Your comments/questions/jokes

- Did you do anything exciting during summer so far?
  - Started teaching PHYS 214

From last semester:

- Why don't we normally detect destructive interference in everyday life?
  - Most of the time, separated wave sources are not *coherent*, so no interference.

- Calculating frequency? It's so easy it Hertz

Some announcements

- We set up the iClicker system to allow you to use the smartphone app.
- There is only one day on which each exam can be taken. The first exam is on Thursday, June 23, and it covers units 1-4 inclusive. Sign up on the CBTF scheduler!
- Can you see each other's responses to the checkpoint? ("student log")

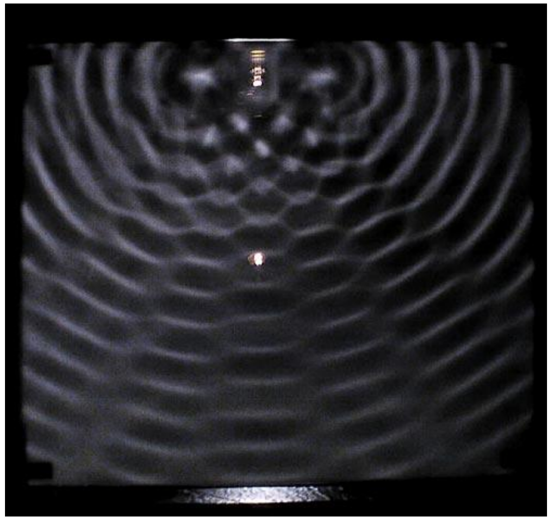
no

Lecture 2: Interference

whoa

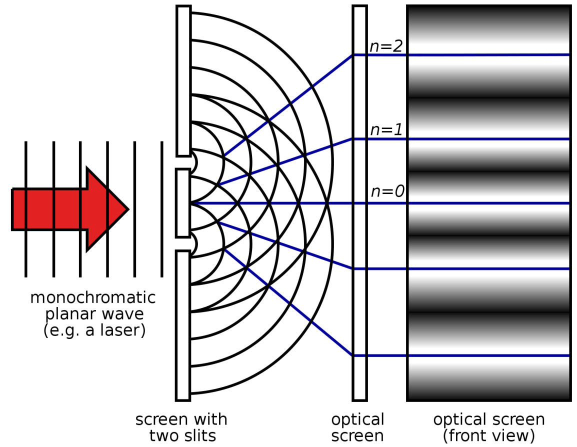
Today

Interference between two *coherent* wave-generating sources



Waves in water

Sound from two speakers



Light traveling through two slits

Two perfectly *coherent* sources have exactly the same phase relationship at all times, e.g. two sinusoidal sources synchronized at the same frequency

"coherent" does not imply "in phase"

but "in phase" implies "coherent"

Phasors are graphical tools that represent sinusoidally oscillating quantities

note on intensity:

$$I_{avg} = \langle [y(t)]^2 \rangle$$

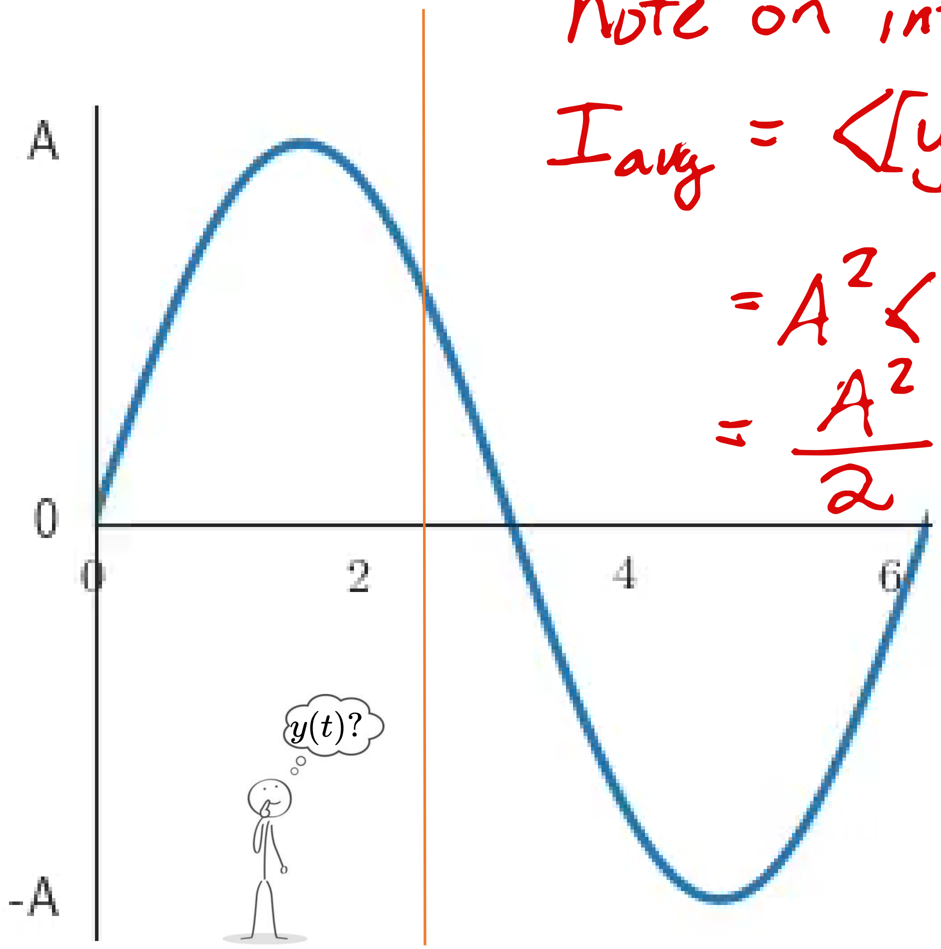
brackets mean "time average"

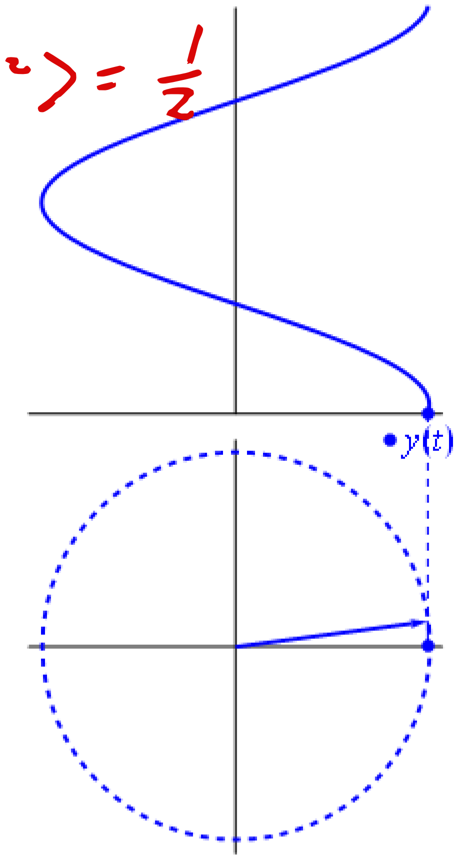
$$y(t) = A\cos(\omega t + \varphi)$$

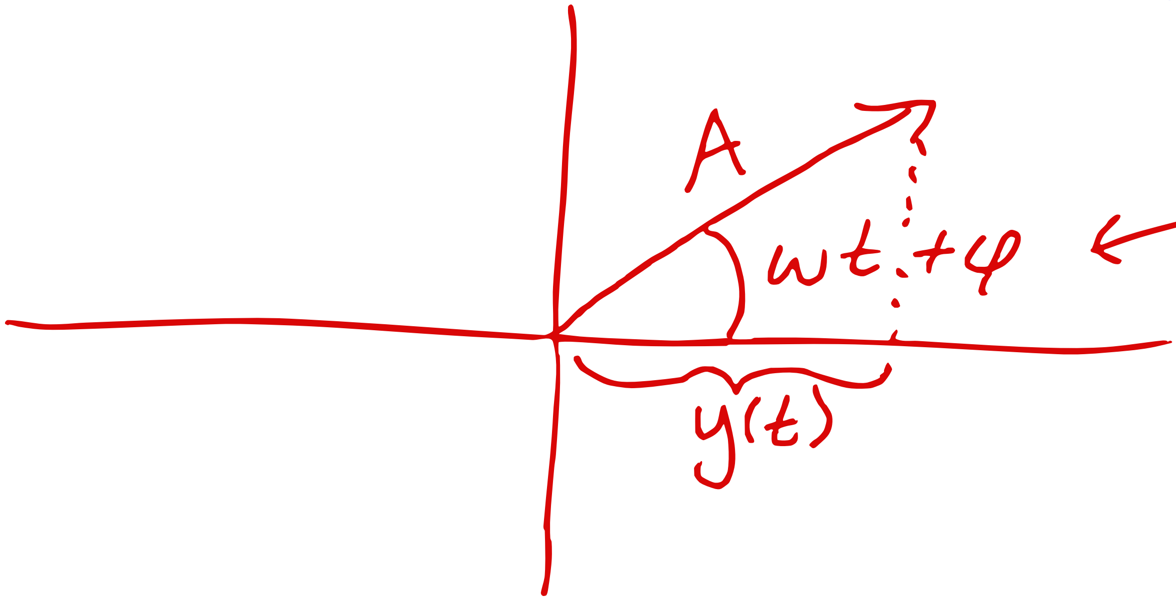
$$= A^2 \langle \cos^2 \rangle$$

$$\langle \cos^2 \rangle = \frac{1}{2}$$

$$= \frac{A^2}{2}.$$

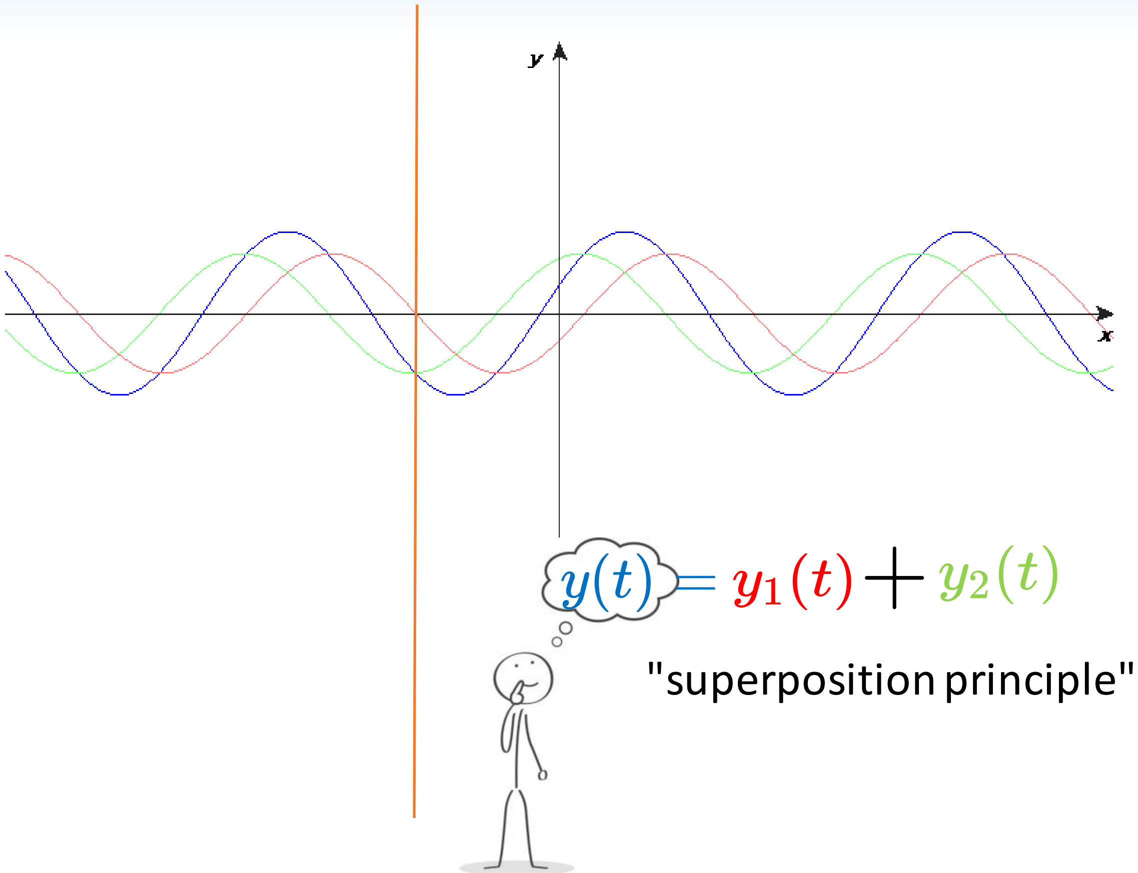






as t increases, phasor rotates CCW

Adding harmonic waves is easier with phasors



$$y_1(t) = A_1 \cos(\omega t + \varphi_1)$$

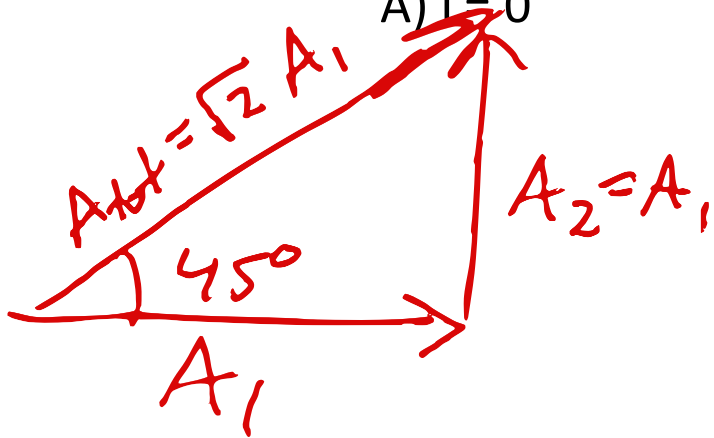
$$y_2(t) = A_2 \cos(\omega t + \varphi_2)$$

What is the intensity experienced by the observer?

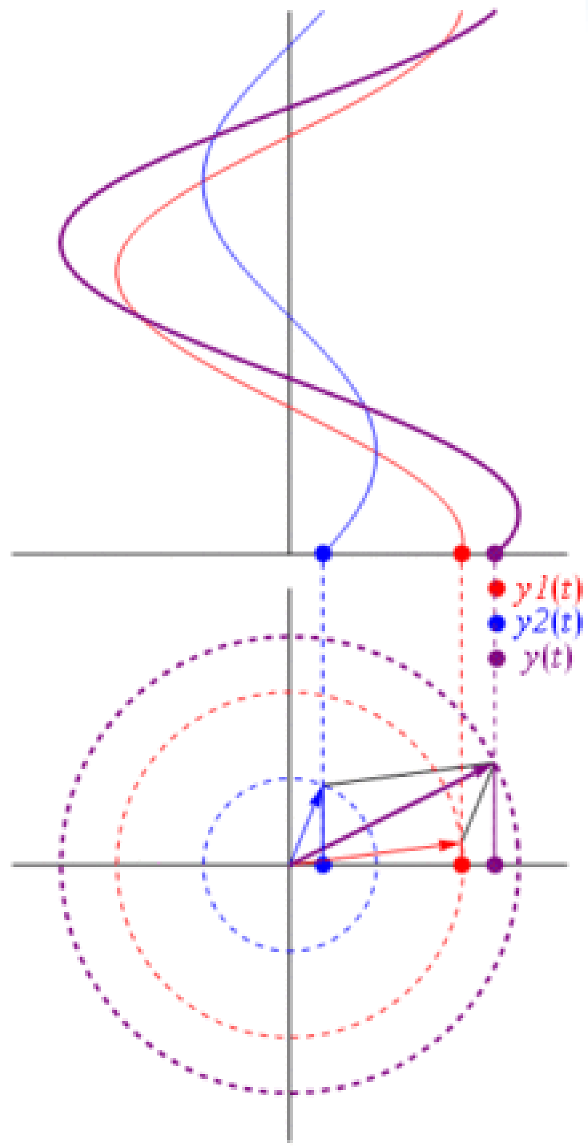
1. Draw and label the phasor diagram
2. Add the phasors like vectors
3. Find the length of the resultant
4. Take the square: $I = \dfrac{A^2}{2}$

If $A_1 = A_2$, what is the intensity at a point where $\varphi_1 = 0$ and $\varphi_2 = \pi/2$?

A) I = 0 B) I = 2 $(A_1)^2$ C) I = $(A_1)^2$

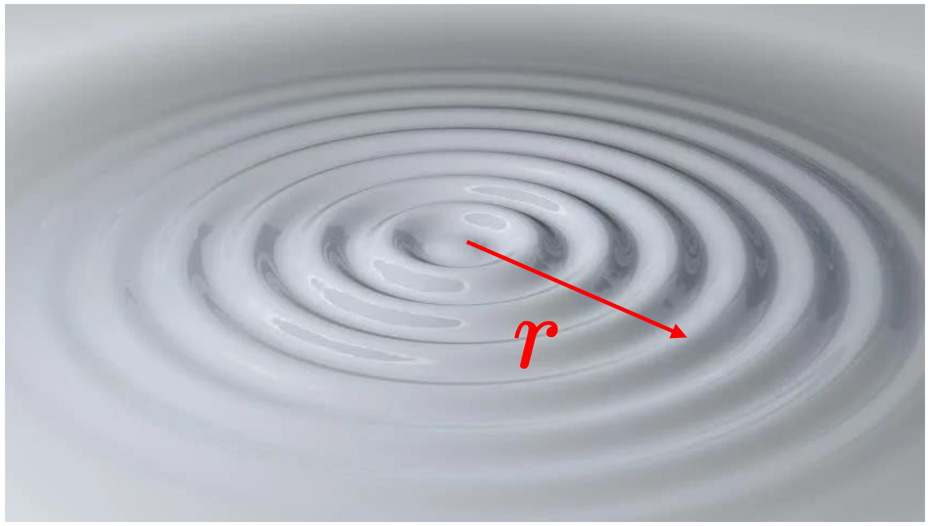


$$I = \frac{A_{tot}^2}{2} = \frac{2A_1^2}{2}$$



$y(t)$ has same ω as y_1 and y_2, but typically different phase offset.

Circular waves, two-source interference setup

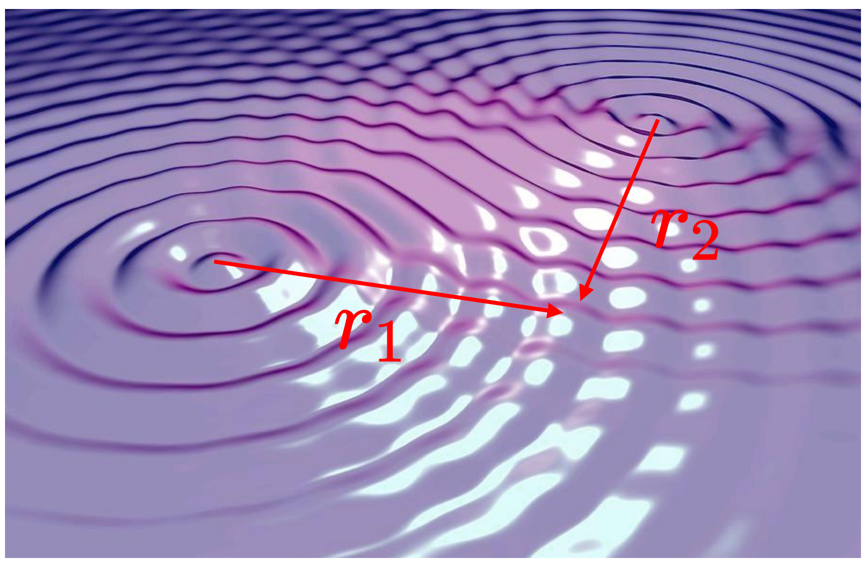


$$y = A\cos(kr - \omega t + \phi)$$

Amplitude decreases with r (why must it?)
but we ignore that

energy conservation

energy spreads out as waves move away, so intensity must be lower at larger r



$$y_1 = A_1\cos(kr_1 - \omega t + \phi_1)$$

$$y_2 = A_2\cos(kr_2 - \omega t + \phi_2)$$

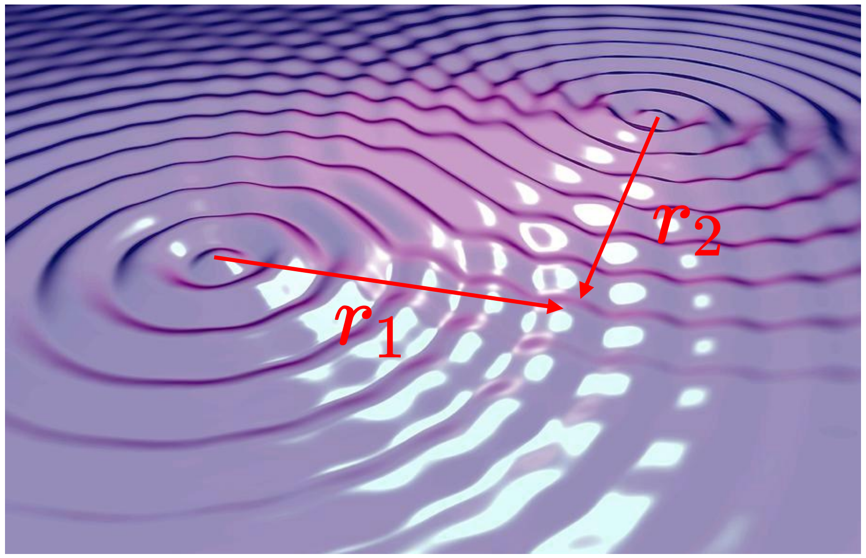
$$y = y_1 + y_2$$ superposition principle

not valid for all types of waves, but for many important cases

(shallow water waves, light, sound, quantum mechanical matter waves)

Ripple tank demo

two-source interference solution



$$y_1 = A_1 \cos(kr_1 - \omega t + \phi_1)$$

$\varphi_2 = -(\phi_2 + kr_2)$

$$y_2 = A_2 \cos(kr_2 - \omega t + \phi_2)$$

$$y = y_1 + y_2$$ superposition principle

From trig identities, *if $A_1 = A_2$* we could show

$$y = \underbrace{2A_1 \cos\left(\frac{k(r_1 - r_2) + \phi_1 - \phi_2}{2}\right)}_{\text{amplitude at observer}} \underbrace{\cos\left(\frac{k(r_1 + r_2) + \phi_1 + \phi_2}{2} - \omega t\right)}_{\text{Oscillating part}}$$

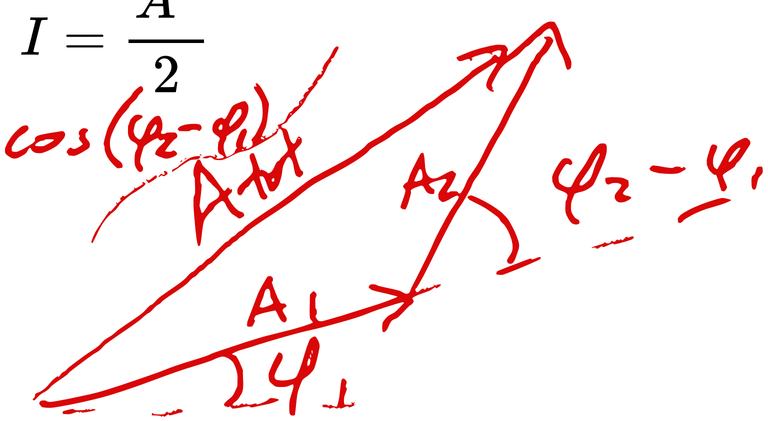
Intensity is proportional to (that)^2

If A_1 different from A_2, the easiest way is with phasors.

1. Draw and label the phasor diagram
2. Add the phasors like vectors
3. Find the length of the resultant
4. Take the square: $I = \dfrac{A^2}{2}$

$A_{tot}^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\varphi_2 - \varphi_1)$

Law of cosines

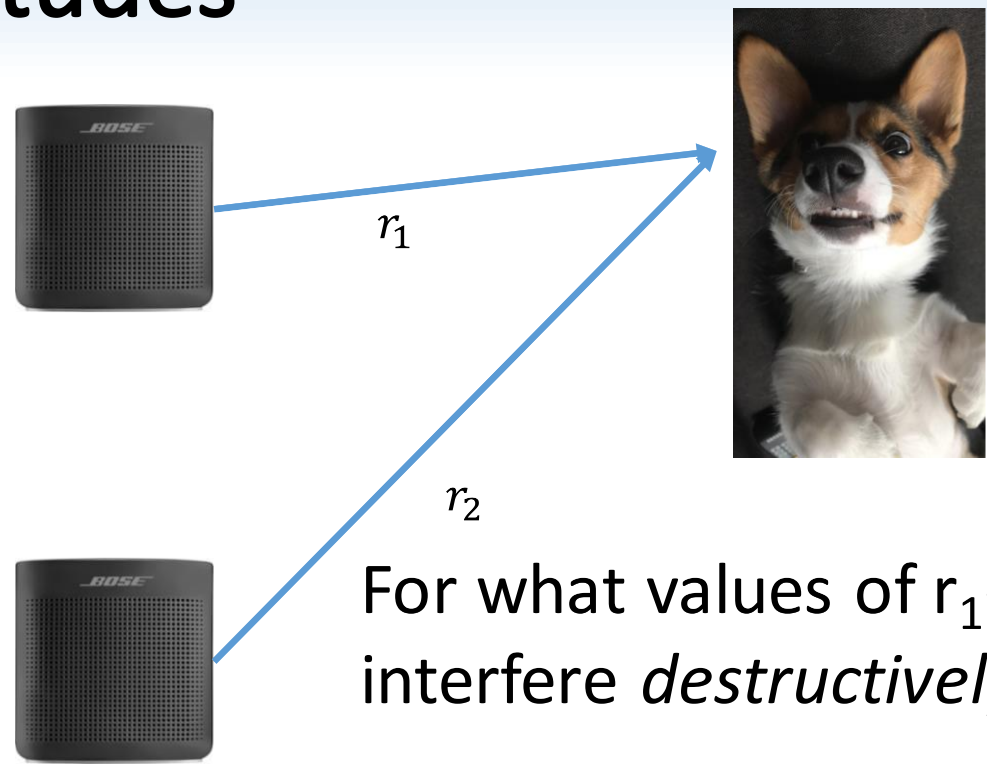


$\varphi_1 = -(\phi_1 + kr_1)$

$\varphi_2 = -(\phi_2 + kr_2)$

curly φ vs regular ϕ notation is <u>not</u> standard!

Two speakers emitting in phase with equal amplitudes



$$f_1 = A\cos(\overbrace{\frac{2\pi r_1}{\lambda} - \omega t}^{arg_1})$$

$$f_2 = A\cos(\underbrace{\frac{2\pi r_2}{\lambda} - \omega t}_{arg_2})$$

For what values of r_1-r_2 will the waves interfere *destructively* at the observer?

A) r_1-r_2 = $0, \pm\lambda, \pm 2\lambda, \ldots$

B) r_1-r_2 = $\pm\frac{\lambda}{2}, \pm\frac{3\lambda}{2}, \ldots$

C) r_1-r_2 = $0, \pm 2\lambda, \pm 4\lambda, \ldots$

D) r_1-r_2 = $\pm\frac{\lambda}{4}, \pm\frac{\lambda}{2}, \ldots$

for a crest of 1 to meet a trough of 2, we need

$arg_2 - arg_1 = (2n+1)\pi$ (an odd multiple of π)

$\Rightarrow \frac{2\pi}{\lambda}(r_2 - r_1) = (2n+1)\pi \Rightarrow r_2 - r_1 = \frac{2n+1}{2}\lambda = 0, \pm\frac{\lambda}{2}, \pm\frac{3\lambda}{2}, \ldots$

Checkpoint

An observer is a distance r_1 and r_2 respectively from two wave sources that emit in phase.

Which quantity alone would allow you to compute whether the waves interfere destructively or constructively?

a) The wavelength λ.

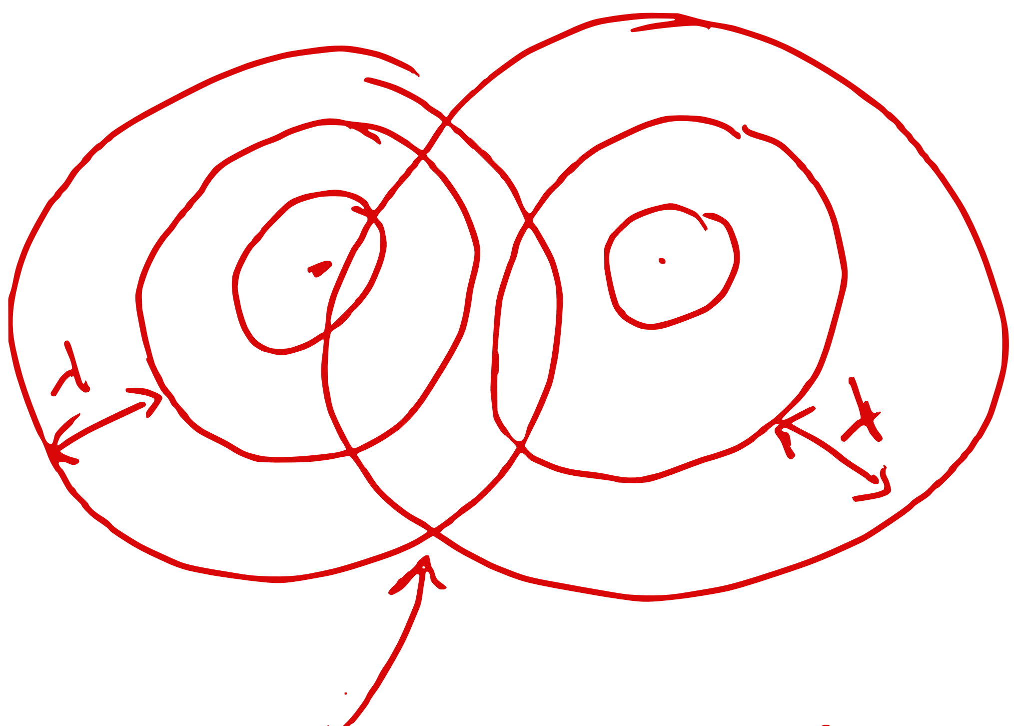
b) The frequency f

c) The amplitudes A_1 and A_2

d) λ **and** f

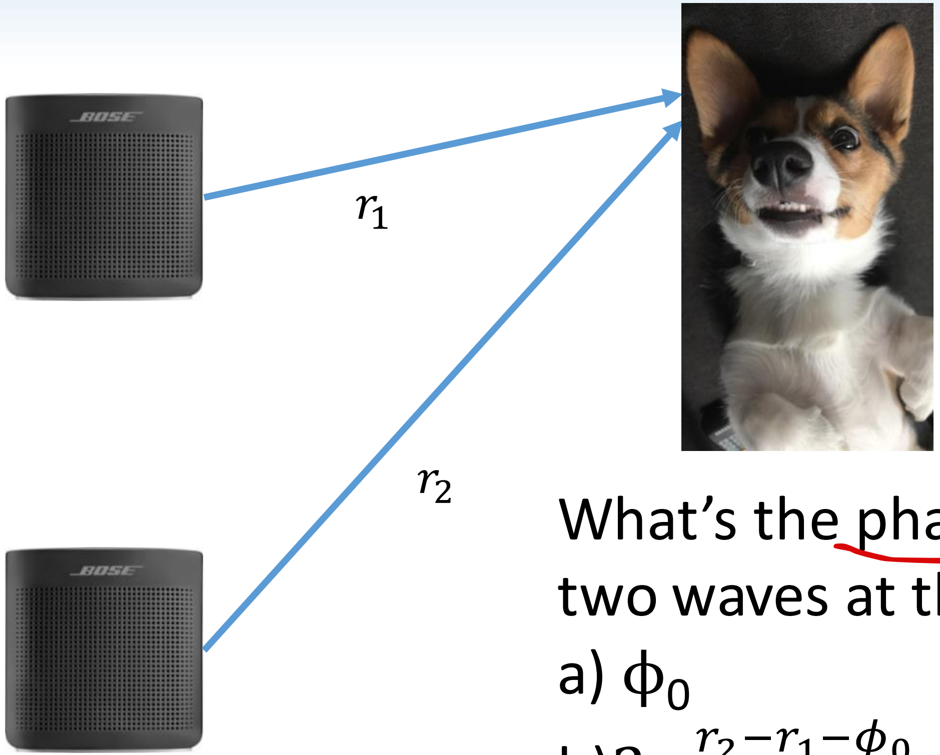
e) λ **and** A_1 and A_2

see previous page!



constructive interference when crest meets crest or trough meets trough.

What if we change the phase offset?



$$f_1 = A\cos\Big(\underbrace{\boxed{\frac{2\pi r_1}{\lambda} - \omega t}}_{\text{arg}_1}\Big)$$

$$f_2 = A\cos\Big(\underbrace{\boxed{\frac{2\pi r_2}{\lambda} - \omega t + \phi_0}}_{\text{arg}_2}\Big)$$

arg₂ − arg₁ (or arg₁ − arg₂)

What's the phase difference between these two waves at the listener's position?

a) ϕ_0

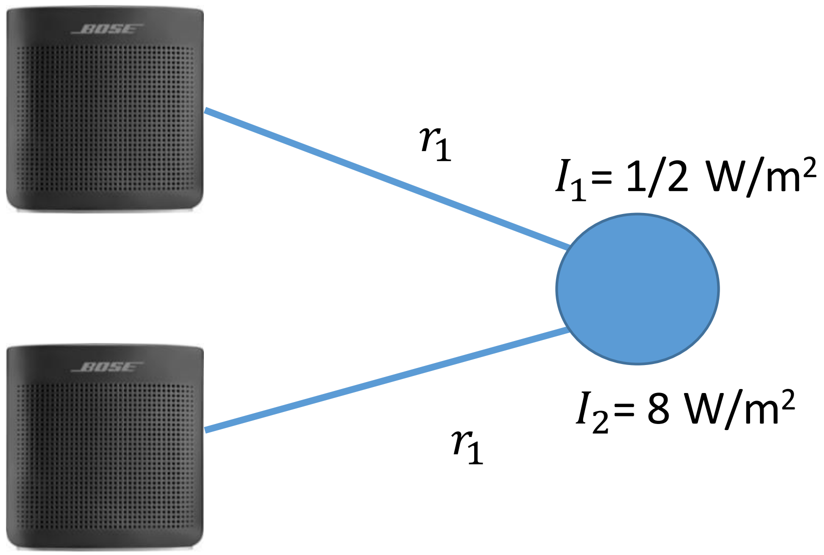
b) $2\pi\frac{r_2 - r_1 - \phi_0}{\lambda}$

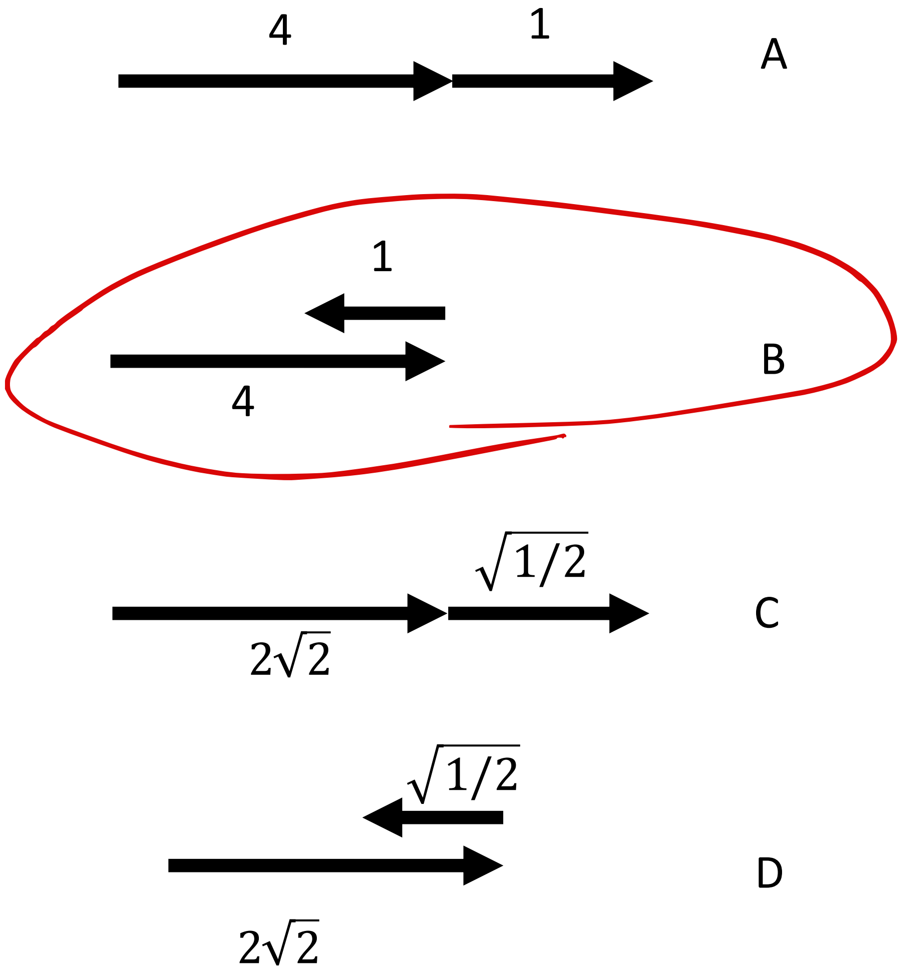
c) $2\pi\frac{r_2 - r_1 + \phi_0}{\lambda}$

d) $2\pi\frac{(r_2 - r_1)}{\lambda} - \phi_0$

e) $2\pi\frac{(r_2 - r_1)}{\lambda} + \phi_0$

Combining phasors practice

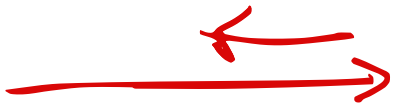




harmonic waves, same frequency, coherent,

Two speakers, different amplitudes. Same distance, π phase difference at observer.

What's the phasor diagram?



(180° angle between the A_1 & A_2 phasors)

$$I_1 = \frac{A_1^2}{2}$$

$$\Rightarrow A_1 = \sqrt{2 I_1} = 1$$

similarly $A_2 = 4$ $\Bigg\} \Rightarrow$

btw, $A_{tot} = 3$...

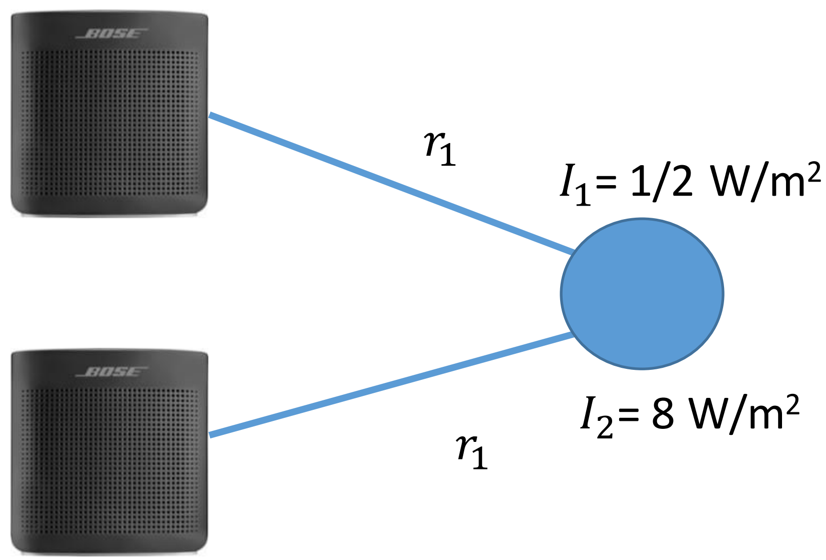
Combining phasors practice

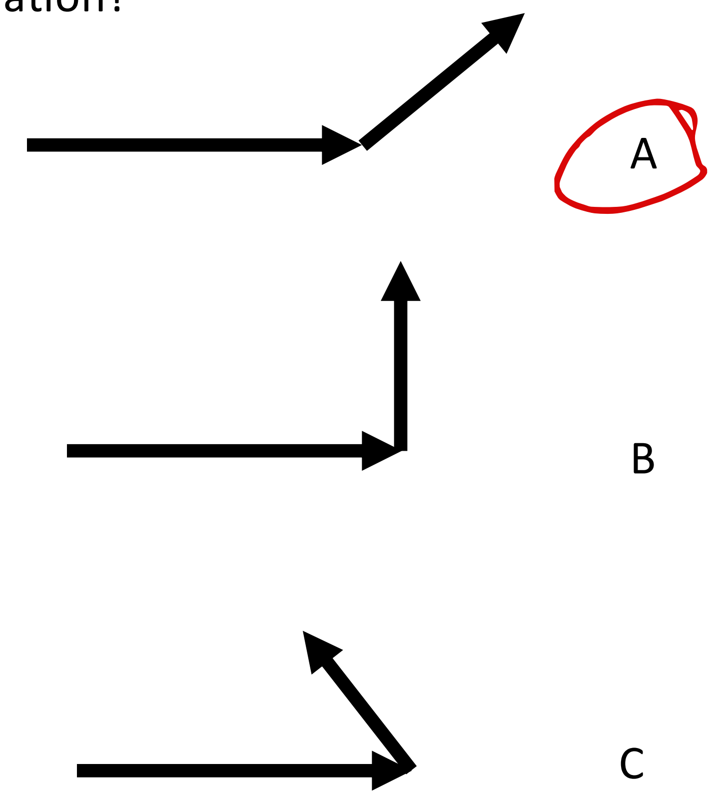
harmonic waves, same frequency, coherent,

Two speakers, different amplitudes.

Same distance, $\frac{\pi}{4}$ phase difference at observer.

Which phasor diagram corresponds to this situation?

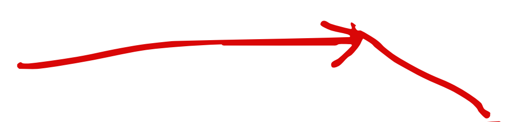




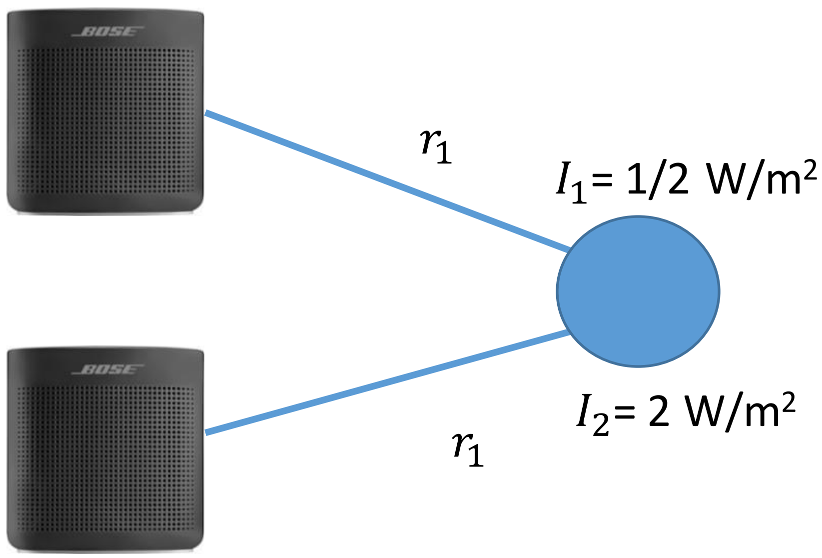
$$f_1 = A_1\cos\left(\frac{2\pi r_1}{\lambda} - \omega t + \frac{\pi}{4}\right)$$

$$f_2 = A_2\cos\left(\frac{2\pi r_1}{\lambda} - \omega t\right)$$

or



Combining amplitudes practice



harmonic waves, same frequency, coherent,

Two speakers, different amplitudes.
Same distance, same phase at the source (in phase).

What's the intensity at the observer?

a) 0.5 W/m²
b) 2.5 W/m²
c) 3 W/m²
d) 4.5 W/m²
e) 9 W/m²

same phase @ source and same distance ⇒ same arg at observer

i.e. zero phase diff. at observer,

$A_1 = \sqrt{2I_1} = 1$ $\quad A_2 = \sqrt{2I_2} = 2$ $\quad \Rightarrow A_{tot} = 3$

$$I_{tot} = \frac{A_{tot}^2}{2} = \frac{9}{2} = 4.5$$

Intermediate summary

If two waves are same amplitude and same wavelength:

$$I = 2A^2 \cos^2 \frac{\varphi}{2}$$

$$\varphi = \frac{2\pi\delta}{\lambda} + \Delta\phi_0$$

$$\delta = r_1 - r_2$$

Maxima at $\varphi = 2\pi m$

Minima at $\varphi = 2\pi\left(m + \frac{1}{2}\right)$

(m is any integer)

For more sources, or for different amplitudes:

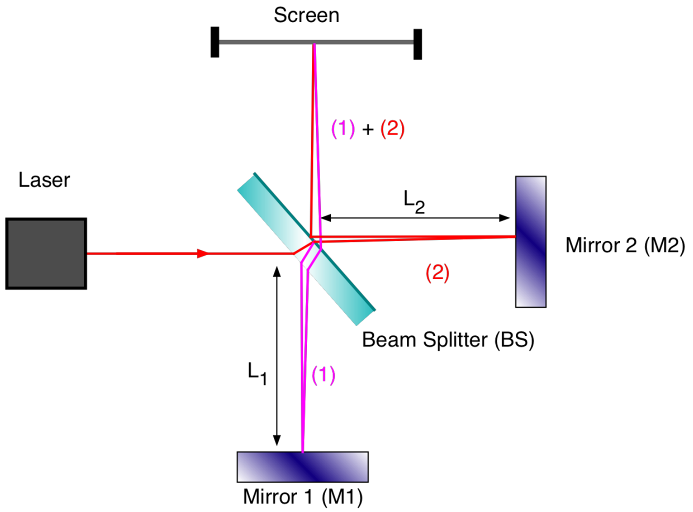
Amplitude <-> length of vector

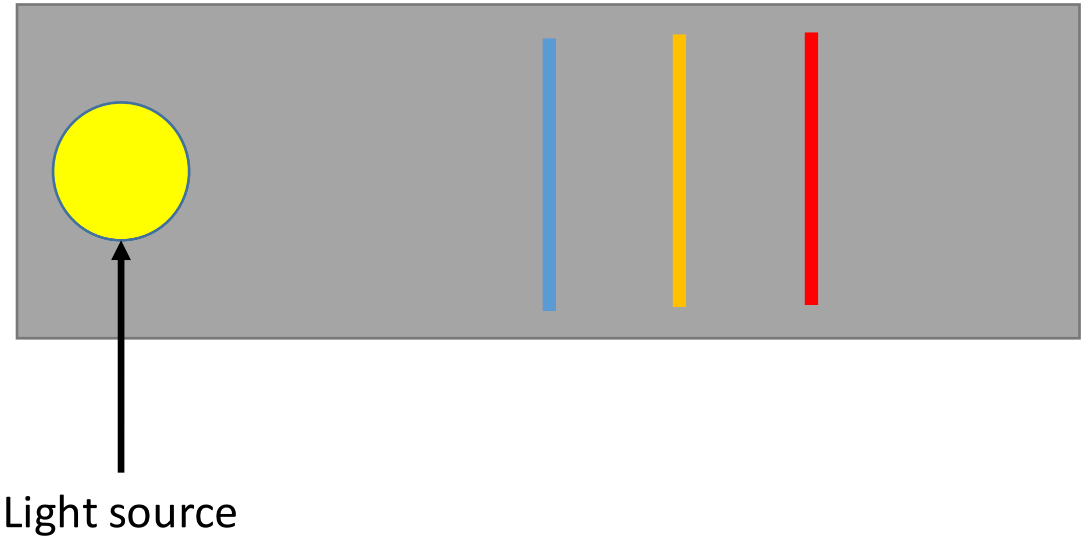
Phase offset <-> angle of vector with respect to horizontal axis at t=0

Use law of cosines:

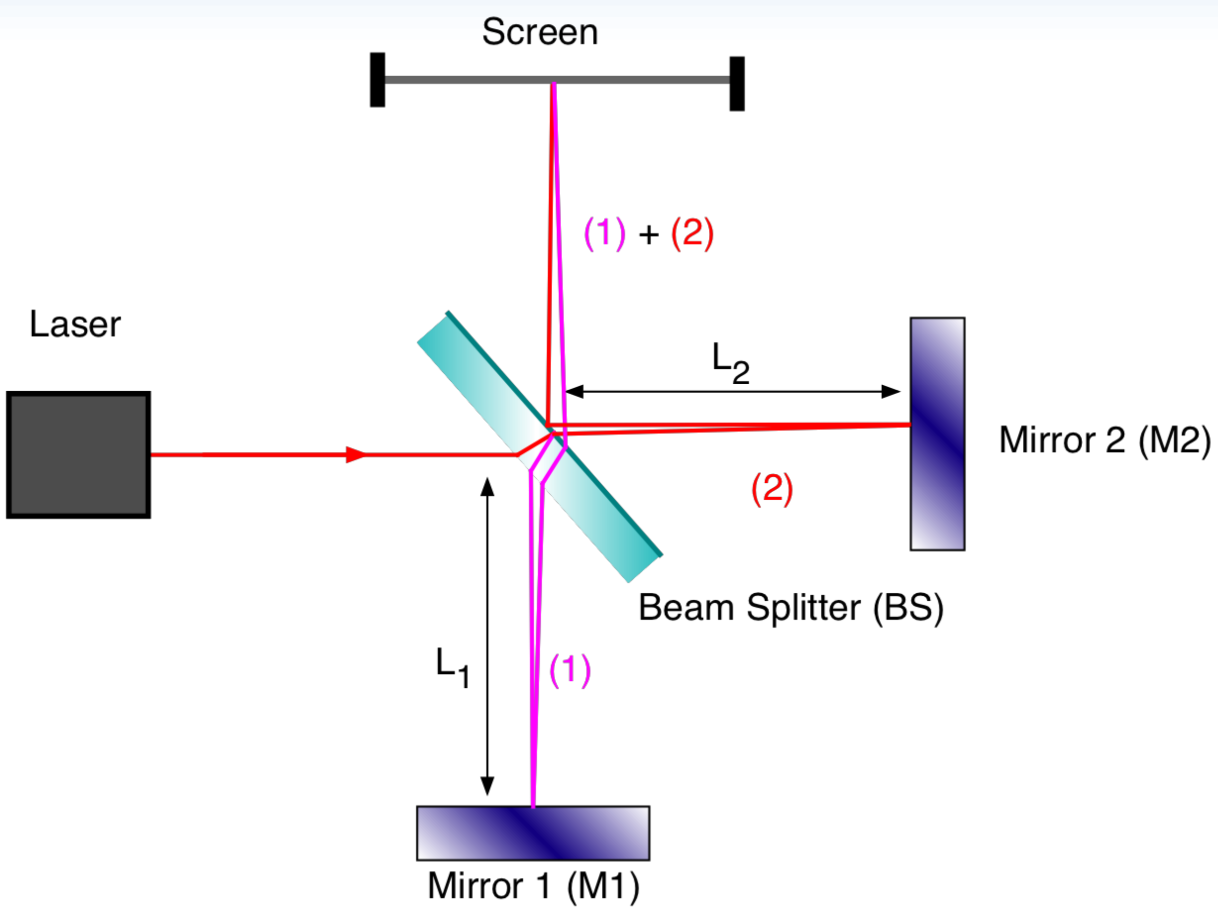
$$c^2 = a^2 + b^2 + 2ab\cos\theta$$

Practice with interference





Interferometer

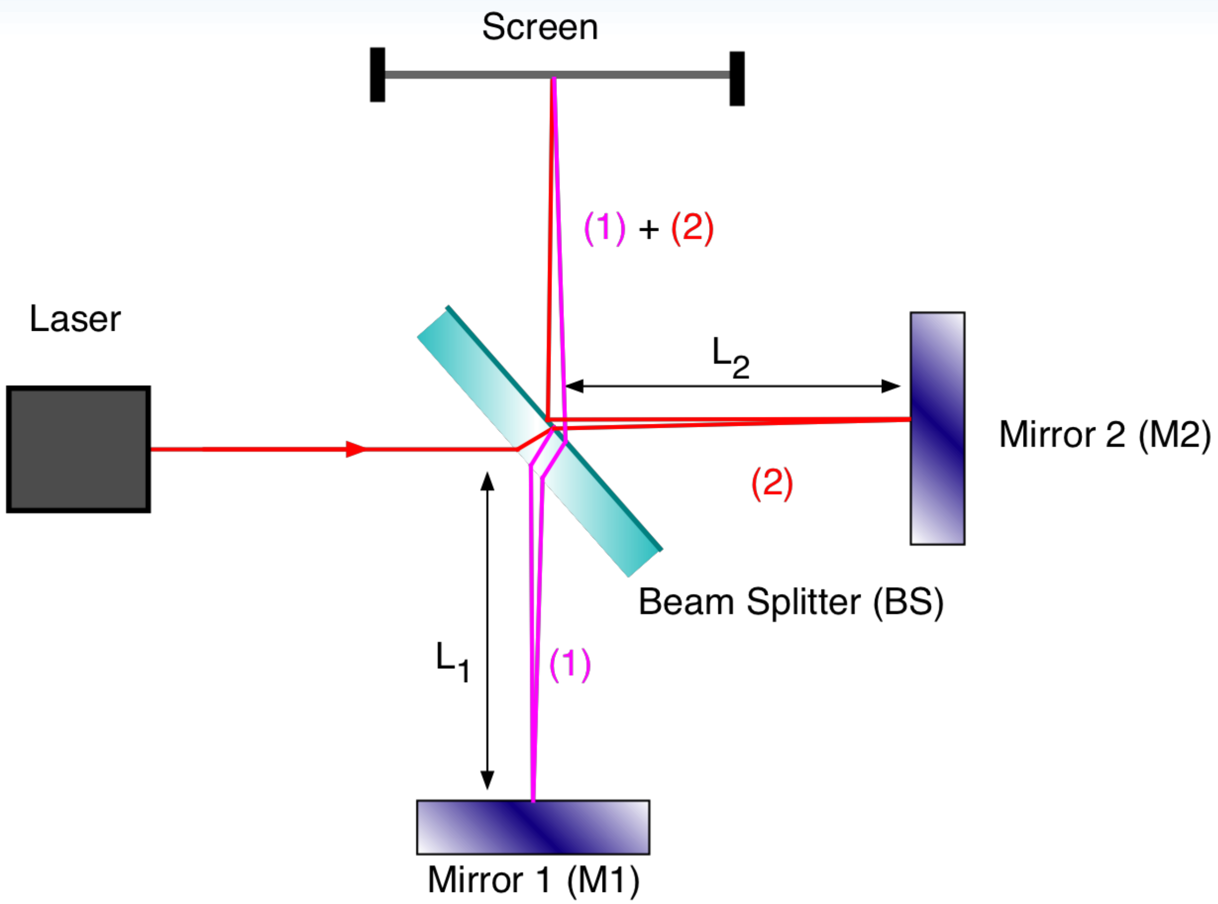


Example of path-length dependent interference. We know the lengths of the arms L_2 and L_1.

What is $r_2 - r_1$?

$a) L_2 - L_1$
$b) 2L_2 - 2L_1$
$c) d \sin\theta$
$d) 2d \sin\theta$
e) Need the path from the laser

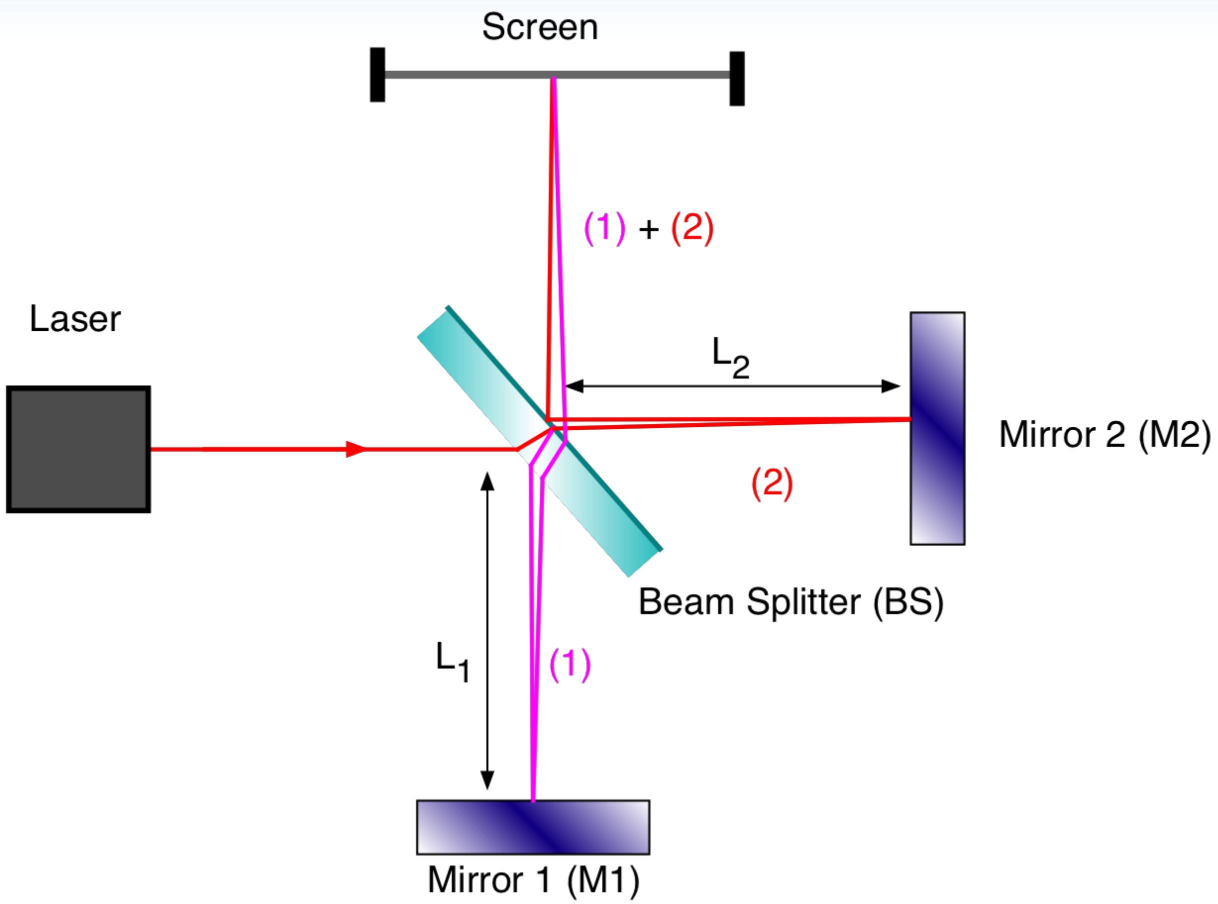
Interferometer problem



Laser has wavelength λ. We observe maximal intensity at the screen. What is the **phase difference** between the two paths?

a) $2\pi n$
b) λn
c) $2\pi(n + \frac{1}{2})$
d) $\lambda(n + \frac{1}{2})$

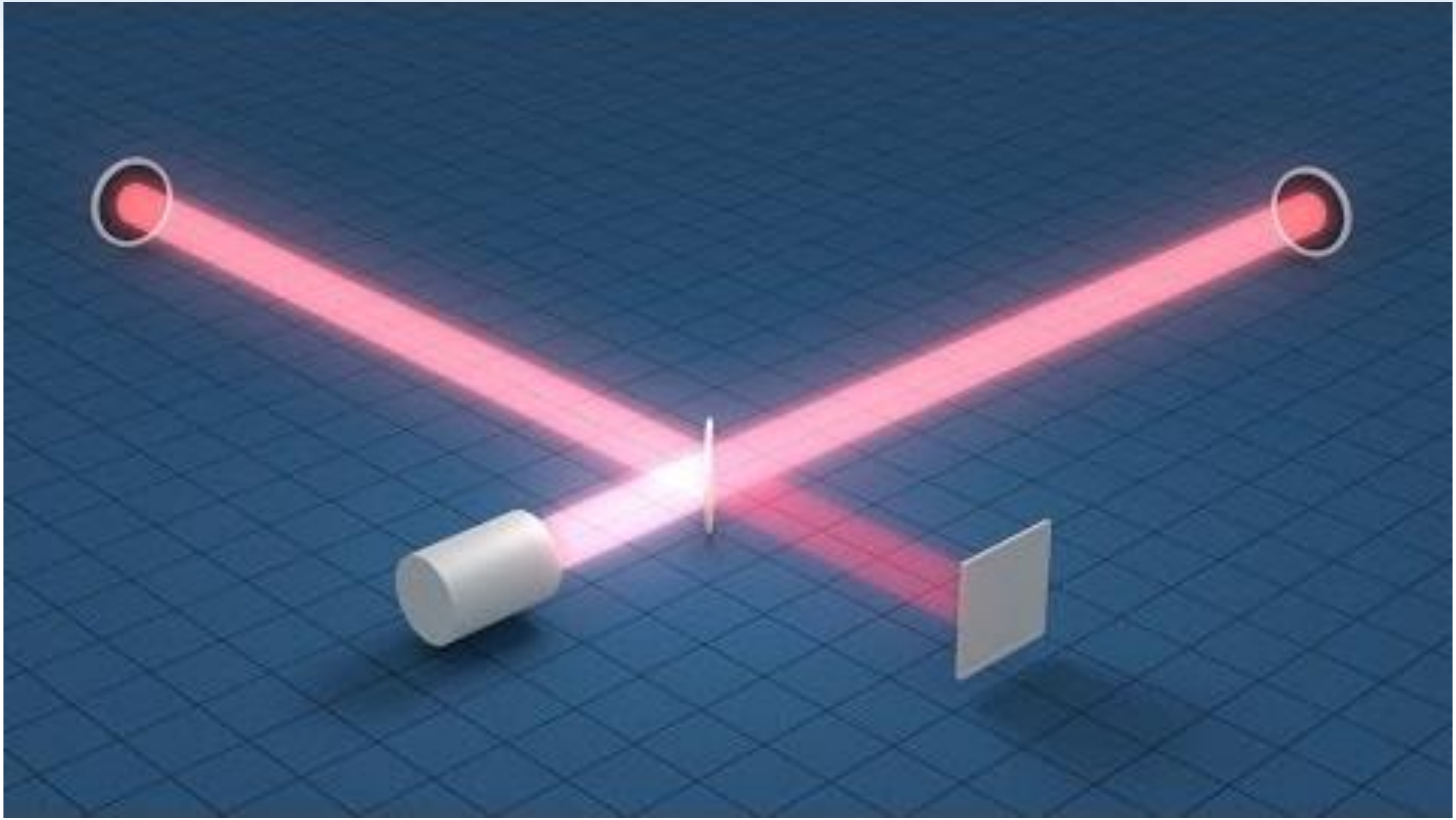
Interferometer problem



Laser has wavelength λ. We observe maximal intensity at the screen. How far must we move mirror 1 to find the next maximum?

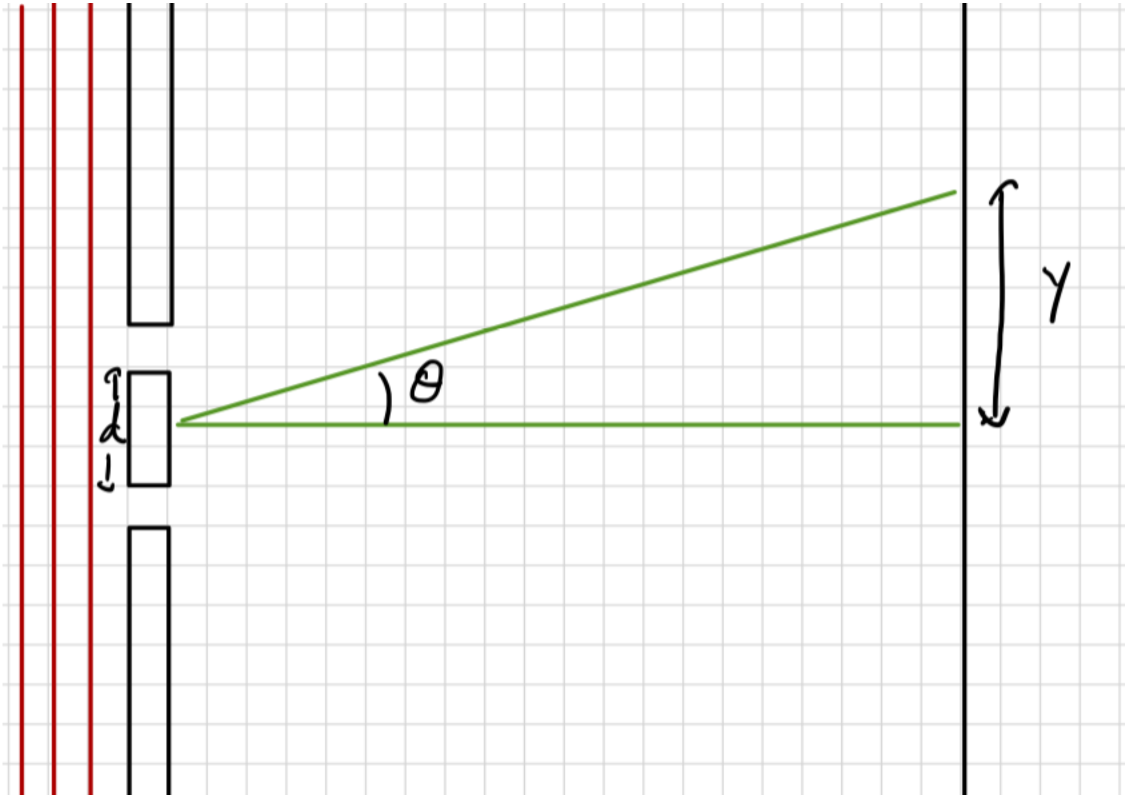
a) 2π
b) λ
c) π
d) $\frac{\lambda}{2}$

Application: LIGO



https://youtu.be/tQ_teIUb3tE

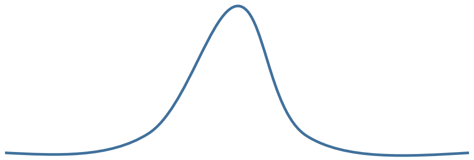
Two-slit experiment



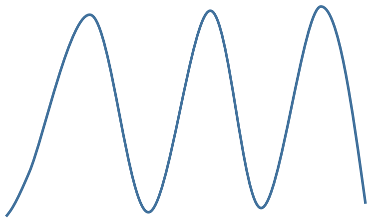
coherent, monochromatic

Each slit acts as a source for light. They are in phase with each other.

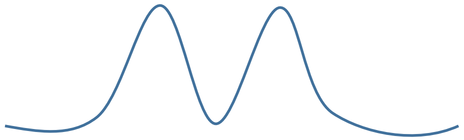
What do you expect to see for $I(y)$?



a) Isolated peak

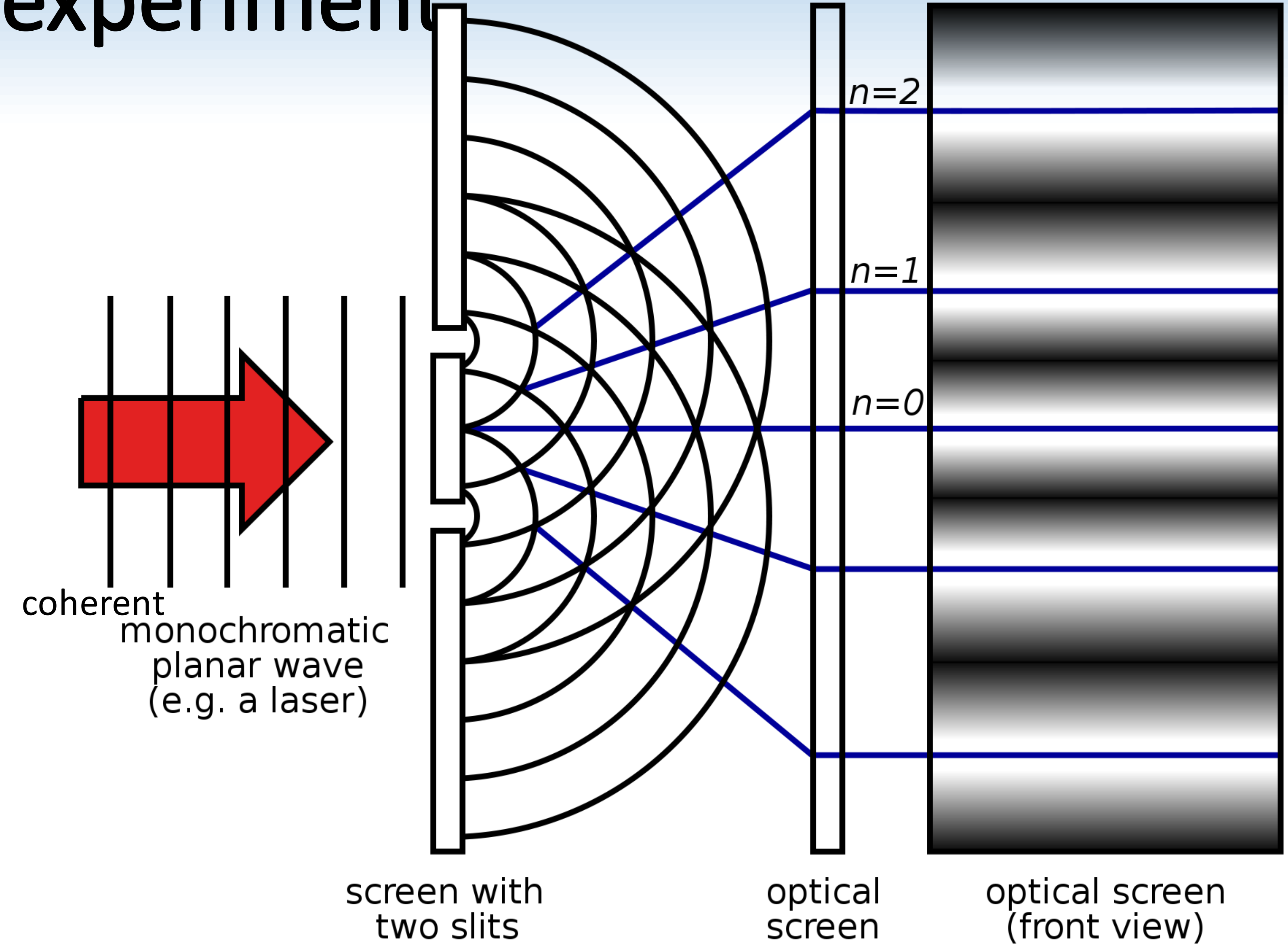


b) Repeating peaks

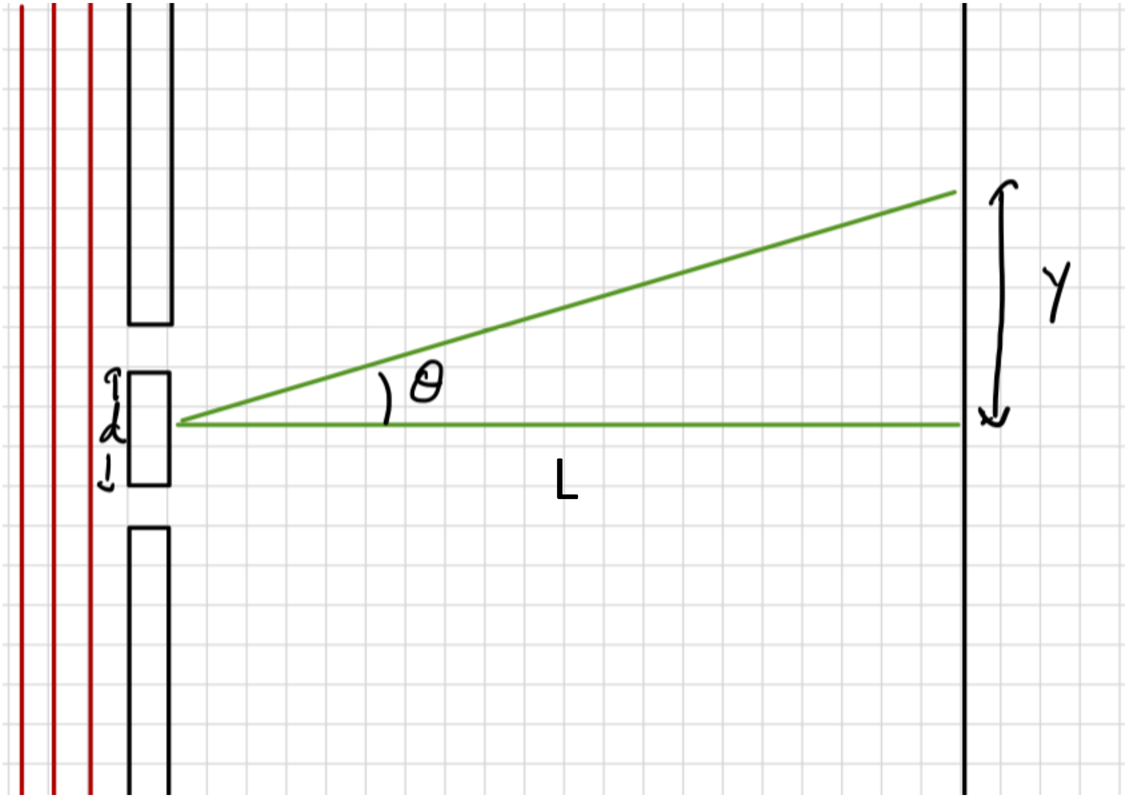


c) Two isolated peaks

Two-slit experiment



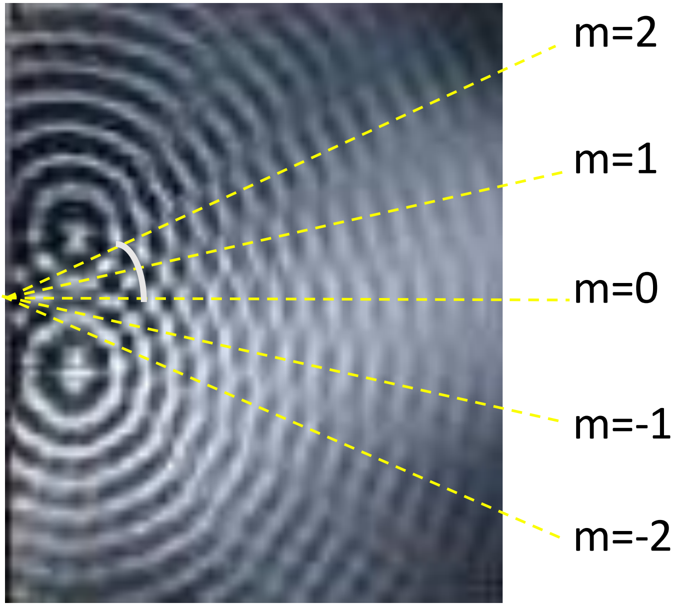
Two-slit experiment: distances



$$E_1(y,t) = E\cos(\omega t - kr_1)$$
$$E_2(y,t) = E\cos(\omega t - kr_2)$$

Approximation: L is very big compared to d and y.

Constructive interference



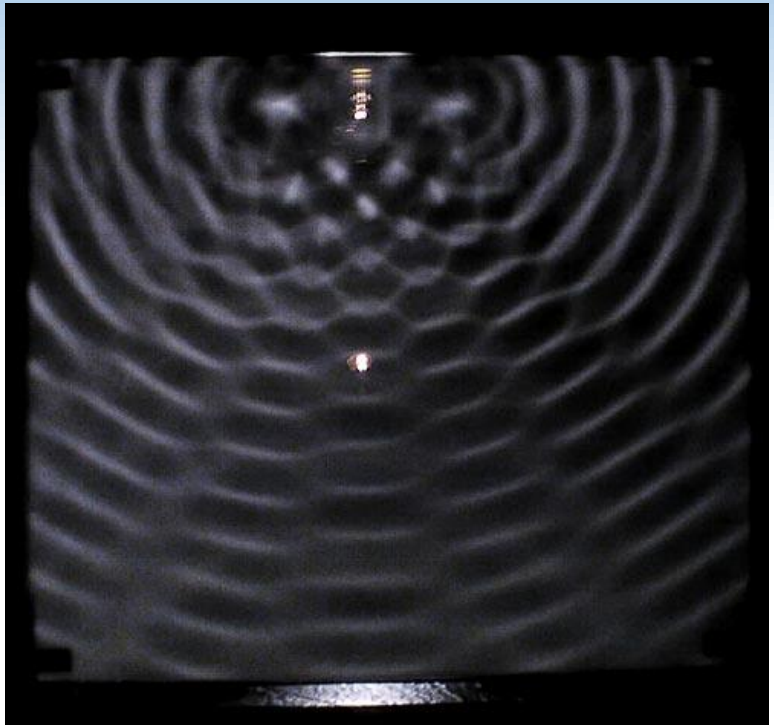
Constructive interference:

$$\phi = 2m\pi = \frac{2\pi d \sin\theta}{\lambda}$$

What is the formula for $d \sin\theta_{max}$?

$a)\, 2m\lambda$
$b)\, 2\lambda$
$c)\, 2m\pi\lambda$
$d)\, m\lambda$

Summary



Waves generated from two sources at distance r_1 and r_2.

To an observer, looks like

$$y_1(x,t) = A_1 \cos(kr_1 - \omega \mathrm{t} + \varphi_1)$$

$$y_2(x,t) = A_2 \cos(kr_2 - \omega \mathrm{t} + \varphi_2)$$

To find $y_1 + y_2$, use phasors!

The $kr_i + \varphi_i$ term tells us the angle of the phasor. The relative angle is most important for interference.