UNIT 4: PHOTONS

Here's the conclusion: it turns out that light comes in discrete packets (quanta) of energy. We call one of these discrete packets a photon. For a given frequency of light, the quantum of energy is given by $E = hf$, or $\hbar\omega$, where h is a fundamental physical constant equal to 6.626×10^{-34} m^2 kg/s and $\hbar = h/2\pi$. The momentum of a photon is $p = h/\lambda = \hbar k$. In this unit, we will explore how we know this to be true.

After this unit, you should be able to

- Explain why only certain wavelengths of light cause electrons to be ejected from materials
- Compute the kinetic energy of electrons ejected from a material
- Use the energy of a photon to compute the number of photons that arrive per second for a given intensity of light.
- Use the momentum of a photon to solve simple kinematics problems involving photons.
- Solve problems that use the relationships between wavelength, frequency, momentum, and energy of photons.

Photoelectric effect

The photoelectric effect is one of the simplest physical situations in which we can observe the quantization of light. The experiment is as follows. Light is incident on a metal or other material, and there is a metallic electrode nearby. Electrons that are emitted from the material hit the electrode and cause a current to be measured.

Figure 1 shows the current observed for a fixed intensity of light, as a function of the frequency. We will explain three observations using the concept of photons:

1. There is a threshold below which no current is emitted, and then current appears.
2. For a fixed intensity of light, the current is maximal at the lowest frequency, and decreases as $1/f$ as the frequency increases.
3. However, the *kinetic energy* of emitted electrons depends linearly on the frequency of light.

It turns out that one can explain this and many other experiments by proposing that energy can be added and removed to the electromagnetic field in **quanta** with energy $\hbar\omega = hf$. We call these discrete packets photons.

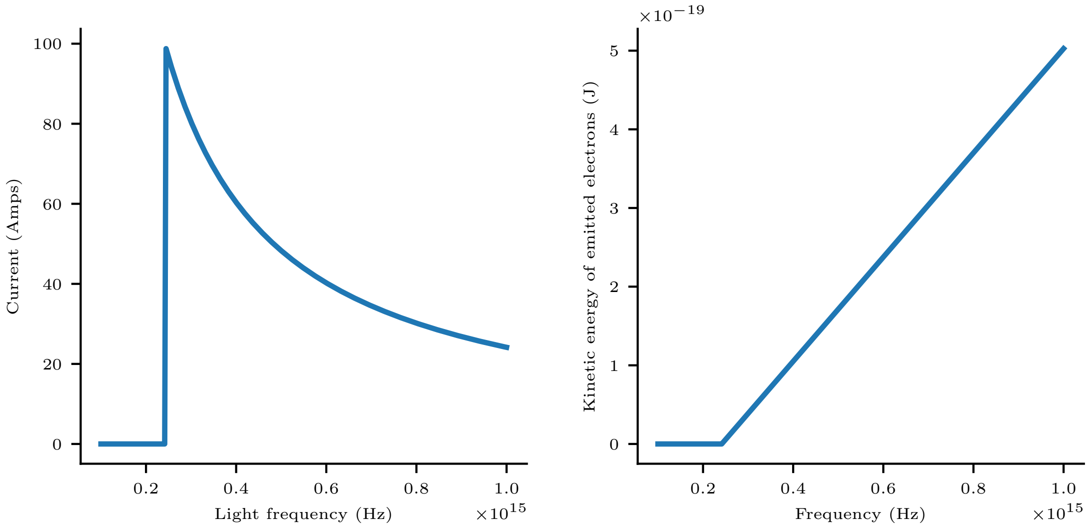

Figure 1: Sketches of the photoelectric effect. (left) The current observed for a fixed intensity of light of a given frequency. (right) The kinetic energy of ejected electrons as a function of the frequency of incident light.

The **threshold effect** is explained because takes a certain amount of energy to eject an electron from a metal. This energy is called the **work function**, Φ. The energy balance equation is

$$E_{\text{initial}} = hf - \Phi \tag{1}$$

$$E_{\text{final}} = KE_{\text{electron}}, \tag{2}$$

where KE_{electron} is the kinetic energy of the ejected electron. If $hf < \Phi$, then the kinetic energy is negative, which is impossible, so the electron cannot make it out of the material. We begin to get a current when $hf = \Phi$, so that is where the current goes from zero to non-zero. The **linear relationship between kinetic energy and frequency** is also explained by Eqn 2. To compute h, we can plot the kinetic energy of the ejected electrons versus the frequency of the light. The slope is h.

The **decrease in current** is explained because each electron is ejected by a single photon. The number of photons arriving per second in a light beam with power P is given by $\frac{P}{hf}$. Therefore, for a constant P, the electrons per second ejected will go as $1/f$, as shown in Fig 1.

Photons can independently be verified by setting up a very sensitive photodetector and finding that the energy comes in discrete packets; for light of frequency f, the energy always arrives in amounts of hf. [1]

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[1] For more experiments that show the existence of photons, look up blackbody radiation and Compton scattering. Lasers are also based on the properties of photons!

Table 1: Ways to write the momentum and energy of the photon

Energy	E	$hf = \hbar\omega = \frac{hc}{\lambda} = \hbar ck$
Momentum	p	$h/\lambda = \hbar k = \frac{hf}{c} = \frac{\hbar\omega}{c}$

Energy and momentum of a photon

We determined that the quantum of energy for light is equal to hf. There is also a corresponding momentum associated with that energy, which is given by $p = h/\lambda$. This can be determined by performing similar types of experiments to those that determined the energy of the photon. There are various relationships that are valid for a photon's momentum and energy, summarized in Table 1.

The relation between intensity and number of photons

We can use unit analysis to determine how many photons are incident on an object. Suppose that light of frequency f is incident on an object, with observed intensity I. In SI units, I is given in $\mathrm{W/m^2}$. We can compute the total power incident by multiplying I by the area of the object, so $P = IA$, which has units of W. One photon has energy hf, so the number of photons incident *per second* is $N = \frac{IA}{hf}$.

There is a force exerted by this light on the object, since the photons also have momentum. Newton's second law is $F = \frac{dp}{dt}$ ($F = ma$ is only true if mass is constant, but photons don't have mass!). Suppose that the photons are absorbed. Then the momentum imparted per second is $pN = \frac{hN}{\lambda} = F$. If the photons are reflected (say a mirror), then the force is twice that.