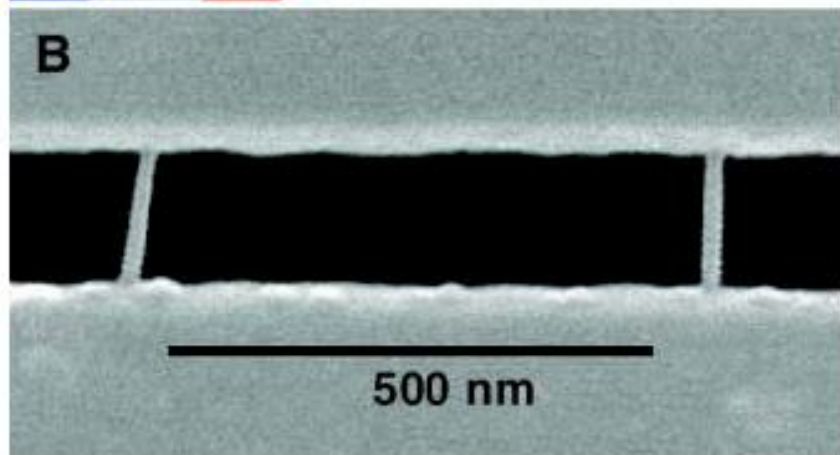
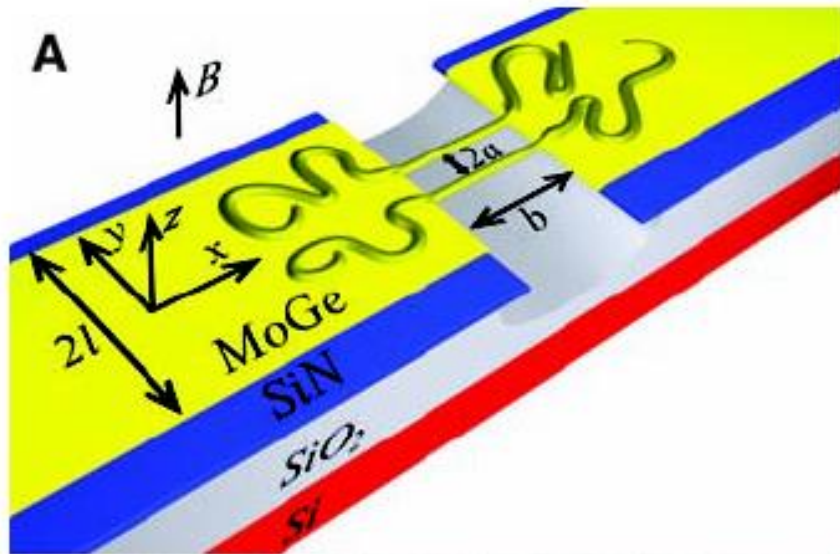
FIG. 2. Measured distribution widths  $\sigma$  vs  $T$  for two junctions with current sweep of  $\sim 400 \mu\text{A}/\text{sec}$ . Curve  $a$  is lower current density junction data and curve  $b$  is higher density junction data. The traces adjacent to the plots are the corresponding  $I$ - $V$  characteristics at  $4.2$  K. The scales are the same for both traces.



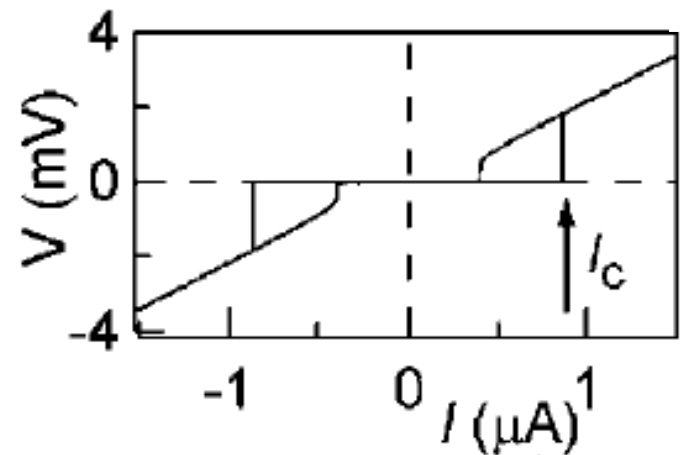
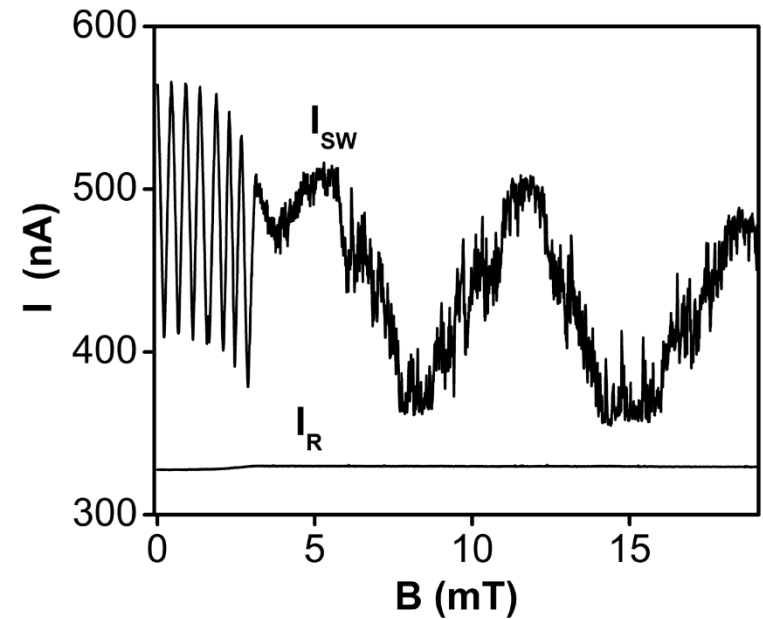
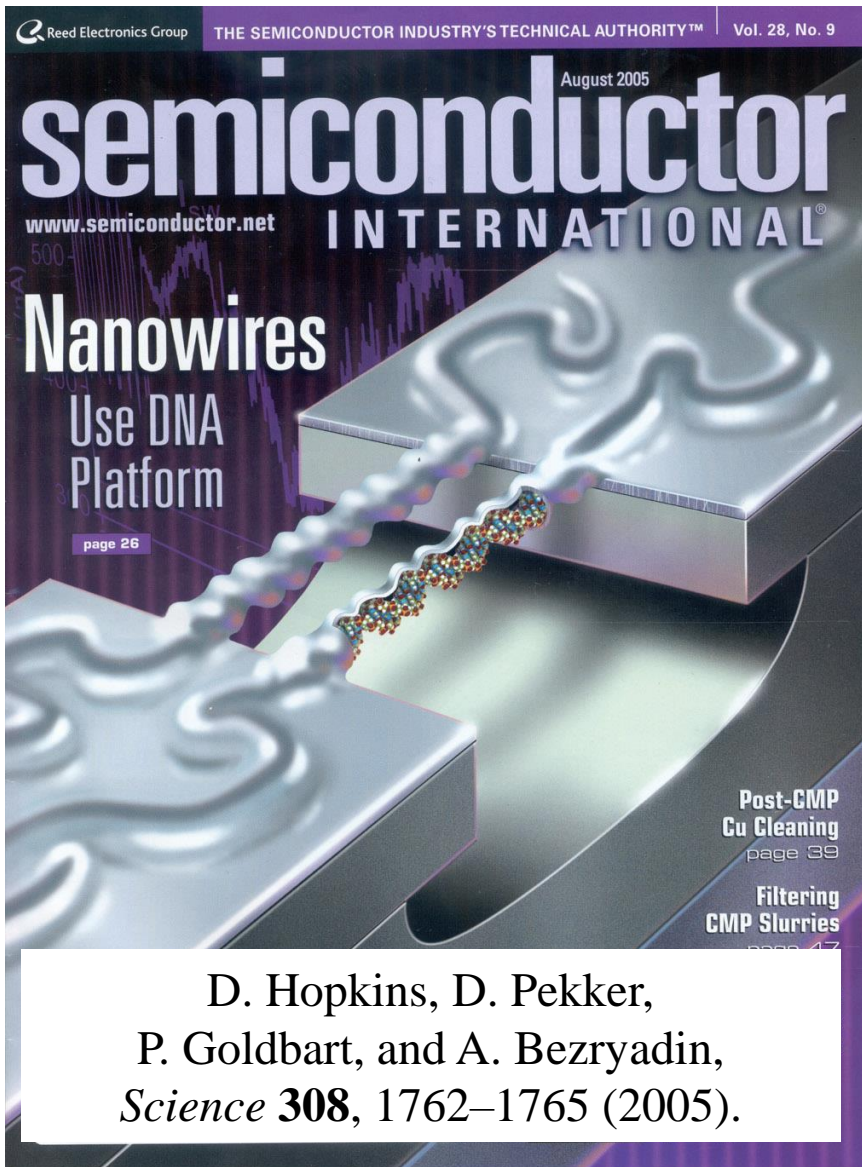
# What determines the period of oscillation?

(A simple guess for the period would be  $\Delta B \sim \Phi_0/2ab$ .

This prediction deviates from the result by a factor 100!)

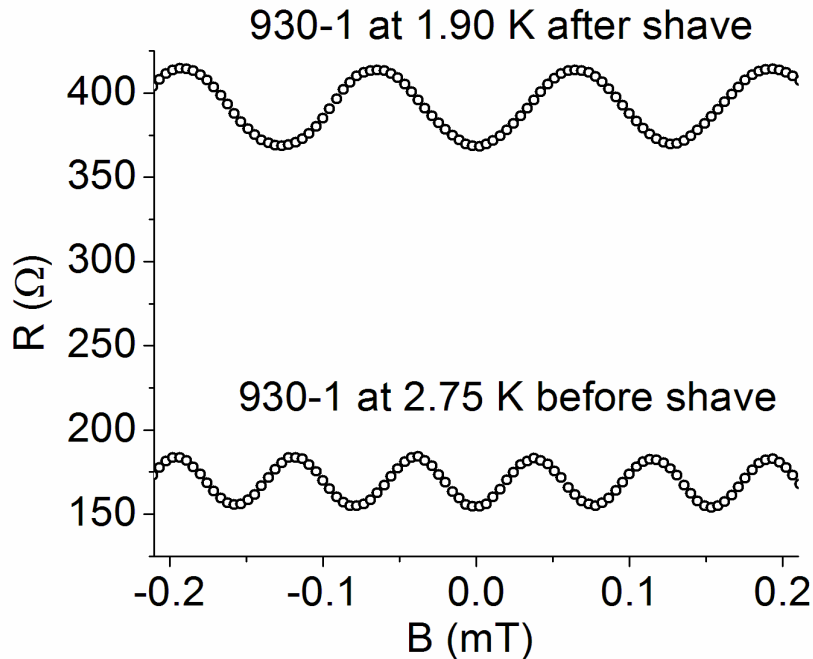


# Phase gradiometers templated by DNA



# Little-Parks effect.

The period of the oscillation is inversely proportional to the width of the electrodes



The width of the leads was changed from 14480 nm to 8930 nm

The period changed from 77.5  $\mu\text{T}$  to 128  $\mu\text{T}$

usual SQUID estimate:

$$Period = \frac{\Phi_0}{2ab} \sim 10mT$$

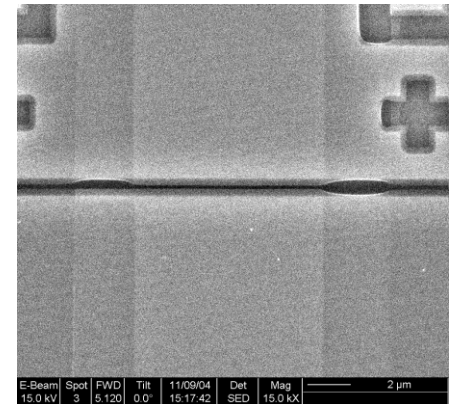
here  $2a$ -distance between the wires;

$b$ - length of wires

**Correct field period:**

$$\Delta B = \frac{\pi^2 \Phi_0}{8G 4al}$$

here  $2l$  - the width of the leads



$G = .916$  is the Catalan number



# SQUID – superconducting quantum interference device

## SQUID helmet project at Los Alamos



Magnetic field scales:

Earth field:  $\sim 1\text{G}$

Fields inside animals:  
 $\sim 0.01\text{G}-0.00001\text{G}$

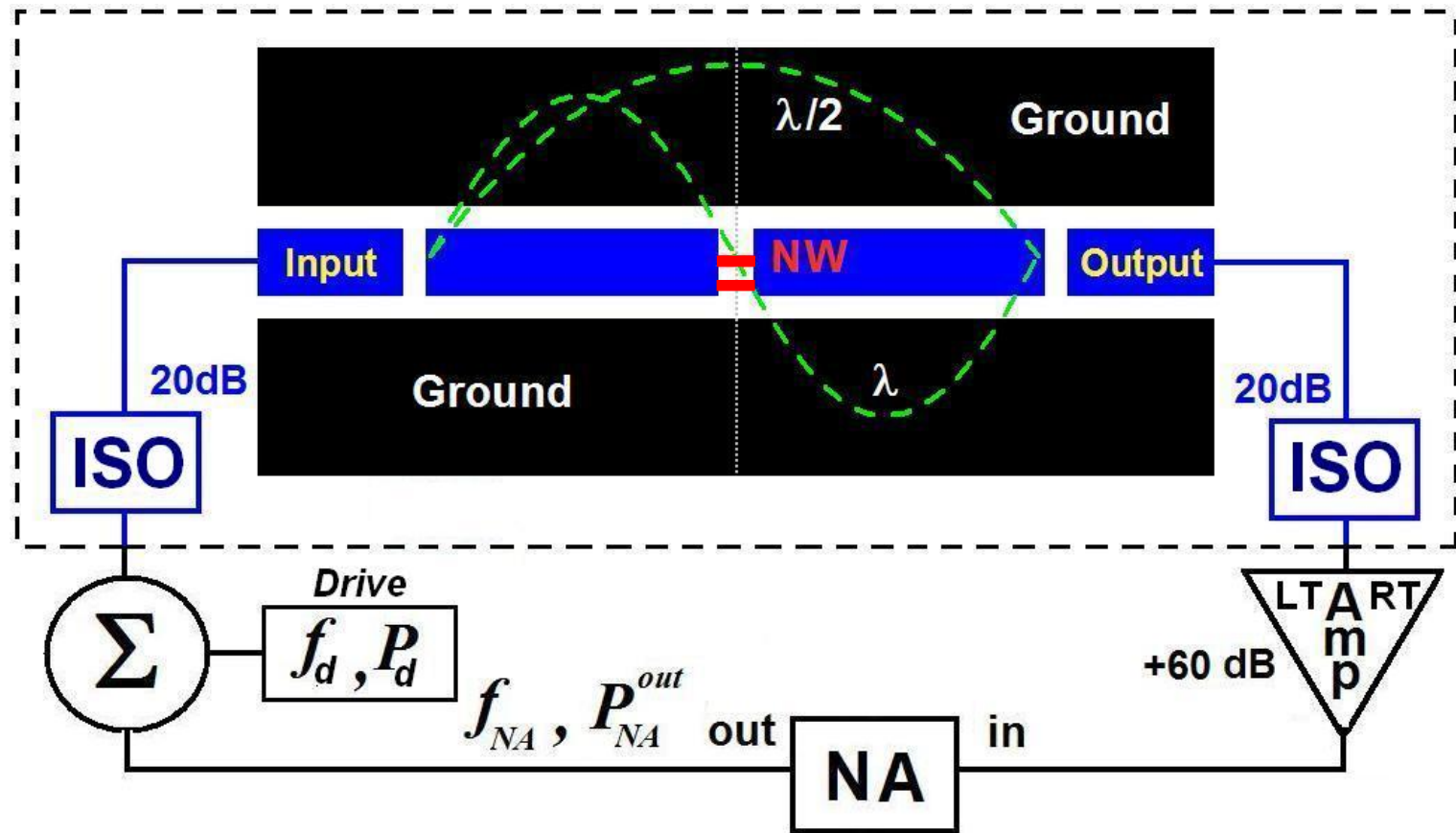
Fields on the **human brain**:  
 $\sim 0.3\text{nG}$

This is less than a hundred-millionth of the Earth's magnetic field.

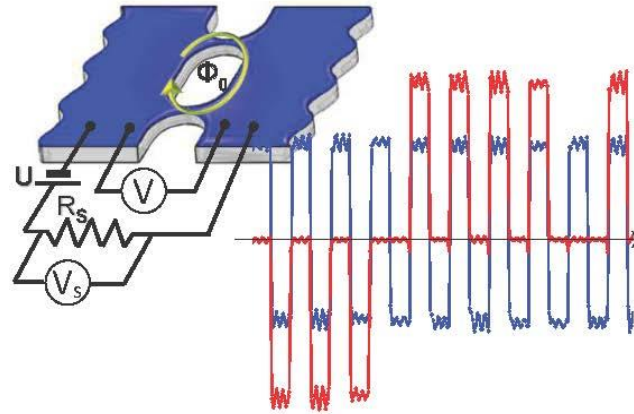
SQUIDs, or Superconducting Quantum Interference Devices, invented in 1964 by Robert Jaklevic, John Lambe, Arnold Silver, and James Mercereau of Ford Scientific Laboratories, are used to measure extremely small magnetic fields. They are currently the most sensitive magnetometers known, with the noise level as low as  $3\text{ fT}\cdot\text{Hz}^{-1/2}$ . While, for example, the Earth magnet field is only about  $0.0001\text{ Tesla}$ , some electrical processes in animals produce very small magnetic fields, typically between  $0.000001\text{ Tesla}$  and  $0.000000001\text{ Tesla}$ . SQUIDs are especially well suited for studying magnetic fields this small.

Measuring the brain's magnetic fields is even much more difficult because just above the skull the strength of the magnetic field is only about  $0.3\text{ picoTesla}$  ( $0.000000000000003\text{ Tesla}$ ). This is less than a hundred-millionth of Earth's magnetic field. In fact, brain fields can be measured only with the most sensitive magnetic-field sensor, i.e. with the superconducting quantum interference device, or SQUID.

# Measuring nanowires within GHz resonators. Detection of individual phase slips.







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## Supercurrent-controlled kinetic inductance superconducting memory element

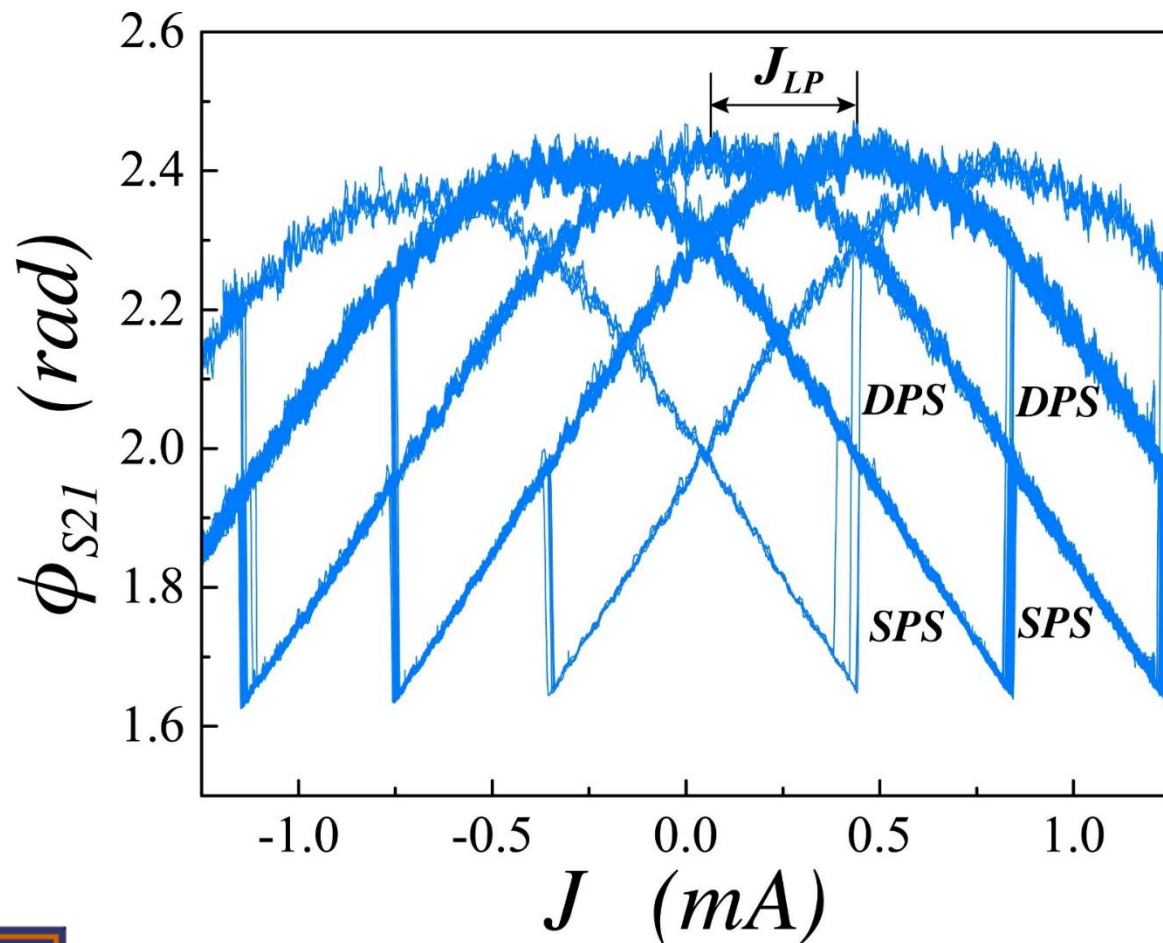
Appl. Phys. Lett. **118**, 112603 (2021); doi: 10.1063/5.0040563

Eduard Ilin, Xiangyu Song, Irina Burkova, Andrew Silge, Ziang Guo, Konstantin Ilin, and Alexey Bezryadin

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# Resonators used to detect single phase slips (SPS) and double phase slips (DPS)



$T = 360$  mK  
 $f = f_0(H=0)$



# Conclusions

- Superconductivity is fun and might be useful

