Unit 3: Diffraction

Consider light passing through a single slit, incident on a screen a distance L away. Previously, we assumed that the slits acted as if they were infinitesimally thin and acted as point sources. For slits of a finite width, *each location* along the slit acts as if it were a point source; this principle is known as **Huygen's principle**. This will create a spot on the screen due to the interference between all the points on the slit. Thinner slits lead to larger spots and thicker slits lead to smaller spots.

After this unit, you should be able to

- Compute the size of spots that a single slit or a single circular aperture makes on a screen.

- Be able to construct phasor diagrams that lead to destructive interference and compute the angle between the phasors.

- For a diffraction-limited optical setup, suggest changes that will decrease/increase the size of the spots.

Phasors for more than two sources

Recall that the phasor diagram for two sources in constructive interference looks like $\longrightarrow\longrightarrow$. The angle between the phasors is $2\pi m$, with m an integer. This is achieved when $k(r_2 - r_1) = 2\pi m$, assuming the sources were in phase. For the two-slit experiment, $r_1 - r_2 = d\sin\theta$, which combined with before gives us the relationship $d\sin\theta = m\lambda$.

Phasors become particularly useful when there are more than two sources. Suppose that there are now three slits equally spaced d apart. Then $r_1 - r_2 = d\sin\theta$ and $r_2 - r_3 = d\sin\theta$. Then we can get constructive interference between *all three* if we have a phasor diagram like the following: $\longrightarrow\longrightarrow\longrightarrow$. In analogy to the two-slit problem, this happens when $k(r_2 - r_1) = 2\pi m$ *and* $k(r_3 - r_2) = 2\pi m$, for the same m. This can happen if $d\sin\theta = m\lambda$, the same condition for maxima of the two slit situation. You can likely see that the same condition will hold no matter how many evenly spaced slits we have.

It turns out (we will not prove this in this course) that the maxima get sharper the more slits are participating. This fact is used to produce **diffraction gratings**, which have many slits and are used to perform spectroscopy. Spectroscopy[1] allows us to measure precisely what wavelengths are present in a given light source.

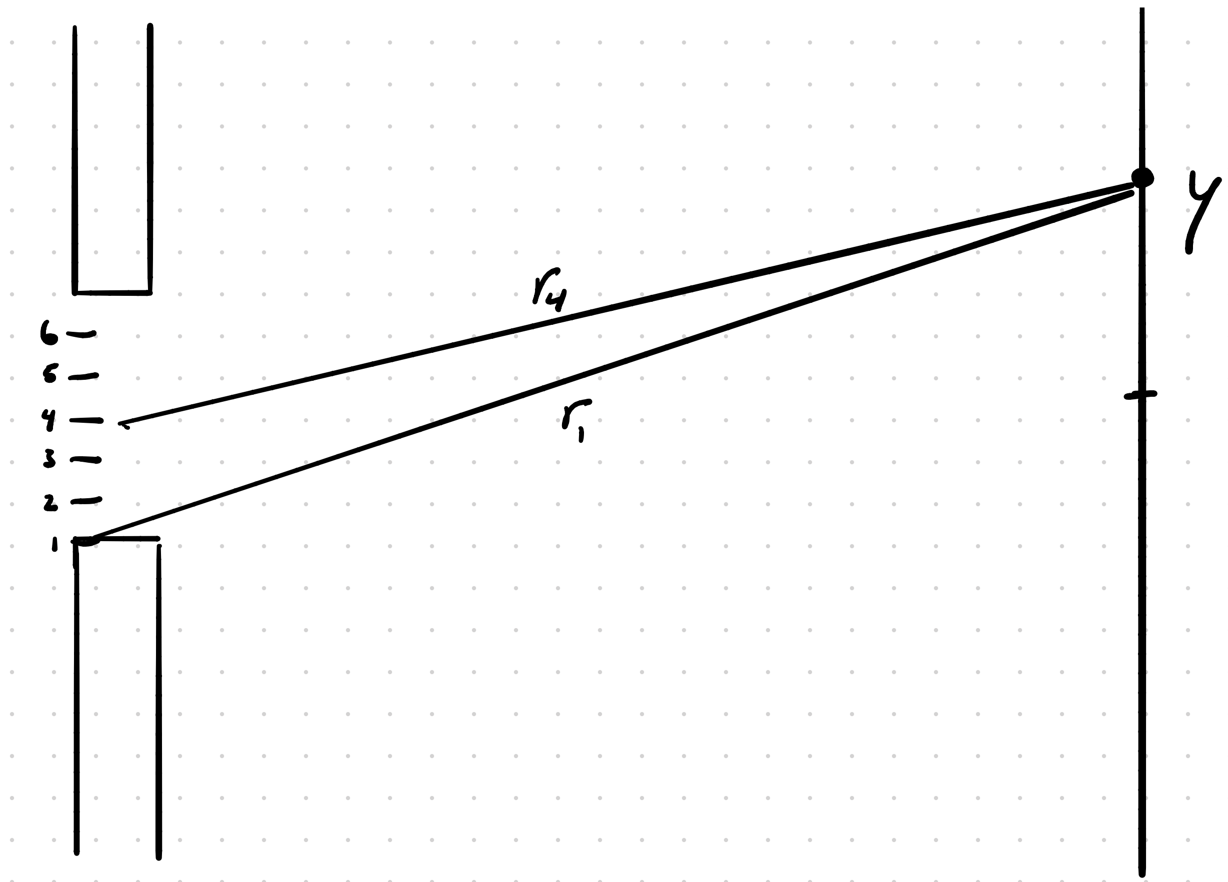

Figure 1: The partitioning of a single slit into 6 point sources.

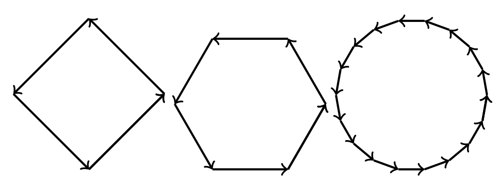

Figure 2: Phasor diagrams that lead to destructive interference for different numbers of point sources.

Diffraction from a single slit

Our goal in this section* will be to compute the intensity of light at a given position y on the screen. For our purposes, we will really just be satisfied in finding where the intensity goes to zero the first time, so we know how big the spot is. It's easier to consider the *angle* θ_0 at which the intensity is zero. To do this, let's pretend that the single slit (width a) is actually made up of N point sources, separated by a distance $d = a/N$. We will take the limit as $N \to \infty$.

There is complete destructive interference when the phasor diagrams complete a full loop, since the sum of all the phasors is zero (the sum ends up back where we started, so the total is zero). This is shown in Fig 2 for a few different values of N.

We will follow the same strategy as before; first we need to know what angle $\phi_0(N)$ leads to a closed phasor diagram. This is $2\pi/N$; that way N angles add up to one complete rotation, 2π radians. There is destructive interference when

$$k(r_2 - r_1) = \frac{2\pi}{N} \tag{1}$$

Plugging in $2\pi/\lambda$ for k and $a/N \sin\theta$ for $(r_2 - r_1)$, we get

$$a \sin\theta_0 = \lambda \tag{2}$$

Note that this result does not depend on N at all! So the limit as $N \to \infty$ does not change the result. Remember that this is the angle of the first zero in the spot.

The position of the zero on the screen is given by $y_0 = L \tan\theta_0$. The **size** of the spot is $2y_0$, since it extends in the positive and negative direction.

Diffraction for a circular aperture

For a slit geometry, the zero of the spot satisfies the equation $a \sin\theta_0 = \lambda$. For a circular geometry, the derivation is very similar to the single slit case. We will not derive this quantity in this class; it's a little complicated and does not offer much insight. The result is that for a circle of diameter D, the first zero in the pattern is $D \sin\theta_0 = 1.22\lambda$.[2]

Diffraction-limited optics

The presence of diffraction puts fundamental physical limits on how tightly we can focus light. Consider a point source of light, which is far away. The light from the point source is incident on a lens, which focuses the light onto a screen. This is how telescopes and cameras capture their images. The main point here is that even if a lens is used, diffraction will occur, so a point source will create on the screen a spot as large as we derived above.

[1] In Latin, spectrum means "image" and the -scopy relates to the study of a subject.

[2] If you want to know more, this is called the Airy disk

Example: diffraction and lithography

Lithography[3] is the technique used to draw tiny nanometer resolution circuits on semiconductors to create computers, phones, and other electronics. A laser is sent through a lens to focus it onto a given spot, which removes material from the silicon. The important question is how large the spot will be; this determines how narrow the lines are and how closely they can be drawn to one another.

Suppose the lens has diameter D and is a distance L from the silicon. The wavelength of the laser is λ. Then the first zero is at $\theta_0 = \sin^{-1}(1.22\lambda/D)$. The thickness of the line is $2L\tan\theta_0$. Plot these functions; you can see that if λ is increased, then the thickness of the line increases, while if D increases, then the thickness decreases. Similarly, L should be as small as possible to make the line as thin as possible.

[3] The litho prefix means 'stone,' and the -graph suffix means to write or draw, so lithography literally means 'to draw on stone.'