

USEFUL EQUATIONS

Physical constants

speed of light	c	2.998×10^8 m/s
Planck constant	h	6.626×10^{-34} J s
		4.135×10^{-15} eV s
	\hbar	1.054×10^{-34} J s
		0.658×10^{-15} eV s
electron volt	eV	1.602×10^{-19} J
electron charge	e	1.602×10^{-19} C
Bohr radius	a_0	0.05292 nm
electron mass	m_e	9.109×10^{-31} kg
		0.511 MeV/ c^2
proton mass	m_p	1.673×10^{-27} kg
		938.3 MeV/ c^2
neutron mass	m_n	1.675×10^{-27} kg
		939.6 MeV/ c^2
hydrogen mass	m_H	1.674×10^{-27} kg

Waves

Symbol	Name	SI units
k	Wave number	m^{-1}
λ	Wavelength	m
ω	Angular frequency	rad/s
ϕ	Phase	radians
T	Period	s
f	Frequency	s^{-1}
I	Intensity	W/m^2
A	Amplitude	$\sqrt{\text{W}/\text{m}^2}$

$$f = \omega/2\pi = 1/T$$

$$k = 2\pi/\lambda$$

$$I_{avg} = \frac{|A|^2}{2}$$

Trigonometric identities

$$\cos \alpha + \cos \beta = 2 \cos \left(\frac{\alpha - \beta}{2} \right) \cos \left(\frac{\alpha + \beta}{2} \right)$$

$$\cos^2 \alpha + \sin^2 \alpha = 1$$

$$A^2 + B^2 + 2AB \cos \phi = C^2$$

Intensity of superposition of two waves of equal magnitude

$$I_{total} = 2A^2 \cos^2 \left(\frac{kr_1 + \phi_1 - kr_2 - \phi_2}{2} \right)$$

Diffraction

$$a \sin \theta_0 = \lambda$$

$$D \sin \theta_0 = 1.22\lambda$$

Complex numbers

$$i = \sqrt{-1}$$

$$e^{i\theta} = \cos(\theta) + i \sin(\theta)$$

$$z = x + iy$$

$$z^* = x - iy$$

$$|z|^2 = zz^* = x^2 + y^2$$

Photons

$$p = \hbar k = h/\lambda$$

$$E = hf = \hbar\omega$$

$$\sin a \sin b = \frac{1}{2} [\cos(a-b) - \cos(a+b)]$$

$$\cos a \cos b = \frac{1}{2} [\cos(a+b) + \cos(a-b)]$$

$$\sin a \cos b = \frac{1}{2} [\sin(a+b) + \sin(a-b)]$$

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\cos \left(\frac{\theta}{2} \right) = \sqrt{\frac{1 + \cos \theta}{2}} \quad \sin \left(\frac{\theta}{2} \right) = \sqrt{\frac{1 - \cos \theta}{2}}$$

Wave functions

$$\rho(x, t) = \Psi(x, t)\Psi^*(x, t)$$

$$P(a < x < b) = \int_a^b \rho(x, t) dx$$

$$\int_{-\infty}^{\infty} \rho(x, t) dx = 1$$

Measurement rule

$$\Psi = a\Psi_1 + b\Psi_2$$

$$P(1) = \frac{|a|^2}{|a|^2 + |b|^2}$$

Quantum matter

Wave function of momentum p

$$\Psi(x) = Ae^{ikx}$$

$$p = \hbar k$$

Heisenberg Uncertainty Principle

$$\Delta x \Delta p \geq \hbar/2$$

Schrödinger equation

$$-\frac{\hbar^2}{2m} \frac{d^2\Psi(x)}{dx^2} + U(x)\Psi(x) = E\Psi(x)$$

Infinite square well eigenstates

$$\Psi_n(x) = \begin{cases} \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) & \text{if } 0 < x < L \\ 0 & \text{otherwise,} \end{cases}$$

$$E_n = \frac{\hbar^2 n^2 \pi^2}{2mL^2}$$

Harmonic oscillator

Ground state

$$\Psi_0(x) = Ae^{-\alpha x^2}$$

$$\alpha = \frac{1}{2\hbar} \sqrt{mk}$$

$$E = \frac{\hbar^2 \alpha}{m} = \frac{\hbar}{2m} \sqrt{mk}$$

Spectrum

$$E_n = \left(n + \frac{1}{2}\right) \hbar \omega, n = 0, 1, 2, \dots$$

Time dependent Schrödinger equation

$$i\hbar \frac{\partial \Psi(x, t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi(x, t)}{\partial x^2} + U(x)\Psi(x, t)$$

Time dependence of energy eigenstates

$$\Psi(x, t) = e^{-i\omega_n t} \Psi_n(x)$$

$$E_n = \hbar \omega_n$$

Double wells

$$\Delta E = Ce^{-\kappa \sqrt{mV}L},$$