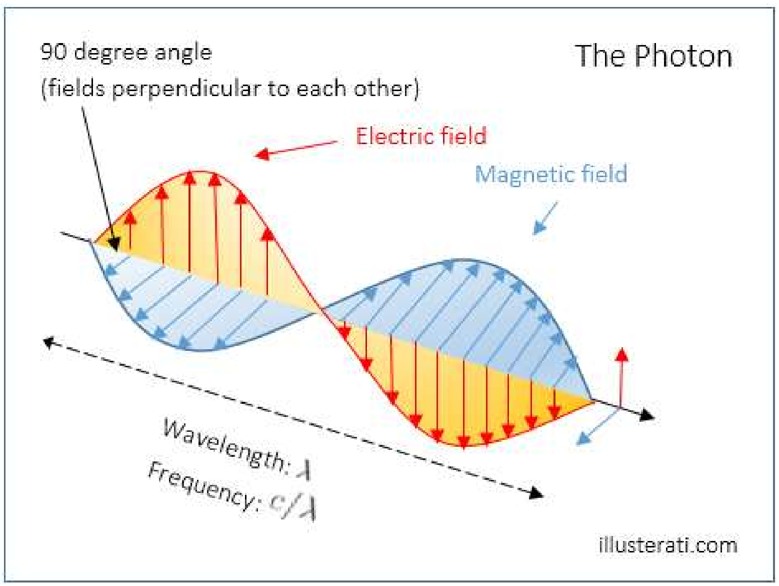
Part 1: Polarization

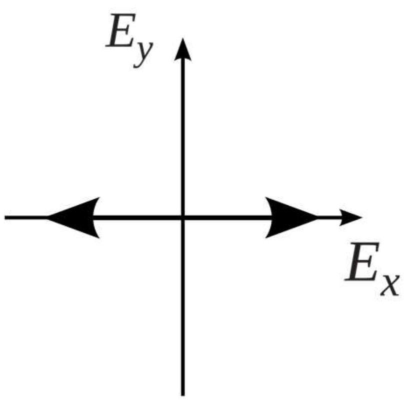


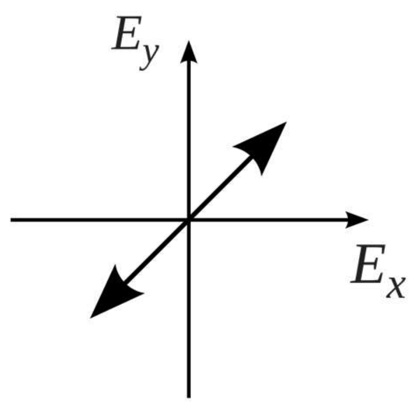
What we will learn today

ψ_h photon is polarized in the horizontal direction

ψ_v: photon is polarized in vertical direction

$$\psi = a\psi_v + b\psi_h$$





Probability of passing through a vertical filter:

$$P(v) = \frac{|a|^2}{|a|^2 + |b|^2}$$

Probability of passing through a horizontal filter:

$$P(h) = \frac{|b|^2}{|a|^2 + |b|^2}$$

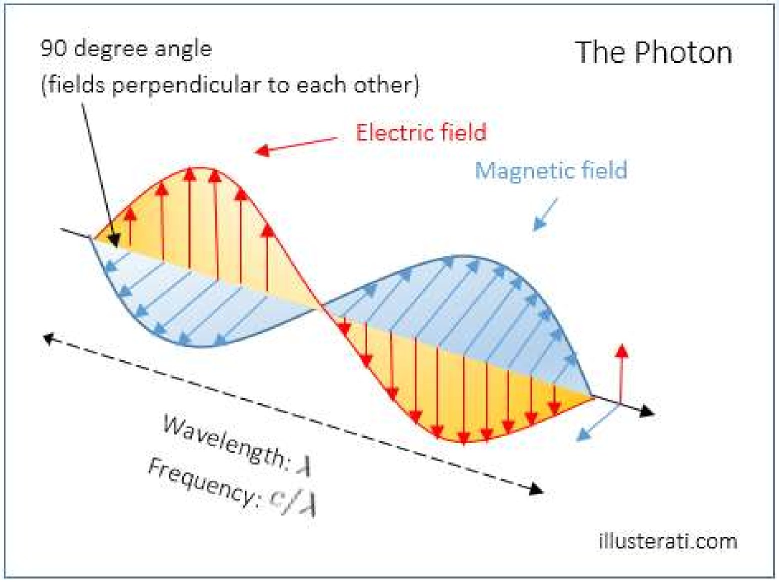
Polarization as a quantum state

Separate from the position wave function.

Basis states:

ψ_h photon is polarized in the horizontal direction

ψ_v: photon is polarized in vertical direction



Making horizontally polarized photons

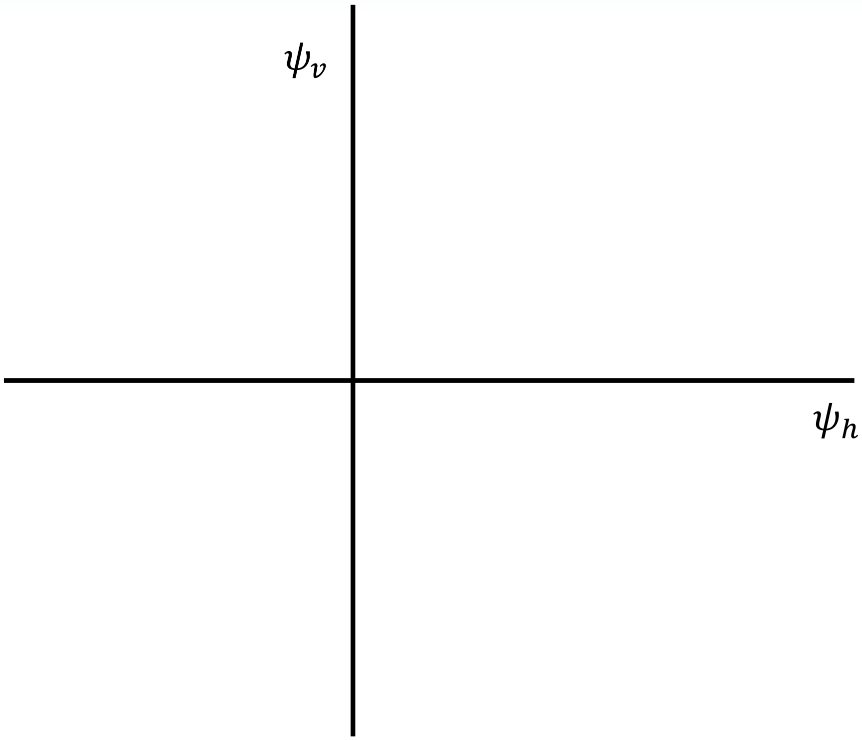
Photons with random polarization, pass through a horizontal filter, all photons that pass through are horizontally polarized.

Suppose we pass horizontally polarized photons through a vertical filter.

What is the probability that the photons will pass through?

a) 0
b) ¼
c) ½
d) ¾
e) 1

Representing other polarizations



Diagonal polarization:

$$\frac{\psi_v + \psi_h}{\sqrt{2}}$$

Measurement rule

Wave function can be a superposition of these two states:

$$\psi = a\psi_v + b\psi_h$$

If we measure vertical polarization, we will get (for example):

$$P(v) = \frac{a^2}{a^2 + b^2}$$

The state after will be

$$\psi_v$$

For a general polarizer, the probability of measuring photons with orientation S is given by

$$P(S) = |S^*.\psi|^2$$

Checkpoint

Light with polarization wave function $\frac{\psi_v+\psi_h}{\sqrt{2}}$ passes through a horizontal filter, then passes through a second horizontal filter.

What's the probability that the photon will be transmitted through both filters?

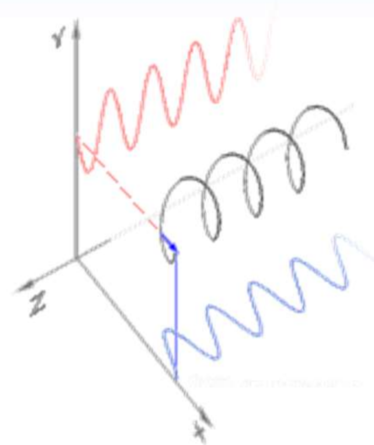
a) 0
b) ¼
c) ½
d) ¾
e) 1

What's the state once it's passed through the filters?

a) $\frac{\psi_v+\psi_h}{\sqrt{2}}$
b) ψ_v
c) ψ_h

Circular polarization

$$\psi = \frac{\psi_v + i\psi_h}{\sqrt{2}}$$
$$\psi = \frac{\psi_v - i\psi_h}{\sqrt{2}}$$



Not going to go too much into this!

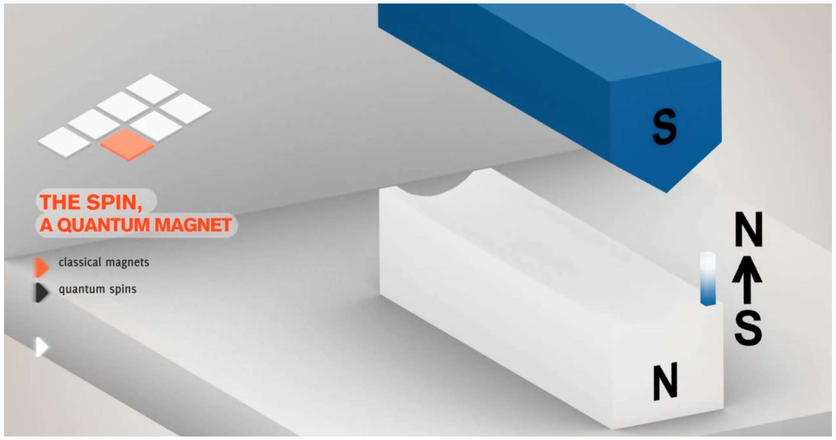
Third axis of polarization.

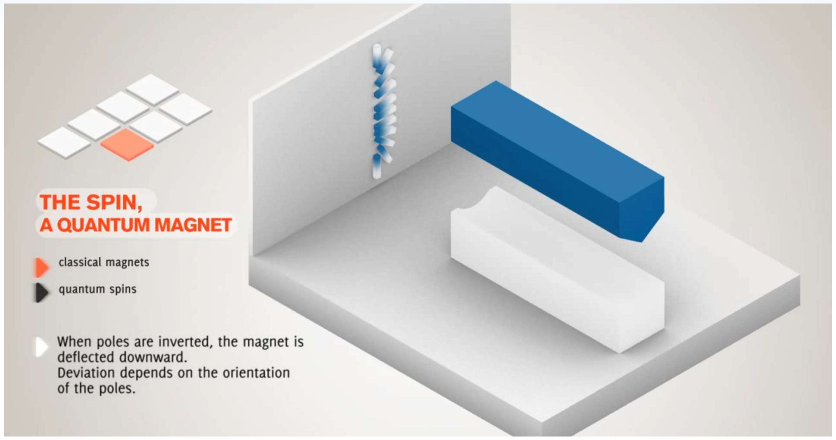
Interaction with linear polarizers: always 50% probability to go through, no matter the orientation.

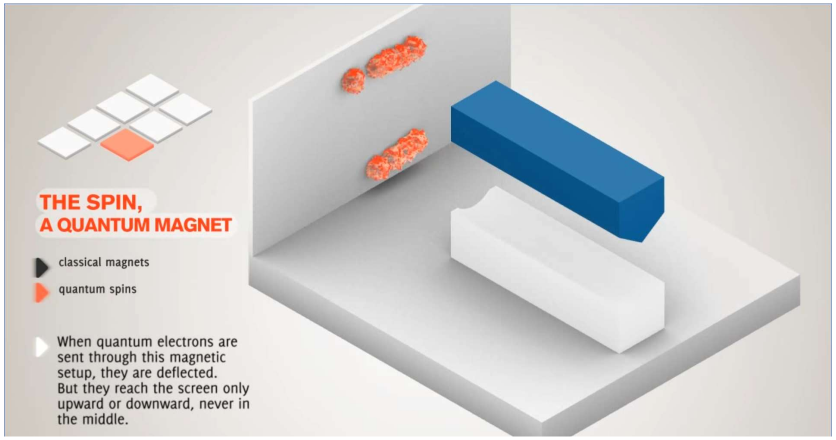
With circular polarizers: 1 for the same handedness, 0 otherwise.

Part II: Spin

Stern Gerlach Experiment: Explaining Spin

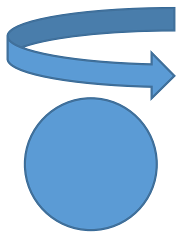




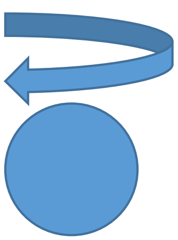


Electronic spin: two options

This is the origin of the two electron/state rule! Each state can get a spin up and spin down



Spin up



Spin down

Angular momentum of an electron is:

$$\pm \frac{1}{2}\hbar$$

No matter which axis we choose, we will never measure it with zero angular momentum…

Magnetic moment is:

$$\mu = \pm \frac{1}{2} g \frac{e\hbar}{2m_e}$$

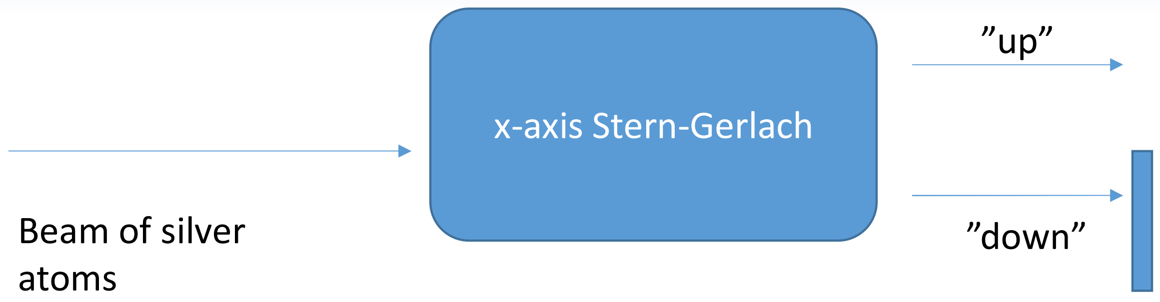
$$g = 2.002319304361(2)$$

Measuring spin: Stern Gerlach filter

Spin direction	State
$\hat{z}$	$\uparrow$
$-\hat{z}$	$\downarrow$
$\hat{x}$	$\frac{1}{\sqrt{2}}(\uparrow + \downarrow)$
$-\hat{x}$	$\frac{1}{\sqrt{2}}(\uparrow - \downarrow)$
$\hat{y}$	$\frac{1}{\sqrt{2}}(\uparrow + i \downarrow)$
$-\hat{y}$	$\frac{1}{\sqrt{2}}(\uparrow - i \downarrow)$

What is the state of the silver atoms that pass?

a) Random
b) $\uparrow$
c) $\downarrow$



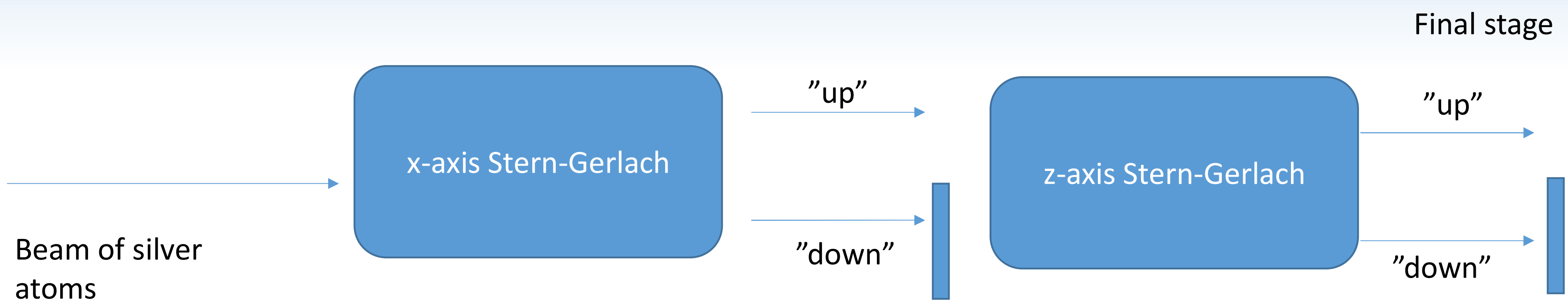
Spin direction	State
$\hat{z}$	$\uparrow$
$-\hat{z}$	$\downarrow$
$\hat{x}$	$\frac{1}{\sqrt{2}}(\uparrow + \downarrow)$
$-\hat{x}$	$\frac{1}{\sqrt{2}}(\uparrow - \downarrow)$
$\hat{y}$	$\frac{1}{\sqrt{2}}(\uparrow + i \downarrow)$
$-\hat{y}$	$\frac{1}{\sqrt{2}}(\uparrow - i \downarrow)$

What is the state of the silver atoms that pass?

a) Random

b) $\frac{1}{\sqrt{2}}(\uparrow + \downarrow)$

c) $\frac{1}{\sqrt{2}}(\uparrow - \downarrow)$



What is the state of the silver atoms in the final stage?

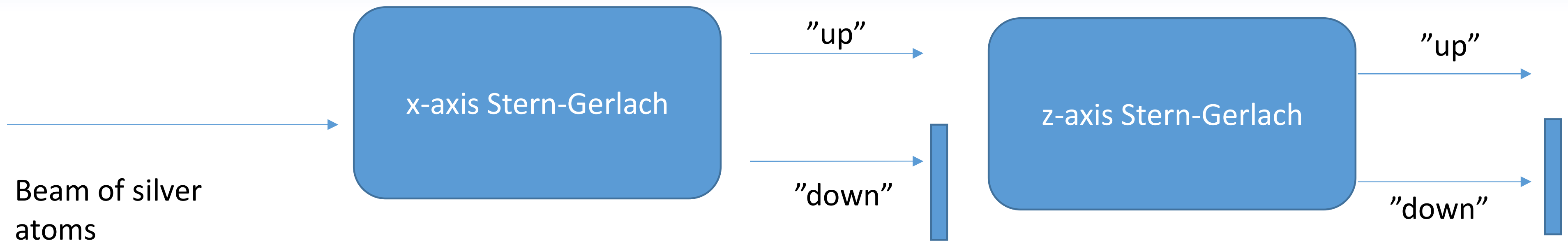
$a)\frac{1}{\sqrt{2}}(\uparrow+\downarrow)$

$b)\frac{1}{\sqrt{2}}(\uparrow-\downarrow)$

$c)\uparrow$

$d)\downarrow$

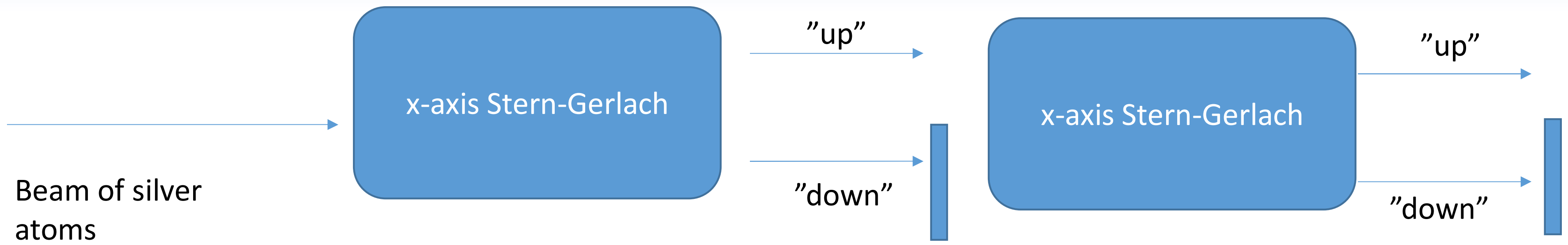
Spin direction	State
$\hat{z}$	$\uparrow$
$-\hat{z}$	$\downarrow$
$\hat{x}$	$\frac{1}{\sqrt{2}}(\uparrow + \downarrow)$
$-\hat{x}$	$\frac{1}{\sqrt{2}}(\uparrow - \downarrow)$
$\hat{y}$	$\frac{1}{\sqrt{2}}(\uparrow + i \downarrow)$
$-\hat{y}$	$\frac{1}{\sqrt{2}}(\uparrow - i \downarrow)$



What is the probability that an electron passes through the z-axis filter after it has passed through the x-axis filter?

a) 0.25
b) 0.5
c) 0.75
d) 1.0

Spin direction	State
$\hat{z}$	$\uparrow$
$-\hat{z}$	$\downarrow$
$\hat{x}$	$\frac{1}{\sqrt{2}}(\uparrow + \downarrow)$
$-\hat{x}$	$\frac{1}{\sqrt{2}}(\uparrow - \downarrow)$
$\hat{y}$	$\frac{1}{\sqrt{2}}(\uparrow + i \downarrow)$
$-\hat{y}$	$\frac{1}{\sqrt{2}}(\uparrow - i \downarrow)$



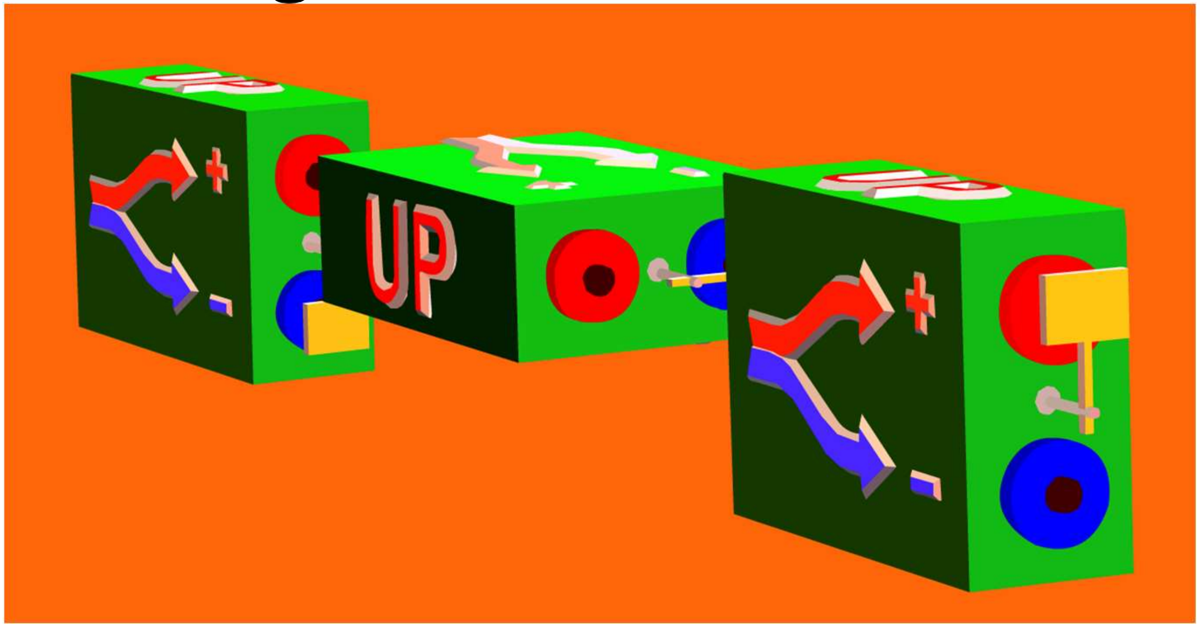
What is the probability that an electron passes through the x-axis filter after it has passed through the x-axis filter?

a) 0.25
b) 0.5
c) 0.75
d) 1.0

Spin direction	State
$\hat{z}$	$\uparrow$
$-\hat{z}$	$\downarrow$
$\hat{x}$	$\frac{1}{\sqrt{2}}(\uparrow + \downarrow)$
$-\hat{x}$	$\frac{1}{\sqrt{2}}(\uparrow - \downarrow)$
$\hat{y}$	$\frac{1}{\sqrt{2}}(\uparrow + i \downarrow)$
$-\hat{y}$	$\frac{1}{\sqrt{2}}(\uparrow - i \downarrow)$

What percentage of atoms will make it through?

Spin direction	State
$\hat{z}$	$\uparrow$
$-\hat{z}$	$\downarrow$
$\hat{x}$	$\frac{1}{\sqrt{2}}(\uparrow + \downarrow)$
$-\hat{x}$	$\frac{1}{\sqrt{2}}(\uparrow - \downarrow)$
$\hat{y}$	$\frac{1}{\sqrt{2}}(\uparrow + i \downarrow)$
$-\hat{y}$	$\frac{1}{\sqrt{2}}(\uparrow - i \downarrow)$



a) 100%
b) 50%
c) 25%
d) 12.5%
e) 0%

Spin

$$\psi_\theta = \cos\theta\,\psi_\uparrow + \sin\theta\,\psi_\downarrow$$

What's the probability that we measure the electron to have spin ↑?

a) $\cos^2\theta$
b) $\sin^2\theta$
c) 0
d) 1
e) ½