

USEFUL EQUATIONS

Physical constants

speed of light	c	2.998×10^8 m/s
Planck constant	h	6.626×10^{-34} J s
		4.135×10^{-15} eV s
	$\hbar$	1.054×10^{-34} J s
		0.658×10^{-15} eV s
electron volt	eV	1.602×10^{-19} J
electron charge	e	1.602×10^{-19} C
Bohr radius	a_0	0.05292 nm
electron mass	m_e	9.109×10^{-31} kg
		0.511 MeV/ c^2
proton mass	m_p	1.673×10^{-27} kg
		938.3 MeV/ c^2
neutron mass	m_n	1.675×10^{-27} kg
		939.6 MeV/ c^2
hydrogen mass	m_H	1.674×10^{-27} kg

Trigonometric identities

$$\cos \alpha + \cos \beta = 2 \cos \left(\frac{\alpha - \beta}{2} \right) \cos \left(\frac{\alpha + \beta}{2} \right)$$

$$\cos^2 \alpha + \sin^2 \alpha = 1$$

$$A^2 + B^2 + 2AB \cos \phi = C^2$$

$$\sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha + \beta) + \cos(\alpha - \beta)]$$

$$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\cos \frac{\theta}{2} = \sqrt{\frac{1 + \cos \theta}{2}}, \sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}}$$

Complex numbers

$$i = \sqrt{-1}$$

$$e^{i\theta} = \cos(\theta) + i \sin(\theta)$$

$$z = x + iy$$

$$z^* = x - iy$$

$$|z|^2 = zz^* = x^2 + y^2$$

Waves

Symbol	Name	SI units
k	Wave number	m^{-1}
λ	Wavelength	m
ω	Angular frequency	rad/s
ϕ	Phase	radians
T	Period	s
f	Frequency	s^{-1}
I	Intensity	W/m^2
A	Amplitude	$\sqrt{\text{W}/\text{m}^2}$

$$f = \omega/2\pi = 1/T$$

$$k = 2\pi/\lambda$$

$$I_{avg} = \frac{|A|^2}{2}$$

$$f\lambda = v$$

Intensity of superposition of two waves of equal magnitude

$$I_{total} = 2A^2 \cos^2 \left(\frac{kr_1 + \phi_1 - kr_2 - \phi_2}{2} \right)$$

Path length difference for two slits

$$\delta r = d \sin \theta$$

Path length difference for interferometer

$$\delta r = 2(L_1 - L_2)$$

Diffraction

$$a \sin \theta_0 = \lambda$$

$$D \sin \theta_0 = 1.22\lambda$$

Photons

$$p = \hbar k = h/\lambda$$
$$E = hf = \hbar\omega = \frac{1240 \text{ eV nm}}{\lambda}$$

Force: $F = \frac{dp}{dt}$

Energy/momentum relationship: $E = pc$

Photoelectric effect: $eV_{stop} = KE = hf - \Phi$

Wave functions

$$\rho(x, t) = \Psi(x, t)\Psi^*(x, t)$$
$$P(a < x < b) = \int_a^b \rho(x, t) dx$$
$$\int_{-\infty}^{\infty} \rho(x, t) dx = 1$$

Measurement rule

$$\Psi = a\Psi_1 + b\Psi_2 + \dots$$
$$P(1) = \frac{|a|^2}{|a|^2 + |b|^2 + \dots}$$

Quantum matter

Wave function of momentum p

$$\Psi(x) = Ae^{ikx}$$
$$p = \hbar k$$

Energy/momentum relationship for
slow, massive particles: $E = \frac{p^2}{2m}$
Heisenberg Uncertainty Principle

$$\Delta x \Delta p \geq \hbar/2$$

Schrödinger equation

$$-\frac{\hbar^2}{2m} \frac{d^2\Psi(x)}{dx^2} + U(x)\Psi(x) = E\Psi(x)$$

Infinite square well eigenstates

$$\Psi_n(x) = \begin{cases} \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) & \text{if } 0 < x < L \\ 0 & \text{otherwise,} \end{cases}$$
$$E_n = \frac{\hbar^2 n^2 \pi^2}{2mL^2}$$

Harmonic oscillator eigenstates

Ground state

$$\Psi_0(x) = Ae^{-\alpha x^2}$$
$$\alpha = \frac{1}{2\hbar} \sqrt{mk}$$
$$E = \frac{\hbar^2 \alpha}{m} = \frac{\hbar}{2m} \sqrt{mk}$$

Spectrum

$$E_n = \left(n + \frac{1}{2}\right) \hbar\omega, n = 0, 1, 2, \dots$$

Multiple non-interacting electrons

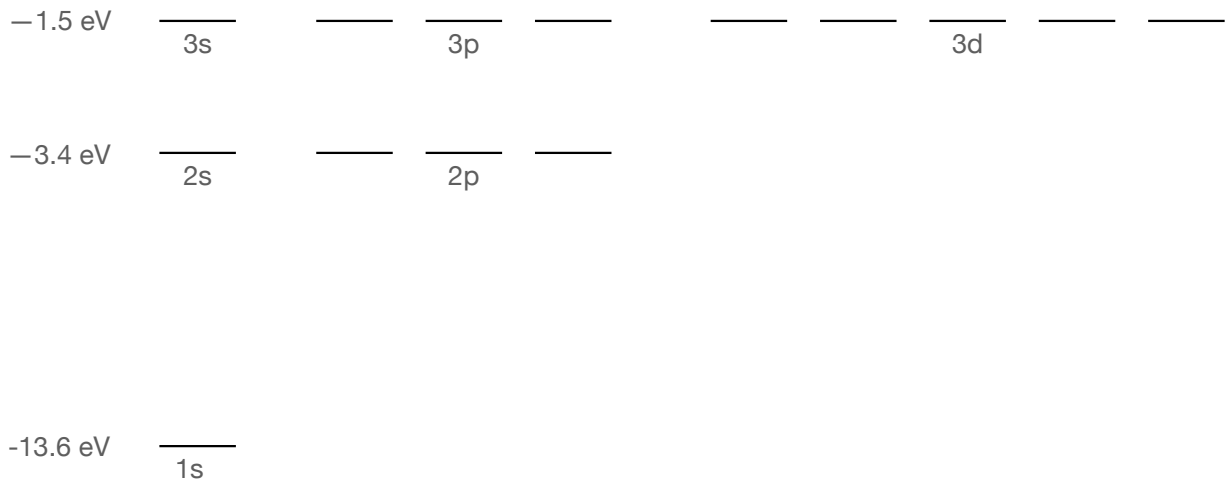
- Ground state: Two electrons per energy level, starting from the bottom.
- Excited states: Other fillings, keeping to the 2 electrons/level rule.
- Energy is the sum of the occupied levels.

Band structure

- Metal: no gap
- Insulator: big gap
- Semiconductor: small gap

Hydrogen energy levels

$$E_n = \frac{-13.6Z^2}{n^2} \text{ eV}$$



Two-state wave functions

Polarization direction	State
Vertical	Ψ_v
Horizontal	Ψ_h
Diagonal (45 degrees)	$\frac{1}{\sqrt{2}}(\Psi_h + \Psi_v)$
Diagonal (-45 degrees)	$\frac{1}{\sqrt{2}}(\Psi_h - \Psi_v)$
Circular (right-handed)	$\frac{1}{\sqrt{2}}(\Psi_h + i\Psi_v)$
Circular (left-handed)	$\frac{1}{\sqrt{2}}(\Psi_h - i\Psi_v)$
Spin direction	State
$\hat{z}$	$\uparrow$
$-\hat{z}$	$\downarrow$
$\hat{x}$	$\frac{1}{\sqrt{2}}(\uparrow + \downarrow)$
$-\hat{x}$	$\frac{1}{\sqrt{2}}(\uparrow - \downarrow)$
$\hat{y}$	$\frac{1}{\sqrt{2}}(\uparrow + i\downarrow)$
$-\hat{y}$	$\frac{1}{\sqrt{2}}(\uparrow - i\downarrow)$

Measurement rule for a particle with wave function Ψ and a state with definite direction S : $P(S) = |S^* \cdot \Psi|^2$

Nuclear particles

Name	Charge	Lepton number	Spin	Description
p^+	+1	0	$\frac{1}{2}$	A proton. Sometimes just called p .
n^0	0	0	$\frac{1}{2}$	A neutron. Sometimes just called n .
α^{2+}	+2	0	0	A ${}^4_2\text{He}$ nucleus.
β^-	-1	+1	$\frac{1}{2}$	An electron. Also called e^-
β^+	+1	-1	$\frac{1}{2}$	A positron. Also called e^+
ν_e	0	1	$\frac{1}{2}$	electron neutrino
$\bar{\nu}_e$	0	-1	$\frac{1}{2}$	electron antineutrino
γ	0	0	1	A photon with wavelength of order 10^{-12} m