Superconductivity in pictures

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How one can measure superconductivity?

Electrical resistance of some metals drops to zero below a certain temperature which is called "critical temperature“ (H. K. O. 1911)

How to observe superconductivity
- Take Nb wire
- Connect to a voltmeter and a current source
- Put into helium Dewar
- Measure electrical resistance

Heike Kamerling Onnes

How to observe superconductivity:
- Take Nb wire
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Dewar with liquid Helium (4.2K)
Meissner effect – the key signature of superconductivity

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<th>Formula</th>
<th>$T_c$ (K)</th>
<th>$H_c$ (T)</th>
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Magnetic levitation
Magnetic levitation train
Resistance vs current diagrams and "Diagrams of State" have been obtained for 63 contacts between crossed wires of tin. The wires were plated with various thicknesses of the following metals: copper, silver, gold, chromium, iron, cobalt, nickel, and platinum. The contacts became superconducting, or showed a noticeable decrease of their resistance at lower temperatures if the plated films were not too thick. The limiting thicknesses were about $35 \times 10^{-4}$ cm for Cu, Ag, and Au; $7.5 \times 10^{-4}$ cm for Pt, $4 \times 10^{-4}$ cm for Cr, and less than $2 \times 10^{-4}$ cm for the ferromagnetic metals Fe, Co, and Ni. The investigation was extended to measurements of the resistance of contacts between crossed wires of copper or gold plated with various thicknesses of tin. Simultaneous measurements of the (longitudinal) resistance of the tin-plated gold or copper wires showed that these thin films of tin do not become superconducting for thicknesses below certain minimum values. These latter findings are in agreement with previous measurements at Toronto. The measurements at Toronto usually were believed to be unreliable because films of tin evaporated onto quartz substrates can be superconducting at thicknesses as small as $1.6 \times 10^{-6}$ cm. It is now believed that just as superconducting electrons can drift into an adjoining normal conducting layer and make it superconducting, normal electrons can drift into an adjoining superconducting layer and prevent superconductivity.

**Fig. 1.** Resistance vs current diagram of cobalt-plated contact Co 4, representative of diagrams type A.

**Fig. 2.** Resistance vs current diagram of silver-plated contact Ag 2, representative of diagrams type B.
Explanation of the supercurrent in SNS junctions --- Andreev reflection

Andreev reflection

A.F. Andreev, 1964
Searching for an explanation: Little-Parks effect ('62)
The basic idea: magnetic field induces non-zero vector-potential, which produces non-zero superfluid velocity, thus reducing the Tc.
Superconducting vortices produced by magnetic field

Quantum trapping

magnetic field penetrates in the form of quantum flux tubes
Vortices introduce electrical resistance to otherwise superconducting materials

Magnetic field creates vortices--
Vortices cause dissipation (i.e. a non-zero electrical resistance)!

\[ B \text{-magnetic field} \]

\[ \Psi = |\Psi| \exp(-i\phi) \]

The order parameter:

- **amplitude**
- **phase**

Vortex core: normal, not superconducting; diameter $\xi \sim 10$ nm
DC transport measurement schematic

Phase slip events are shown as red dots

$I_{bias}$

V

filter

filter

filter

filter
Superconductivity: very basic introduction

Electrical resistance is zero only if current is not too strong

$V$ (voltage)

$I_c$

$I$ (current)

$Normal \ state--Ohms \ law$

$Superconducting \ regime$
How to use voltage to figure out the rate of phase slips?

2eV = ℏ dφ/dt

According to Shrodinger, φ = Et/ ℏ
Remember iℏ(dΨ/dt) = E Ψ
But for BCS pairs E = 2eV,
where V is the electric potential.
Thus the equation above can be obtained:

2eV = ℏ dφ/dt
Transport properties: Little’s Phase Slip

\[ \Delta(x) = |\Delta(x)| \exp[i\phi] \]

Two types of phase slips (PS) can be expected:
1. The usual, thermally activated PS (TAPS)
2. Quantum phase slip (QPS)

Fabrication of nanowires

Method of Molecular Templating

Si/ SiO₂/SiN substrate with undercut
- ~ 0.5 mm Si wafer
- 500 nm SiO₂
- 60 nm SiN
- Width of the trenches ~ 50 - 500 nm

HF wet etch for ~10 seconds to form undercut

Sample Fabrication

TEM image of a wire shows amorphous morphology. Nominal MoGe thickness = 3 nm

Schematic picture of the pattern
Nanowire + Film Electrodes used in transport measurements
Measurement Scheme

Circuit Diagram

Sample mounted on the $^3$He insert.
Tony Bollinger's sample-mounting procedure in winter in Urbana

Procedure (~75% Success)
- Put on gloves
  - Put grounded socket for mounting in vise with grounded indium dot tool connected
  - Spray high-backed black chair all over and about 1 m square meter of ground with anti-static spray
    - DO NOT use green chair
    - Not sure about short-backed black chairs
  - Sit down
  - Spray bottom of feet with anti-static spray
  - Plant feet on the ground. **Do not move your feet again for any reason until mounting is finished.**
- Mount sample
- Keep sample in grounded socket until last possible moment
- Test samples in dipstick at ~1 nA
Possible Origin of Quantum Phase Slips

He3

He4

origin of quantum phase slips
Search for QPS at high bias currents, by measuring the fluctuations of the switching current

\[ \text{Slope} = R_N = 3.28 k\Omega \]
**Dichotomy in nanowires:**
Evidence for superconductor-insulator transition (SIT)

\[ R = \frac{V}{I} \quad I \sim 3 \text{nA} \]

The difference between samples is the amount of the deposited Mo79Ge21.

\[ R_{\text{sheet}} = 100 - 400 \Omega \]

Can the insulating behavior be due to Anderson localization of the BCS condensate?

Useful Expression for the Free Energy of a Phase Slip

“Arrhenius-Little” formula for the wire resistance:

\[ R_{AL} \approx R_N \exp[-\Delta F(T)/k_B T] \]

\[ \Delta F = (8\sqrt{2}/3)(H_c(T)^2/8\pi)(A\xi(T)) \]

\[ \frac{\Delta F(0)}{k_B T_c} = \sqrt{6} \frac{\hbar I_c(0)}{2e k_B T_c} = 0.83 \frac{R_q L}{R_n \xi(0)} = 0.83 \frac{R_q}{R_\xi(0)} \]
Linearity of the Schrödinger’s equation

Suppose $\Psi_1$ is a valid solution of the Schrödinger equation:

$$i\hbar \frac{\partial \psi_1}{\partial t} = \frac{\partial^2 \psi_1}{\partial x^2} + U(x)\psi_1$$

And suppose that $\Psi_2$ is another valid solution of the Schrödinger equation:

$$i\hbar \frac{\partial \psi_2}{\partial t} = \frac{\partial^2 \psi_2}{\partial x^2} + U(x)\psi_2$$

Then $(\Psi_1 + \Psi_2)/\sqrt{2}$ is also a valid solution, because:

$$i\hbar \frac{\partial (\psi_1 + \psi_2)}{\partial t} = \frac{\partial^2 (\psi_1 + \psi_2)}{\partial x^2} + U(x)(\psi_1 + \psi_2)$$

The state $(\Psi_1 + \Psi_2)/\sqrt{2}$ is a new combined state which is called “quantum superposition” of state (1) and (2)
Quantum tunneling is possible since quantum superpositions of states are possible.

George Gamow
(He also developed Big Bang theory)
Schrödinger cat –
the ultimate macroscopic quantum phenomenon

E. Schrödinger, Naturwiss. 23 (1935), 807.
Schrödinger cat – thought experiment

Hans Geiger

Geiger counter
What sort of tunneling we will consider?

- Red color represents some strong current in the superconducting wire loop
- Blue color represents no current or a much smaller current in the loop
Previous results relate loops with insulating interruptions (SQUIDs)

- Red color represents some strong current in the superconducting loop
- Blue color represents no current or very little current in the superconducting loop
Leggett’s prediction for macroscopic quantum tunneling (MQT) in SQUIDs

80 Supplement of the Progress of Theoretical Physics, No. 69, 1980

Macroscopic Quantum Systems and the Quantum Theory of Measurement

A. J. LEGGETT

School of Mathematical and Physical Sciences
University of Sussex, Brighton BN1 9QH

(Received August 27, 1980)

It is this property which makes a SQUID the most promising candidate to date for observing macroscopic quantum tunnelling; if it should ever become possible to observe macroscopic quantum coherence, the low entropy and consequent lack of dissipation will be absolutely essential.
FIG. 1. Measured distribution for $T = 1.6 \, \text{K}$ for small high-current-density junction. The solid line is a fit by the CL theory for $R = 20 \, \Omega$, $C = 8 \, \text{fF}$, and $I_{CFF} = 310.5 \, \mu\text{A}$. The inset is $U(\phi)$ for $x = 0.8$ with barrier $\Delta E$.

FIG. 2. Measured distribution widths $\sigma$ vs $T$ for two junctions with current sweep of $\sim 400 \, \mu\text{A/}\text{sec}$. Curve $a$ is lower current density junction data and curve $b$ is higher density junction data. The traces adjacent to the plots are the corresponding $I-V$ characteristics at 4.2 K. The scales are the same for both traces.
What determines the period of oscillation?
(A simple guess for the period would be $\Delta B \sim \Phi_0/2ab$. This prediction deviates from the result by a factor 100!)
Phase gradiometers templated by DNA

Little-Parks effect.
The period of the oscillation is inversely proportional to the width of the electrodes.

The width of the leads was changed from 14480 nm to 8930 nm.
The period changed from 77.5 $\mu$T to 128 $\mu$T.

The usual SQUID estimate:

$$ Period = \frac{\Phi_0}{2ab} \sim 10mT $$

Here $2a$-distance between the wires;

$b$- length of wires

**Correct field period:**

$$ \Delta B = \frac{\pi^2 \Phi_0}{8G 4al} $$

$G = .916$ is the Catalan number.
SQUID – superconducting quantum interference device

SQUID helmet project at Los Alamos

Magnetic field scales:

Earth field: ~1G

Fields inside animals:
~0.01G-0.00001G

Fields on the human brain:
~0.3nG
This is less than a hundred-millionth of the Earth’s magnetic field.

SQUIDs, or Superconducting Quantum Interference Devices, invented in 1964 by Robert Jaklevic, John Lambe, Arnold Silver, and James Mercereau of Ford Scientific Laboratories, are used to measure extremely small magnetic fields. They are currently the most sensitive magnetometers known, with the noise level as low as 3 fT•Hz−½. While, for example, the Earth magnet field is only about 0.0001 Tesla, some electrical processes in animals produce very small magnetic fields, typically between 0.000001 Tesla and 0.000000001 Tesla. SQUIDs are especially well suited for studying magnetic fields this small.

Measuring the brain’s magnetic fields is even much more difficult because just above the skull the strength of the magnetic field is only about 0.3 picoTesla (0.0000000000003 Tesla). This is less than a hundred-millionth of Earth’s magnetic field. In fact, brain fields can be measured only with the most sensitive magnetic-field sensor, i.e. with the superconducting quantum interference device, or SQUID.
Measuring nanowires within GHz resonators. Detection of individual phase slips.

A. Belkin et al, Appl. Phys. Lett. 98, 242504 (2011)
Supercurrent-controlled kinetic inductance superconducting memory element


Eduard Il'in, Xiangyu Song, Irina Burova, Andrew Stiga, Zhang Guo, Konstantin Il'in, and Alexey Bezryadin
Resonators used to detect single phase slips (SPS) and double phase slips (DPS)

$T = 360 \text{ mK}$ 
$f = f_0(H=0)$

A. Belkin et al, PRX 5, 021023 (2015)
Conclusions

- Superconductivity is fun and might be useful