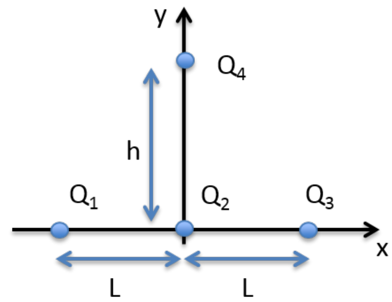
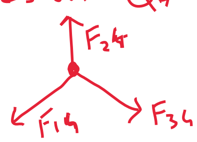
The next two questions pertain to the situation described below.

Three point charges Q_1=7 μC, Q_2=-10.5 μC and Q_3=7 μC are placed a distance L=1.3 meter apart on the x-axis at points (-L, 0), (0,0), and (L, 0) as shown in the figure. A fourth charge Q_4=-10.5 μC is placed at a position (0, h) where h = 2.6 m.



1) What is x-component of the force on Q_4 due to the charges Q_1, Q_2, and Q_3?

Forces on Q_4

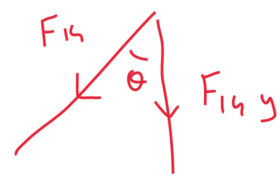


Note that $Q_1 = Q_3$

So $F_{14} = F_{34}$ → the x-components cancel out

a. F_{Q4x} = 0.147 N
b. F_{Q4x} = -0.07 N
c. F_{Q4x} = -0.217 N
d. F_{Q4x} = 0.077 N
e. F_{Q4x} = Zero

2) What is y-component of the force on Q_4 due to the charges Q_1, Q_2, and Q_3?



a. F_{Q4y} = Zero
b. F_{Q4y} = 0.00674 N
c. F_{Q4y} = -0.00979 N
d. F_{Q4y} = 0.147 N
e. F_{Q4y} = -0.287 N

$$F_{Q_4y} = F_{24} - F_{14y} - F_{34y}$$

$$= F_{24} - 2F_{14}\cos\theta \quad , \quad \cos\theta = \frac{h}{\sqrt{h^2+L^2}}$$

$$= \frac{|Q_2 Q_4|}{4\pi\varepsilon_0 h^2} - \frac{2|Q_1 Q_4|}{4\pi\varepsilon_0 (h^2+L^2)} \frac{h}{(h^2+L^2)^{1/2}}$$

$$F_{Q_4y} = \frac{1}{4\pi\varepsilon_0}\left\{ \frac{(10.5\times10^{-6})^2}{2.6^2} - \frac{2(7)(10.5)(10^{-6})^2\, 2.6}{(2.6^2+1.3^2)^{3/2}} \right\}$$

$$= \frac{(10^{-6})^2}{4\pi\varepsilon_0}\left[\frac{10.5^2}{2.6^2} - \frac{2\times7\times10.5\times2.6}{(2.6^2+1.3^2)^{3/2}} \right]$$

$$F_{Q_4y} = 0.00674\,N$$

The next two questions pertain to the situation described below.

Two conducting spheres of radii $r_1 = 20$ mm and $r_2 = 5$ mm are charged with $q_1 = 0.4$ μC and $q_2 = 0.12$ μC respectively. The spheres are separated by a large distance.

3) What is the potential difference between the surfaces of the two spheres?

$$\Delta V = \frac{q_2}{4\pi\varepsilon_0 r_2} - \frac{q_1}{4\pi\varepsilon_0 r_1} = \frac{1}{4\pi\varepsilon_0}\left(\frac{0.12\times10^{-6}}{5\times10^{-3}} - \frac{0.4\times10^{-6}}{20\times10^{-3}}\right)$$

$$\Delta V = 3.6\times10^{4}$$

a. 3.6×10^4 Volts
b. 1.8×10^5 Volts
c. 2.16×10^5 Volts

4) If the spheres are connected by a thin conducting wire, in which direction (if any) would positive charge flow?

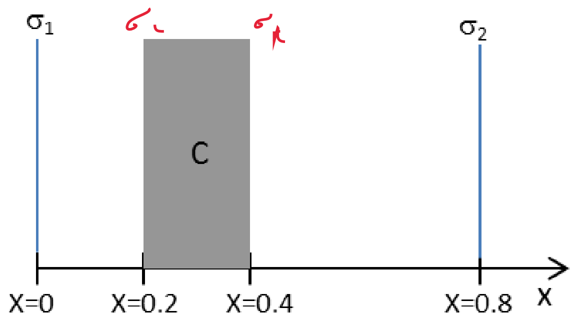
a. no net charge is transferred between the two spheres
b. from sphere 1 to sphere 2
c. from sphere 2 to sphere 1

Note that sphere 2 has a greater potential than sphere 1

So charge flows from sphere 2 to sphere 1

The next two questions pertain to the situation described below.

Two infinite nonconducting sheets of charge and one infinite conducting slab are placed perpendicular to the x direction as shown in the following figure. The conducting slab is electrically neutral and labeled C. The charge densities on the two sheets of charge are σ_1 = +5 μC/m^2 and σ_2 = -9.5 μC/m^2.



for an infinite slab, $E = \dfrac{\sigma}{2\varepsilon_0}$

so at $x = 0.9$, $E = \dfrac{\sigma_2}{2\varepsilon_0} + \dfrac{\sigma_1}{2\varepsilon_0}$

$E = \dfrac{1}{2\varepsilon_0}(-9.5 + 5)\times 10^{-6}$

$E = -0.254 \times 10^6$ V/m

5) The x-component of the electric field at x = 0.9 is:

a. E_x =-0.254 × 10^6 V/m

b. E_x =0.536 × 10^6 V/m

c. E_x = -0.536 × 10^6 V/m

6) The induced charge density on the left side of the conductor (i.e. at x=0.2) is

In the middle of the conductor, $E = 0$

i.e. $\dfrac{\sigma_1}{2\varepsilon_0} + \dfrac{\sigma_L}{2\varepsilon_0} - \dfrac{\sigma_R}{2\varepsilon_0} - \dfrac{\sigma_2}{2\varepsilon_0} = 0$ — (1)

Since the conductor is neutral

$\sigma_L + \sigma_R = 0$ — (2)

So $\sigma_1 + \sigma_L + \sigma_L - \sigma_2 = 0$

$\sigma_L = \dfrac{\sigma_2 - \sigma_1}{2}$

a. σ_L = -7.25 μC/m^2

b. σ_L = -2.25 μC/m^2

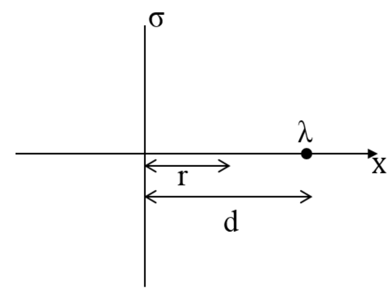
c. σ_L = -5 μC/m^2

d. σ_L = -2.5 μC/m^2

e. σ_L = -14.5 μC/m^2

The next three questions pertain to the situation described below.

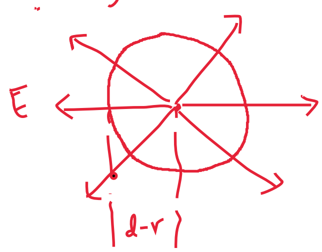
An infinite sheet with charge density per unit area σ is placed along the y axis at x=0. An infinite line of charge with charge density per unit length λ is located at x=d and y=0 and oriented in the z direction (out of page) as shown in the figure.



7) What is the x component of the electric field **due ONLY to the infinite line of charge** at the point on the x axis a distance r to the right of the plane, as shown in the figure?

a. $E_x = \frac{\lambda}{2\pi\epsilon_0 r}$
b. $E_x = \frac{-\lambda}{2\pi\epsilon_0 (d-r)}$
c. $E_x = \frac{-\lambda}{2\pi\epsilon_0 r}$
d. $E_x = \frac{\lambda}{4\pi\epsilon_0 r^2}$
e. $E_x = \frac{\lambda}{2\pi\epsilon_0 (d-r)}$

By Gauss' law → $\oint \vec{E}\cdot d\vec{A} = Q_{enc}/\varepsilon_0$



$$\vec{E} = \frac{-\lambda \cancel{L}}{2\pi(d-r)\varepsilon_0 \cancel{L}}\,\hat{x}$$

$$E_x = \frac{-\lambda}{2\pi(d-r)\varepsilon_0}$$

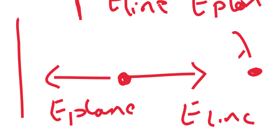
8) You are told that there is a point on the x axis between the charged plane and the line of charge ($0 < r < d$) where the electric field is zero. What can you conclude about the signs of λ and σ?

a. They have the same sign.
b. They are both negative.
c. Nothing.
d. They have the opposite sign.
e. The are both positive.

for $0 < r < d$ if charged plane and line of charge have same sign ⇒

both +ve

both -ve



for these two fields to cancel, λ and σ have the same sign

9) Which expression gives the position along the x axis between the line of charge and the charged plane at which the electric field is zero?

a. $r = \frac{\lambda}{\pi\sigma}$
b. $r = d - \frac{\lambda}{\pi\sigma}$
c. $r = d + \frac{\lambda}{\pi\sigma}$

for E-field to be zero along the x-axis

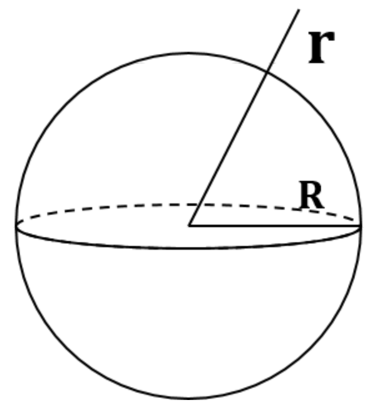
$$|E_{plane}| = |E_{line}|$$

$$\frac{\sigma}{\cancel{2}\cancel{\varepsilon_0}} = \frac{\lambda}{\cancel{2}\pi(d-r)\cancel{\varepsilon_0}} \rightarrow d-r = \frac{\lambda}{\pi\sigma}$$

$$r = d - \frac{\lambda}{\pi\sigma}$$

The next three questions pertain to the situation described below.

An insulating sphere of radius R carries a charge density per unit volume ρ as shown in the figure.



10) What is the magnitude of the electric field at a distance r > R from the center of the sphere?

a. $|E| = \frac{1}{4\pi\epsilon_0}\frac{\rho}{r^2}$

b. $|E| = \frac{1}{3\epsilon_0}\frac{\rho R^2}{r}$

c. $|E| = \frac{1}{4\pi\epsilon_0}\frac{\rho R^3}{r^2}$

d. $|E| = \frac{1}{3\rho\epsilon_0}\frac{R^3}{r^2}$

e. $|E| = \frac{1}{3\epsilon_0}\frac{\rho R^3}{r^2}$

as $\int \vec{E}\cdot d\vec{A} = Q_{enc}/\varepsilon_0$

for $r > R$, $Q_{enc} = \rho \frac{4}{3}\pi R^3$

$$E(4\pi r^2) = \frac{\frac{4}{3}\pi R^3}{\varepsilon_0}\rho$$

$$E = \frac{1}{3\varepsilon_0}\frac{\rho R^3}{r^2}$$

11) What is the magnitude of the electric field at a distance r < R from the center of the sphere?

a. $|E| = \frac{\rho r^2}{3\epsilon_0}$

b. $|E| = \frac{\rho R}{6\epsilon_0}$

c. $|E| = \frac{\rho r}{3\epsilon_0}$

d. $|E| = \frac{\rho R}{3\epsilon_0}$

e. $|E| = \frac{\rho r}{6\epsilon_0}$

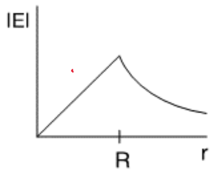
for $r < R$, only the charge enclosed inside a volume of $\frac{4}{3}\pi r^3$ contributes to E-field at r

So, $E(4\pi r^2) = \frac{\rho}{\varepsilon_0}\frac{4}{3}\pi r^3$

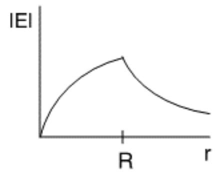
$$E = \frac{\rho r}{3\varepsilon_0}$$

12) Which of the following best describes the magnitude of the |E| field as a function of the distance from the center of the sphere r?

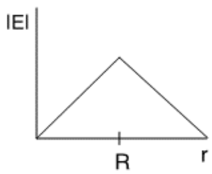
a)



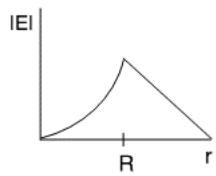
d)



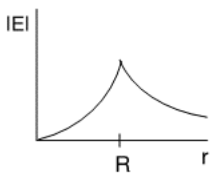
b)



e)



c)



for $r < R$

$E \propto r$

---

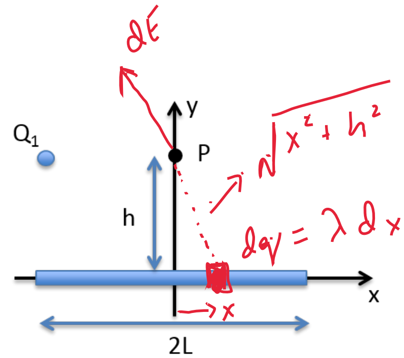
for $r > R$

$E \propto \frac{1}{r^2}$

a. e
b. c
c. d
d. b
e. a

The next three questions pertain to the situation described below.

A charge Q_1 is placed at the point (-L, h) and a rod of length 2 m and total charge charge Q_{rod} = 18 μC distributed uniformly along its length, is placed with its ends at (-L, 0) and (0, L) as shown in the figure.



13) What is the linear charge density of the rod?

$$\lambda = \frac{Q_{rod}}{2L} = \frac{18\ \mu C}{2\ m} = 9\ \mu C/m$$

a. $\lambda = 36\ \mu$C /m
b. $\lambda = 9\ \mu$C/m
c. $\lambda = 4.5\ \mu$C/m

14) Which expression gives the electric field at the point **P** = (0, h) due to the point charge and line of charge?

a. $\vec{E} = \frac{kQ_1}{L^2}\hat{x}$
b. $\vec{E} = k\lambda\int_{-L}^{L}\frac{dx}{(x^2+h^2)}\hat{y} + \frac{kQ_1}{L^2}\hat{x}$
c. $\vec{E} = k\lambda\int_{-L}^{L}\frac{dx}{(x^2+h^2)}\hat{y}$
d. $\vec{E} = k\lambda\int_{-L}^{L}\frac{xdx}{(x^2+h^2)^{\frac{3}{2}}}\hat{y} + \frac{kQ_1}{L^2}\hat{x}$
e. $\vec{E} = k\lambda\int_{-L}^{L}\frac{hdx}{(x^2+h^2)^{\frac{3}{2}}}\hat{y} + \frac{kQ_1}{L^2}\hat{x}$

$\vec{E}_{\text{point charge}} = \frac{KQ_1}{L^2}\hat{x}$

for $\vec{E}_{line}$ consider element dq as shown above. Note that all components of dE along the x-axis cancel.

So $dE = dE_y$

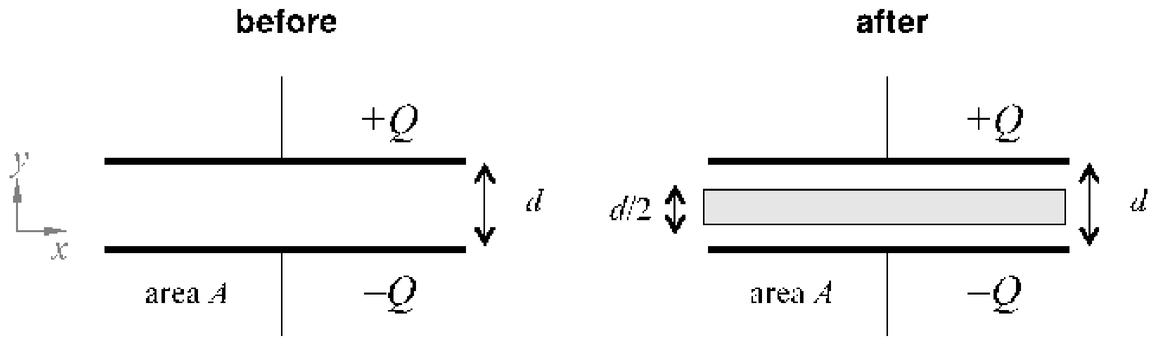
$$dE_y = \frac{Kdq}{x^2+h^2} = \frac{K\lambda dx}{x^2+h^2} \rightarrow \vec{E}_{line} = \int_{-L}^{L}\frac{K\lambda dx}{x^2+h^2}\hat{y}$$

$\vec{E} = \vec{E}_{line} + \vec{E}_{point}$

15) A second charge, Q_2, is placed at (L,h). What value should Q_2 take in order that the **total** electric field at (0, h) is zero

a. $Q_2 = Q_1$
b. It is not possible to make the field at (0, h) vanish by placing a point charge at (L, h).
c. $Q_2 = -Q_1$

The next three questions pertain to the situation described below.



A parallel plate capacitor with a large surface area A compared to the separation between the plates d has charge Q. After a certain time, a conducting slab with the same area A and a thickness of half the separation between the plates d/2 is inserted exactly in the middle of the two plates.

Consider the situation after as two capacitors in series

16) What is the relationship between the capacitance before, C, and after, C'?

$C_1 = \frac{\varepsilon_0 A}{d/4}$, $C_2 = \frac{\varepsilon_0 A}{d/4}$ and $C = \frac{\varepsilon_0 A}{d}$

a. C' = C
b. C' = C/2
c. C' = 2C

$\frac{1}{C'} = \frac{d}{4\varepsilon_0 A} + \frac{d}{4\varepsilon_0 A} \rightarrow C' = \frac{2\varepsilon_0 A}{d} = 2C$

17) What is the relationship between the energy stored in the capacitor before, U, and after, U'?

as $U = \frac{1}{2} Q^2/C$, $U' = \frac{1}{2} Q^2/C' = \frac{1}{2}\frac{Q^2}{2C}$

a. U' = U
b. U' > U
c. U' < U

$U' = \frac{1}{2} U$, $U' < U$

18) Consider the "before" configuration shown above. In what direction can a charge be moved in the field created between the plates without doing any external work on the charge?

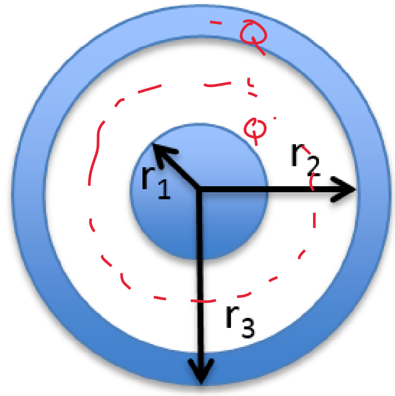
a. parallel to the y-axis
b. external work is always necessary
c. parallel to the x-axis

Note that the electric field b/w the plates is parallel to the y-axis

So no work is done if charge is moved perpendicular to the E-field → along the x-axis

The next two questions pertain to the situation described below.

A solid conducting cylinder of radius r_1 and length L with charge Q is placed inside a hollow conducting cylinder of the same length L with inner radius r_2 and outer radius r_3 and charge –Q.



for $r_1 < r < r_2$

$$E(2\pi r L) = Q/\varepsilon_0$$

$$E = \frac{Q}{2\pi\varepsilon_0 r L}$$

$$V = -\int_{r_2}^{r_1} \frac{Q}{2\pi\varepsilon_0 r L}\, dr$$

$$V = \frac{-Q}{2\pi\varepsilon_0 L} \ln\left[\frac{r_1}{r_2}\right]$$

19) How does the capacitance change if r_2 is decreased slightly keeping L, r_1, and r_3 unchanged.

a. The capacitance decreases.
b. The capacitance increases.
c. The capacitance remains the same.

$$C = \frac{Q}{V} = \frac{2\pi\varepsilon_0 L}{\ln(r_2/r_1)}$$

if r_2 decreases, C increases

20) Suppose the cylinder is submerged in gasoline (ε = 2.0) so that there is gasoline between the plates. How does the capacitance change relative to the capacitance of the previous question?

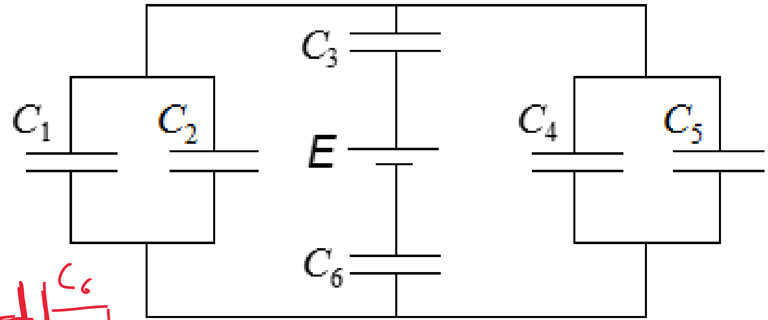
a. $C_1 = C_0/2$
b. $C_1 = C_0$
c. $C_1 = 2\ C_0$

for $\varepsilon = 2$, $V = \frac{-Q}{2\pi\varepsilon\varepsilon_0 L} \ln\left[\frac{r_1}{r_2}\right]$

$$C_1 = \frac{2\pi\varepsilon\varepsilon_0 L}{\ln(r_2/r_1)} \longrightarrow C_1 = 2C_0$$

The next three questions pertain to the situation described below.

Six capacitors are connected to a battery as shown in the circuit diagram. The battery supplies E = 12 V.



$C_1 = 10\ \mu F$
$C_2 = 16\ \mu F$
$C_3 = 50\ \mu F$
$C_4 = 6\ \mu F$
$C_5 = 20\ \mu F$
$C_6 = 40\ \mu F$

$C_{12} = C_1 + C_2$

$C_{45} = C_4 + C_5$

Redraw

$C_{1245} = C_1 + C_2 + C_4 + C_5$
$= 52\ \mu F$

21) What is the equivalent capacitance for the combination of the six capacitors?

a. $C_{123456} = 142\ \mu F$
b. $C_{123456} = 92.6\ \mu F$
c. $C_{123456} = 15.6\ \mu F$

So $\dfrac{1}{C_{123456}} = \dfrac{1}{C_3} + \dfrac{1}{C_{1245}} + \dfrac{1}{C_6}$

$\dfrac{1}{C_{123456}} = \dfrac{1}{50} + \dfrac{1}{52} + \dfrac{1}{40}$

22) Which capacitors have the same charge

a. C_1 and C_4
b. C_3 and C_6
c. C_4 and C_5

Note that C_3 and C_6 are in series
Capacitors in series have the same charge

23) What is the energy stored in capacitor C_3?

$U_3 = \dfrac{1}{2} \dfrac{Q^2}{C_3}$

a. $U_3 = 350\ \mu J$
b. $U_3 = 1120\ \mu J$
c. $U_3 = 3600\ \mu J$

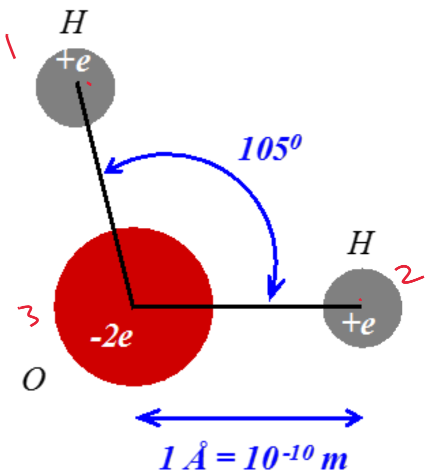
Note that Q is the same for C_3, C_6 which is the same for C_{123456}.

$Q = C_{123456} \times E = 15.6 \times 12 = 187.2\ \mu C$

So $U_3 = \dfrac{1}{2} \dfrac{(187.2)^2}{50}\ \mu J$

$U_3 = 350\ \mu J$

The next two questions pertain to the situation described below.



A water molecule may be crudely approximated as two positively charged hydrogen atoms and a negatively charged oxygen atom, as shown in the figure. Note the electron charge e = 1.6 x 10^{-19} C, and the separation between the two hydrogen atoms is 1.6 x 10^{-10}m.

24) What is the electric potential energy associated with this configuration of charges? (Let 0 corresponds to the three charges being infinitely far apart.)

$U_{total} = U_{12} + U_{23} + U_{13}$

a. 1.45×10^{-18} J
b. -7.76×10^{-18} J
c. -9.22×10^{-18} J

$$U_{total} = \frac{e^2}{4\pi\varepsilon_0 (1.6\times10^{-10})} + \frac{(-2e^2)}{4\pi\varepsilon_0 (10^{-10})} + \frac{(-2e^2)}{4\pi\varepsilon_0 (10^{-10})}$$

$$U_{total} = \left(e^2/4\pi\varepsilon_0\right)\left(\frac{1}{10^{-10}}\right)\left[\frac{1}{1.6} - 2 - 2\right] = -7.76\times10^{-18}\text{ J}$$

25) If the angle between the two hydrogen atoms is increased from 105 degrees to 180 degrees, while keeping the distance between the hydrogen and oxygen atoms fixed at 10^{-10} m, the electric potential energy of the system will

a. decrease
b. remain the same
c. increase

This increases the distance b/w the two hydrogen atoms, since $U \propto \frac{1}{r}$ this decreases the potential energy

U_{total} becomes more negative.