7 Thermodynamic cycles: Engines

- You should be able to compute net work done over a thermodynamic cycle
- Understand the concept of a heat bath and an abstract heat engine
- Carnot efficiency, and how it relates to an engine that produces work from heat flow and also how it affects the creation of temperature differences using work.
- Understand the relationship between entropy and efficiency

7.1 Constructing a simple engine out of thermodynamic cycles

The basic idea of an engine is to construct a sequence of thermodynamic process such that net work is done. For concreteness, we will consider an ideal gas that is inside a cylinder. Imagine that we can heat and cool the gas by putting the cylinder in either a hot environment (could be as simple as boiling water) or a cold environment (could be ice water for example). We idealize both environments as being so large compared to the piston that their temperature doesn't change when we heat or cool the cylinder. Such systems are typically called **reservoirs or baths**. Imagine that we can also either lock the piston in place so that the volume cannot change (isochoric process) or we can allow the piston to move so that the volume can change but at a fixed pressure (isobaric process).

For a very simple example of this, consider the following set of processes, done :

1. Starting at p_1, V_1, allow the piston to move, and heat the gas. The volume will increase, and we can use the force from the piston to push something and do some work.[1]
2. Starting at $p_2 = p_1, V_2 > V_1$, fix the piston's position and cool the gas down by putting it in the ice bath. Now the pressure will decrease but no work is done since the volume doesn't change. Heat flows from the piston to the ice water here.
3. Starting at $p_3 \neq p_2, V_3 = V_2$, keeping it in the ice bath, apply a force to the piston until its volume is the original one V_1. This requires us to do work on the gas.
4. Starting at $p_4 = p_3, V_4 = V_1$, fix the piston in place and put the piston back in the boiling water. Since the volume is fixed, the pressure will increase back to p_1 and the gas has ended up where we started. No work is done because the volume doesn't change. Heat flows from the boiling water to the piston here.

We can count up how much work was done by the gas during this whole cycle. In step 1, the gas expanded at a constant pressure p_2, so the work done was $p_2(V_2 - V_1)$. In Step 3, we compressed the gas back to its original volume at a lower temperature, so the work **on** the gas was $p_3(V_2 - V_1)$. Since $p_2 > p_3$, the total work done **by** the gas was

$$W_{by} = p_2(V_2 - V_1) - p_3(V_2 - V_1) = (p_2 - p_3)(V_2 - V_1). \tag{1}$$

The energy for the work was gotten by transferring heat from the hot reservoir (the boiling water) to the cold reservoir (the ice water). Note that if the temperature of the reservoirs were the same, then it would not be possible to get work out of the setup. It's very important that heat spontaneously would move from one reservoir to another. We are extracting some of that heat to repurpose into work. One might consider the **efficiency** of the engine as follows:

$$\epsilon = \frac{W_{by}}{Q_H}, \tag{2}$$

where W_{by} is the work done in the cycle and Q is the heat transferred from the hot reservoir.

[1] In a car for example it might turn a crank that turns the wheels.

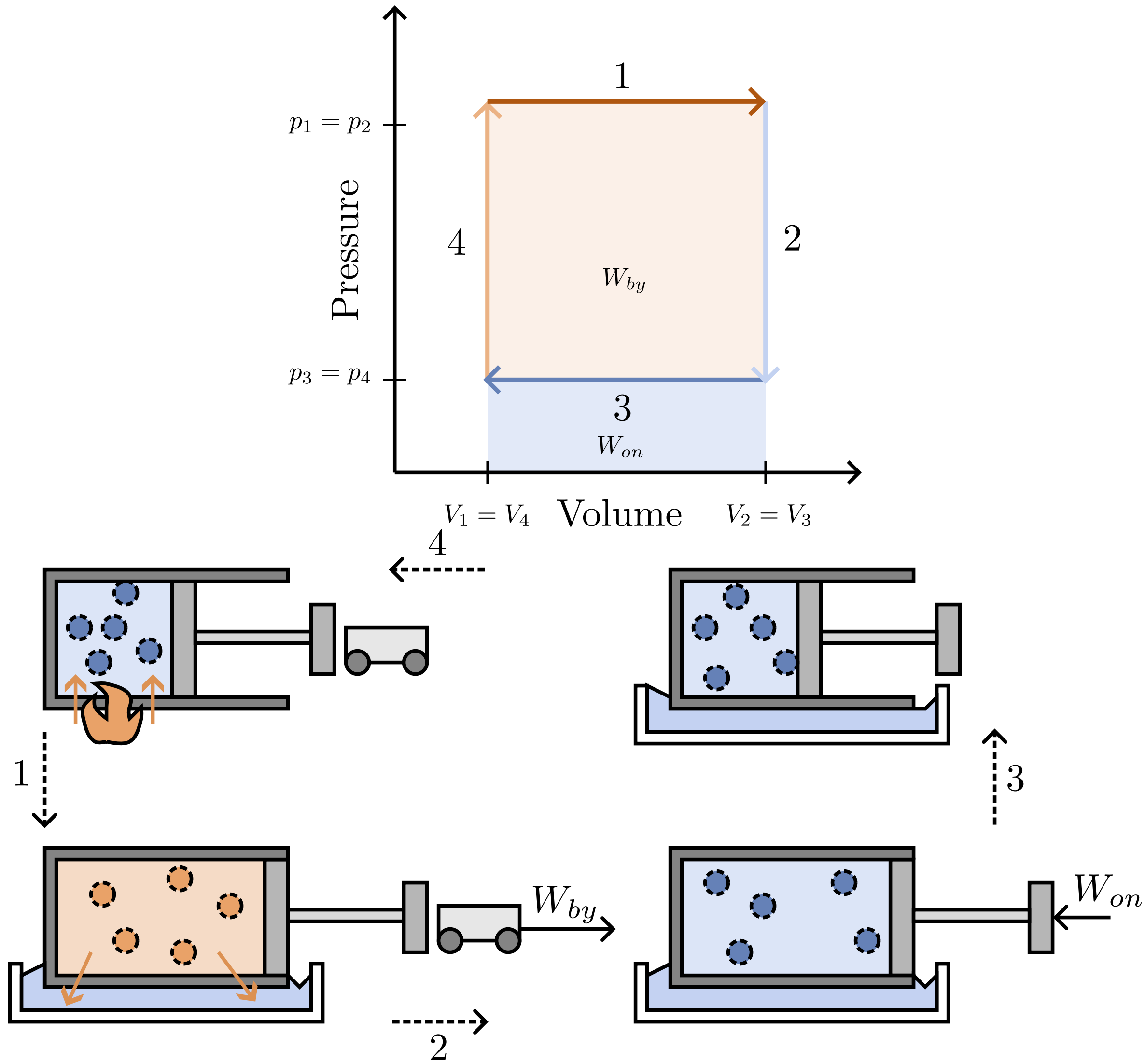

Figure 1: (above) $p--V$ diagram of the thermodynamic cycle in the text 7.1. (below) A diagram of how each process in the cycle might be impelented.

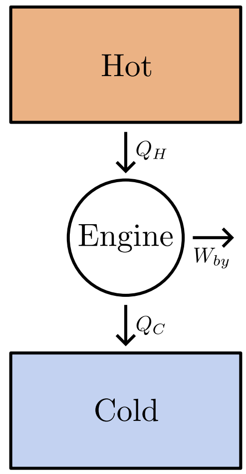

Figure 2: An abstract diagram of a heat engine. The engine uses the energy from heat flow from the hot reservoir to create net work. Waste heat energy is dumped into the cold reservoir.

The idealized engine presented here is not very practical; in reality the gas is typically transported in pipes, rather than moving a piston back and forth. As we will discover, it is also not that efficient. In the next section, we will compute how efficient the engine **could** be while still not violating the second law of thermodynamics.

7.2 Carnot efficiency

The Carnot efficiency is an upper limit on the efficiency attainable. Consider the abstract picture of an engine in Fig **??**. Over the engine cycle, some heat flows from the hot reservoir into the engine, some of that heat energy is turned into work, and finally some heat flows into the cold reservoir.

There are two main equations here. Energy conservation:

$$Q_H = W_{by} + Q_C, \tag{3}$$

and entropy change:

$$\Delta S_{total} = -\frac{Q_H}{T_H} + \frac{Q_C}{T_C} \geq 0 \tag{4}$$

The entropy of the hot reservoir goes down, but the entropy of the cold reservoir must go up at least as much, so

$$\frac{Q_C}{T_C} \geq \frac{Q_H}{T_H} \tag{5}$$

Substituting Eqn 3 into Eqn 5 and rearranging some terms, you can get

$$\epsilon = \frac{W_{by}}{Q_H} \leq 1 - \frac{T_C}{T_H}, \tag{6}$$

which is called the Carnot efficiency.

There are a few assumptions in the Carnot efficiency calculation: the engine itself is perfect, so it does not increase entropy at all. This must mean it is made up only of reversible processes. The engine we designed in the previous section is not perfect! Secondly, the hot and cold reservoirs are large enough that their temperatures do not change; then $\Delta S = \int \frac{dQ}{T} = \frac{Q}{T}$. Relaxing these assumptions generally makes the efficiency go down, so the Carnot efficiency remains an upper theoretical bound to any device that tries to use heat energy flowing between a hot and cold object.

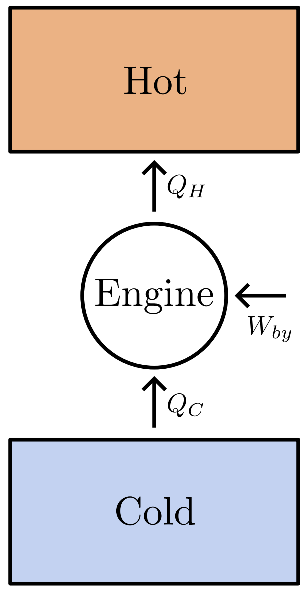

Figure 3: An abstract diagram of a heat pump. Here, work is input to draw heat from the cold reservoir and put it into the hot reservoir. It is exactly the reverse of the heat engine with a different figure of merit.

Relationships like Eqn 6 are very important because they tell us what is possible according to the laws of physics. They also tell us how to get close to a Carnot efficiency; it's very important that the engine itself creates as little entropy as possible. This means that the engine should use reversible or near-reversible processes. The Carnot analysis works not just for engines such as you'd find in a car, but also for power plants based on nuclear heat sources and even solar cells, which use the sun as a hot reservoir and the Earth as a cold reservoir.

7.3 Heat pumps and other applications of engine theory

While an engine is a device that turns heat motion into work, you can turn the system around and turn work into heat motion. This is a heat pump. In the winter, you can use a heat pump to move heat from the outside to the inside. Even though it's cold outside, there is still a lot of internal energy available; usually on Earth it does not get anywhere close to absolute zero. On the other hand, in the summer, you can use a heat pump to move heat from the house to outside. In each case, you are interested in how much heat you added or removed from the house compared to how much work you put into it. Practically the work is usually in the form of electricity running a motor, so the work is essentially the cost of the heating/cooling. The efficiency is typically called the coefficient of performance. For a heater, the coefficient is

$$COP = \frac{Q_H}{W_{on}} \tag{7}$$

while for a refrigerator it is

$$COP = \frac{Q_C}{W_{on}}. \tag{8}$$

All of these efficiencies are basically a ratio between what you get and what you pay.

We can use the exact same analysis as we did for the Carnot efficiency, except some of the signs are reversed (note that $W_{by} = -W_{on}$). Energy conservation is

$$Q_H = W_{on} + Q_C. \tag{9}$$

Then the second law reads

$$-\frac{Q_C}{T_C} + \frac{Q_H}{T_H} \geq 0 \tag{10}$$

or

$$\frac{T_H}{T_C} \geq \frac{Q_H}{Q_C}. \tag{11}$$

Taking the reciprocal, you get

$$\frac{T_C}{T_H} \leq \frac{Q_C}{Q_H}. \tag{12}$$

Let's do the analysis for a heat pump used to heat a home. The coefficient of performance is

$$\frac{Q_H}{W_{on}} = \frac{Q_H}{Q_H - Q_C} = \frac{1}{1 - \frac{Q_C}{Q_H}}. \tag{13}$$

Since $\frac{Q_C}{Q_H} \leq \frac{T_C}{T_H}$, $1 - \frac{Q_C}{Q_H} \geq 1 - \frac{T_C}{T_H}$, and finally

$$\frac{Q_H}{W_{on}} \leq \frac{1}{1 - \frac{T_C}{T_H}}. \tag{14}$$

Another way to do this is just to focus on the maximum efficiency possible and not worry about the inequalities. Note also that this is just the reciprocal of the Carnot efficiency, which makes sense!

Consider the use of electricity for heating, such as a space heater you might buy. One Joule of electrical energy gets turned into one Joule of heat, so the coefficient of performance is exactly 1. Now consider using a heat pump. Let's suppose that the outside temperature T_C is near freezing, 273 K, and inside it is 298 K. Then the maximum coefficient of performance is near 12, which means you get 12 J of heat for every one of electrical energy. In realistic implementations, the coefficient of performance is closer to 4-5 for a high quality device in the 2020's. The fact that no one has made a heat pump with such a large coefficient of performance (12) is further evidence that thermodynamics works.