

Last Name: _____ First Name _____ ID _____

Discussion Section: _____ Discussion TA Name: _____

Instructions—Turn off your cell phone and put it away.

Calculators cannot be shared. Please keep yours on your own desk.

This is a closed book exam. You have 90 minutes to complete it.

This is a multiple choice exam. Use the bubble sheet to record your answers.

1. Use a #2 pencil; do **not** use a mechanical pencil or a pen. Fill in completely (until there is no white space visible) the circle for each intended input – both on the identification side of your answer sheet and on the side on which you mark your answers. If you decide to change an answer, erase vigorously; the scanner sometimes registers incompletely erased marks as intended answers; this can adversely affect your grade. Light marks or marks extending outside the circle may be read improperly by the scanner.
2. Print your last name in the **YOUR LAST NAME** boxes on your answer sheet and print the first letter of your first name in the **FIRST NAME INI** box. Mark (as described above) the corresponding circle below each of these letters.
3. Print your NetID in the **NETWORK ID** boxes, and then mark the corresponding circle below each of the letters or numerals. Note that there are different circles for the letter “I” and the numeral “1” and for the letter “O” and the numeral “0”. **Do not** mark the hyphen circle at the bottom of any of these columns.
4. You may find the version of **this Exam Booklet at the top of page 2**. Mark the version circle in the **TEST FORM** box in the bottom right on the front side of your answer sheet. **DO THIS NOW!**
5. Stop **now** and double-check that you have bubbled-in all the information requested in 2 through 4 above and that your marks meet the criteria in 1 above. Check that you do not have more than one circle marked in any of the columns.
6. Print your UIN# in the **STUDENT NUMBER** designated spaces and mark the corresponding circles. You need not write in or mark the circles in the **SECTION** box.
7. Write in your course on the **COURSE LINE** and on the **SECTION line**, print your **DISCUSSION SECTION**. (You need not fill in the **INSTRUCTOR** line.)
8. Sign (**DO NOT PRINT**) your name on the **STUDENT SIGNATURE line**.

*Before starting work, check to make sure that your test booklet is complete. After these instructions, you should have ****9** numbered pages plus 2 Formula Sheets**.*

On the test booklet:

Write your **NAME**, your **Discussion TA's NAME**, your **DISCUSSION SECTION** and your **NETWORK-ID**. Also, write your **EXAM ROOM** and **SEAT NUMBER**.

When you are finished, you must hand in BOTH the exam booklet AND the answer sheet. Your exam will not be graded unless both are present.

Academic Integrity—Giving assistance to or receiving assistance from another student or using unauthorized materials during a University Examination can be grounds for disciplinary action, up to and including expulsion.

This Exam Booklet is Version A. Mark the **A** circle in the **TEST FORM** box in the bottom right on the front side of your answer sheet. **DO THIS NOW!**

Exam Format & Instructions:

This exam is a combination of

- * Three-Answer Multiple Choice (3 points each)
- * Five-Answer Multiple Choice (6 points each)

There are 23 problems for a maximum possible raw score of 114 points.

Instructions for Three-Answer Multiple Choice Problems:

Indicate on the answer sheet the correct answer to the question (*a*, *b* or *c*).

Each question is worth 3 points. If you mark the wrong answer, or mark more than one answer, you receive 0 points.

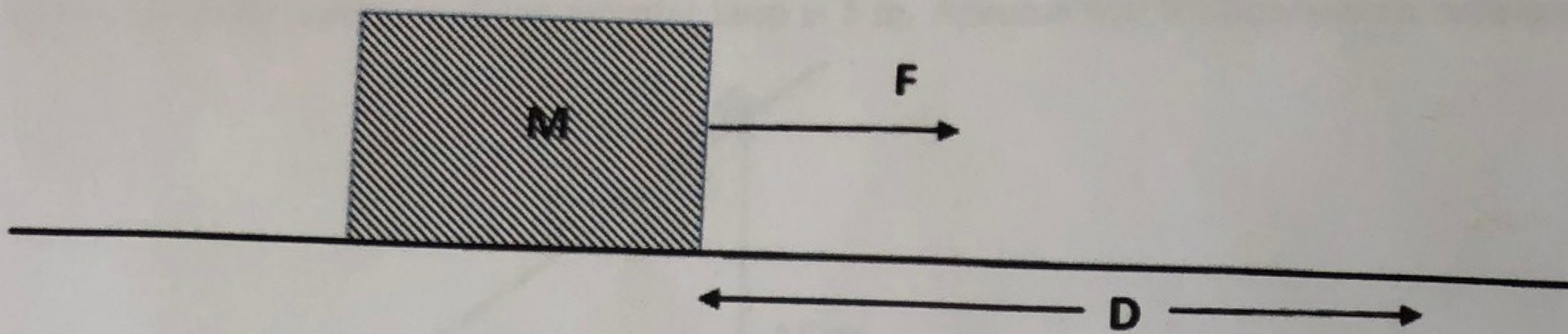
Instructions for Five-Answer Multiple Choice Problems:

Indicate on the answer sheet the correct answer to each question (*a*, *b*, *c*, *d* or *e*).

Credit is awarded in the following way:

- If you mark one answer and it is correct, you will receive 6 points;
- If you mark two answers, and one of them is correct, you will receive 3 points;
- If you mark three answers and one of them is correct, you will receive 2 points.
- If you mark no answer or more than three answers, you will receive 0 points.

The next three questions pertain to the situation described below.



A block of mass $M = 10 \text{ kg}$ is on a rough, horizontal table. A rope pulls the block by applying a horizontal force F and, as a result, the block moves a distance of 1.5 m .

1) If the work done on the block by the rope is 10 Joules , what is the magnitude of the force F ?

- a. 15 N
- b. 6.7 N
- c. 10 N

$$W = Fd \Rightarrow F = \frac{W}{d} = \frac{10 \text{ J}}{1.5 \text{ m}} = 6.7 \text{ N}$$

2) Taking into account the work done on the block by the rope in the question above as well as the work done by friction, the net work done on the block is 5.1 Joules . If the block initially starts from rest, what is its final speed?

- a. 1 m/s
- b. 3.2 m/s
- c. 0.5 m/s
- d. 1.4 m/s
- e. 0.71 m/s

$$W = \Delta KE = \frac{1}{2} m v^2 \Rightarrow v = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2 \times 5.1 \text{ J}}{10 \text{ kg}}} = 1 \text{ m/s}$$

3) Given the information on the work done by the rope and the net work done in the last two questions, the coefficient of kinetic friction μ_k between the block and the table is

- a. 0.1
- b. 0.068
- c. 0.033
- d. 0.017
- e. 0.67

$$W = W_{\text{rope}} + W_{\text{friction}} = 5.1 \text{ J}$$

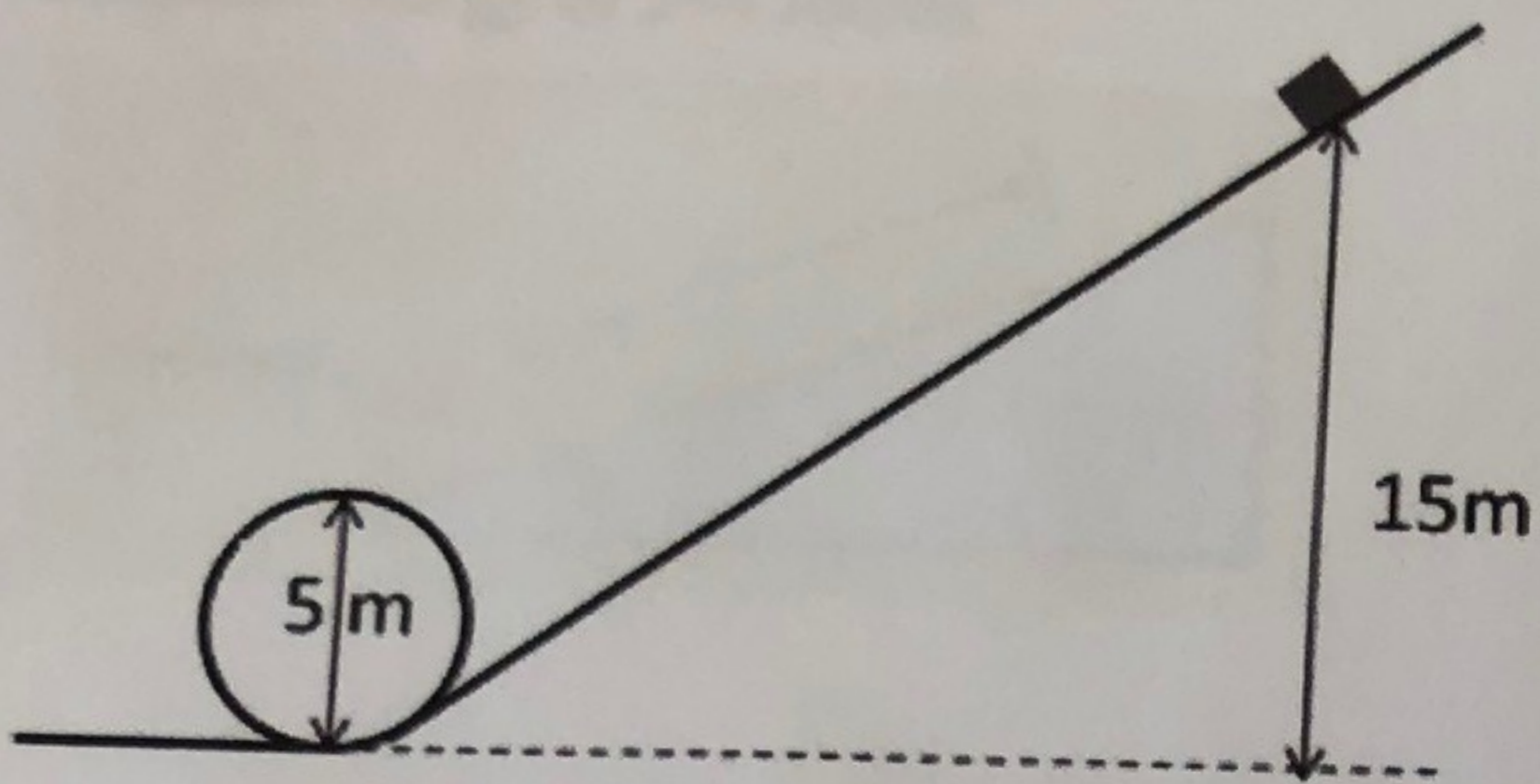
$\begin{matrix} \text{"} & \text{"} \\ 10 \text{ J} & f_k d d \end{matrix}$

friction does } negative work } $\rightarrow -\mu_k m g d$

$$\Rightarrow \mu_k = \frac{10 \text{ J} - 5.1 \text{ J}}{m g d} = \frac{4.9 \text{ J}}{10 \text{ kg} \times 9.8 \text{ m/s}^2 \times 1.5 \text{ m}} = 0.033$$

The next three questions pertain to the situation described below.

A block of mass 50 kg is about to roll down a roller coaster track from rest as shown below. Its initial height is 15 m and the diameter of the circular loop is 5 m. Assume that friction and air resistance are negligible.



4) What is the kinetic energy of the block right at the base of the loop (on the ground)?

- a. 7360 J
- b. 14700 J
- c. 3680 J
- d. 1840 J
- e. 1230 J

$$KE = mgh = 50 \text{ kg} \times 9.8 \text{ m/s}^2 \times 15 \text{ m} = 7350 \text{ J}$$

5) What is v_{top} , the speed of the block at the top of the loop?

- a. 37.7 m/s
- b. 0.3 m/s
- c. 26.9 m/s
- d. 14 m/s
- e. 5.5 m/s

$$\frac{1}{2}mv^2 = mg(15 \text{ m} - 5 \text{ m})$$

$$v = \sqrt{2g \times 10 \text{ m}} = \sqrt{2 \times 9.8 \text{ m/s}^2 \times 10 \text{ m}} = 14 \text{ m/s}$$

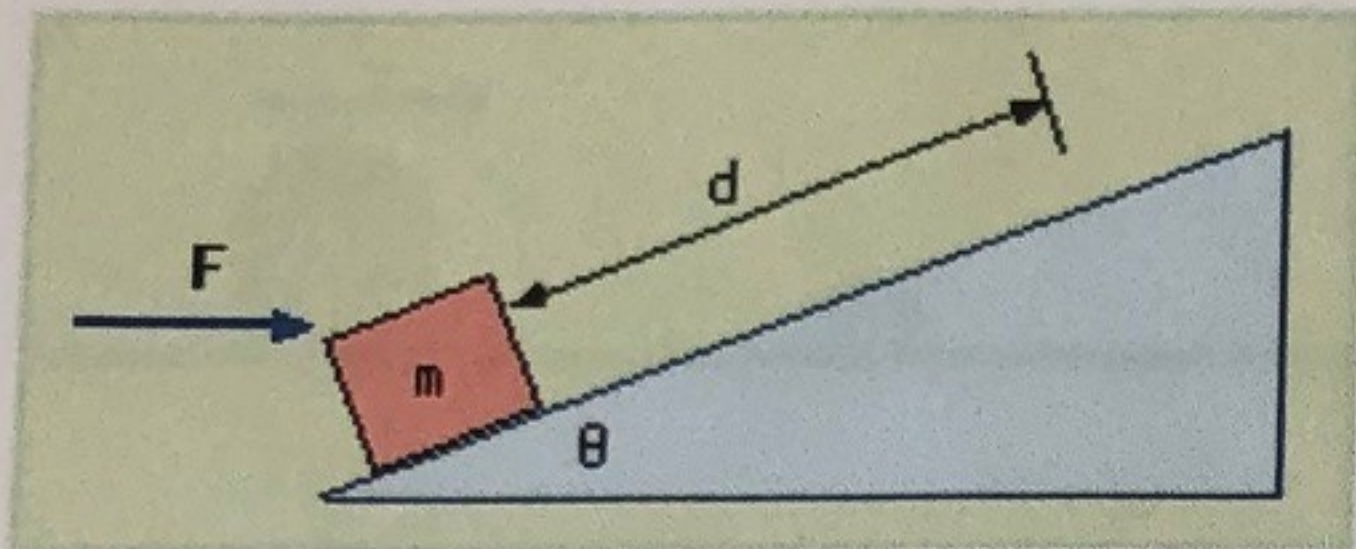
6) At the top of the loop, with regards to the mass, the centripetal force (due to rotation) and force of gravity are acting

- a. In the same direction.
- b. In opposite directions.
- c. Perpendicular to each other.

Downward

The next three questions pertain to the situation described below.

A horizontal force of magnitude F (unknown) is applied to push a block of mass $m = 14$ kg up a frictionless incline of slope 30° . The block starts from rest and travels a distance of $d = 15$ m along the slope and acquires a kinetic energy of 294 Joules.



7) How much work is done on the block by the gravitational force during the displacement?

- a. -577 J
- b. 0 J
- c. -1030 J
- d. -680 J
- e. -2680 J

$$W = Fh = -mg d \sin \theta = -14 \text{ kg} \times 9.8 \text{ m/s}^2 \times 15 \text{ m} \times \sin 30^\circ$$

↑
vertical displacement

$$= -1,029 \text{ J}$$

8) What is the work done by the **normal** force of the slope on the block during this displacement?

- a. 680 J
- b. -135 J
- c. 0 J

9) What is the speed of the block at the end of this displacement?

- a. 12 m/s
- b. 3.2 m/s
- c. 6.5 m/s
- d. 8.6 m/s
- e. 4.6 m/s

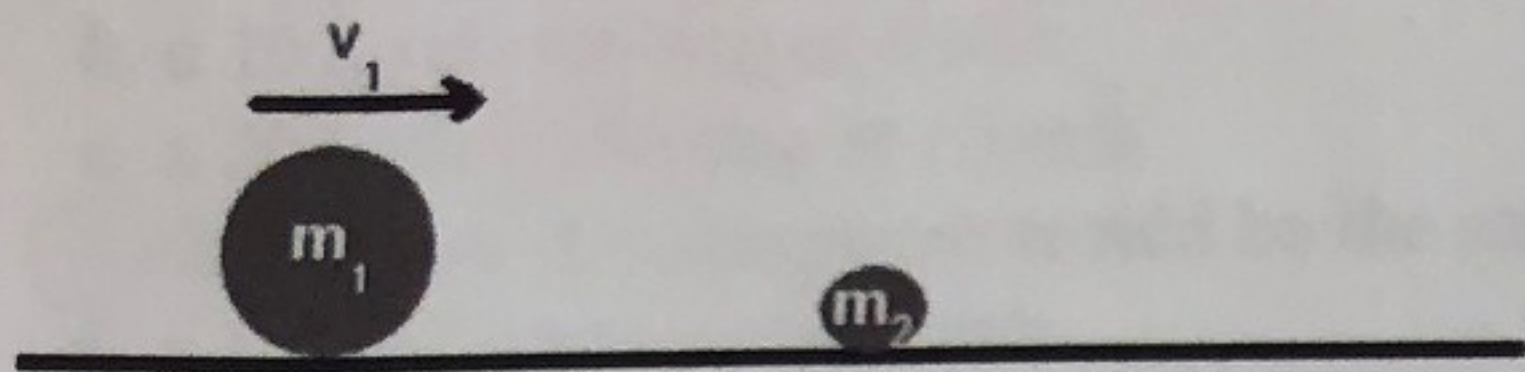
$$\frac{1}{2} m v^2 = 294 \text{ J}$$

$$v = \sqrt{\frac{2}{m} \times 294 \text{ J}} = \sqrt{\frac{2}{14 \text{ kg}} \times 294 \text{ J}}$$

$$= 6.48 \text{ m/s}$$

The next two questions pertain to the situation described below.

A mass $m_1 = 0.14$ kg is moving at a velocity 0.26 m/s towards a smaller stationary mass $m_2 = 0.08$ kg. In the collision, it sticks to it and continues to move in the same direction.



10) The linear momentum of the system after the collision is:

- a. The same as before the collision.
- b. Larger than before the collision.
- c. Smaller than before the collision.

11) Determine the velocity of the combined masses, v , after the collision

- a. 0.26 m/s
- b. 0.61 m/s
- c. 0.17 m/s
- d. 0.52 m/s
- e. 0.21 m/s

$$m_1 v_1 = (m_1 + m_2) V$$

$$V = \frac{m_1}{m_1 + m_2} v_1 = \frac{0.14 \text{ kg}}{(0.14 + 0.08) \text{ kg}} \times 0.26 \text{ m/s}$$

$$= 0.165 \text{ m/s}$$

12) If the same force were applied for the same time to each of the following, the change in momentum would be the greatest for:

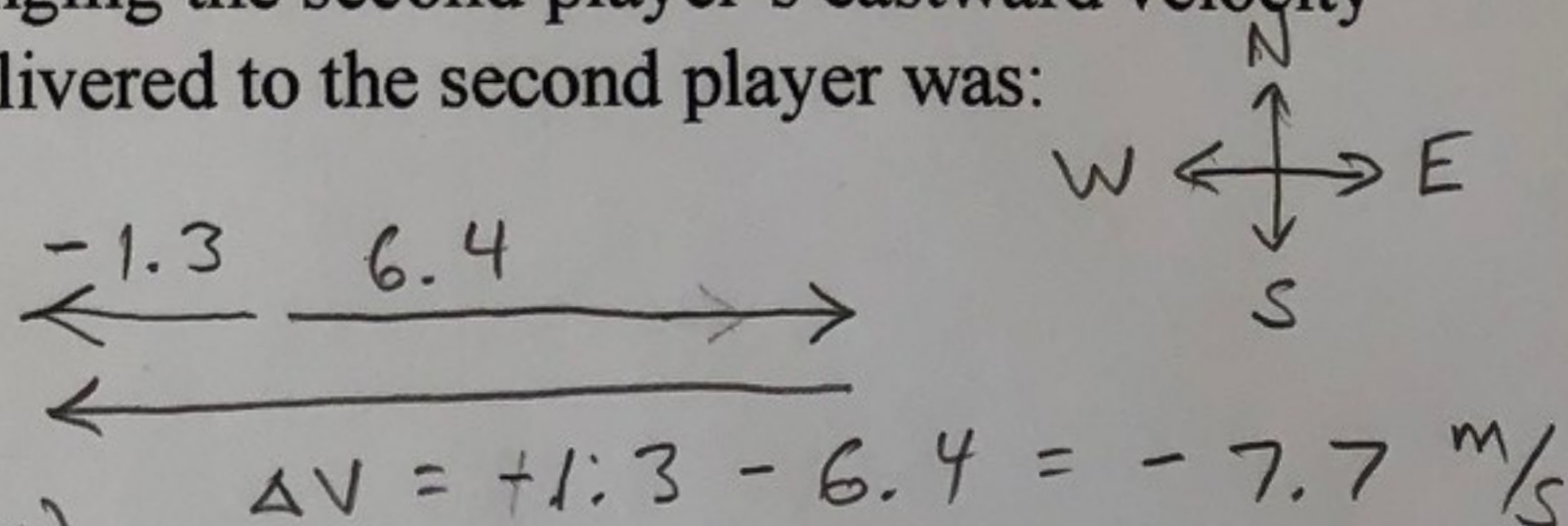
$$F \Delta t = \Delta p$$

- a. a 5 kg ball moving at 9 m/s
- b. a 10 kg ball moving at 5 m/s
- c. a 20 kg ball moving at 10 m/s
- d. the change in momentum would be the same for all
- e. a 7 kg ball moving at 7 m/s

13) A football player delivers a hit to a second 86 kg player, changing the second player's eastward velocity of 6.4 m/s to a westward velocity of 1.3 m/s. The impulse delivered to the second player was:

- a. 440 N.s west
- b. 220 N.s east
- c. 660 N.s west

$$I = \Delta p = m \Delta v$$

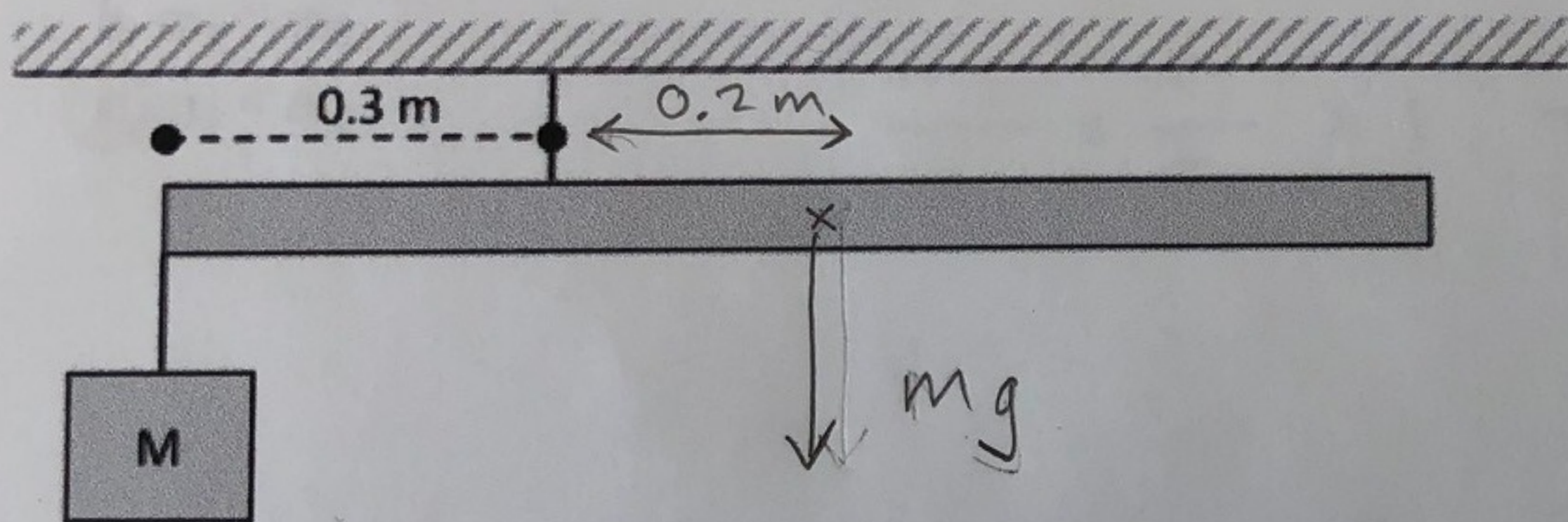


$$= 86 \text{ kg} \times (-7.7 \text{ m/s})$$

$$\Delta v = +1.3 - 6.4 = -7.7 \text{ m/s}$$

$$= 662 \text{ N.s west}$$

14) A one meter board with a mass of 16.5 kg hangs by a rope as shown. A block is hung from the left end of the board. If the board is to be balanced level to the ground, what must be the mass of the block?



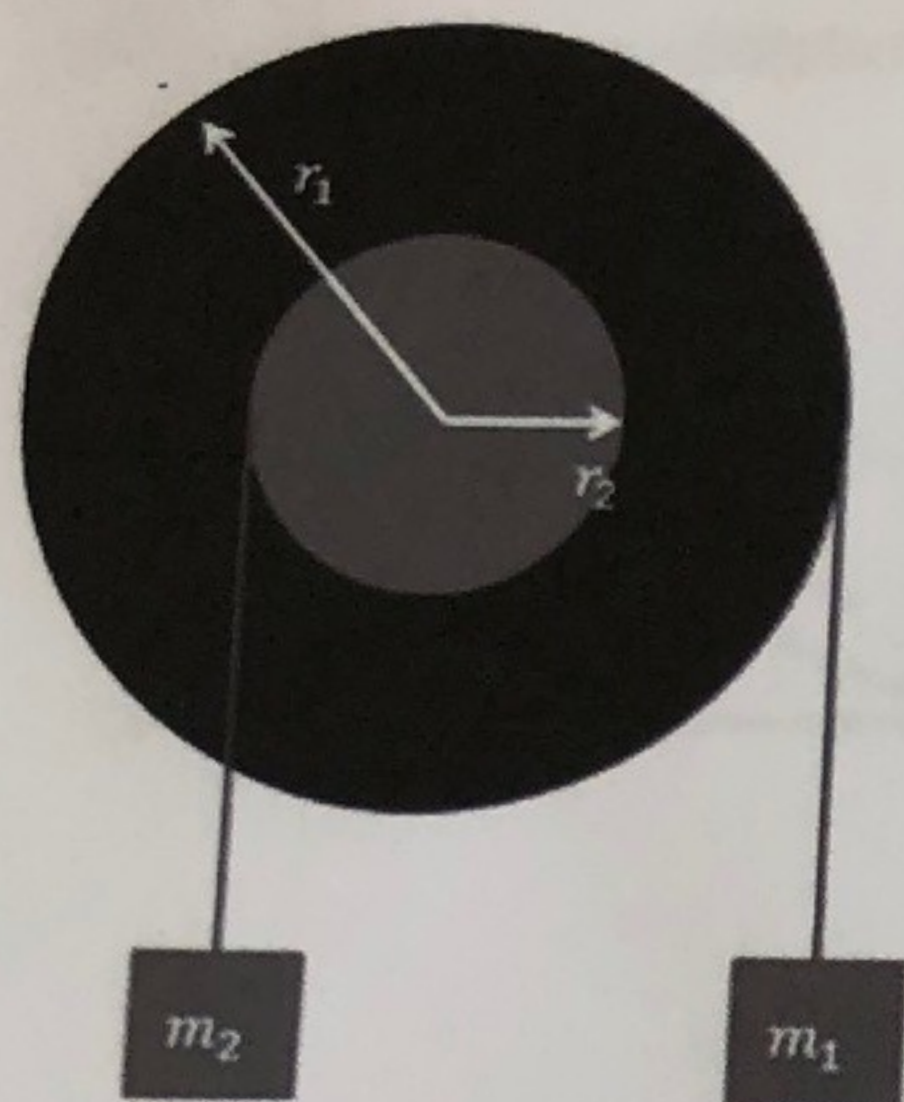
$$Mg \times 0.3 \text{ m} = mg \times 0.2 \text{ m}$$

$$M = \frac{0.2 \text{ m}}{0.3 \text{ m}} \times m = \frac{0.2}{0.3} \times 16.5 \text{ kg}$$

$$= 11 \text{ kg}$$

- a. 20 kg
- b. 11 kg
- c. 5 kg
- d. 4 kg
- e. 6 kg

A wheel has a radius $r_1 = 0.24$ m and mass $M_w = 2.3$ kg with a hub of radius $r_2 = 0.12$ m and mass $M_h = 9.2$ kg as shown in the diagram. Two masses are hung from the wheel-hub system. The mass m_2 , on the left, is hung from the hub. The mass m_1 is hung from the wheel. The system is in equilibrium. The wheel and hub are both solid disks. The moment of inertia for a solid disk is $I_{\text{disk}} = 1/2 mR^2$.



15) Which of the following statements is true:

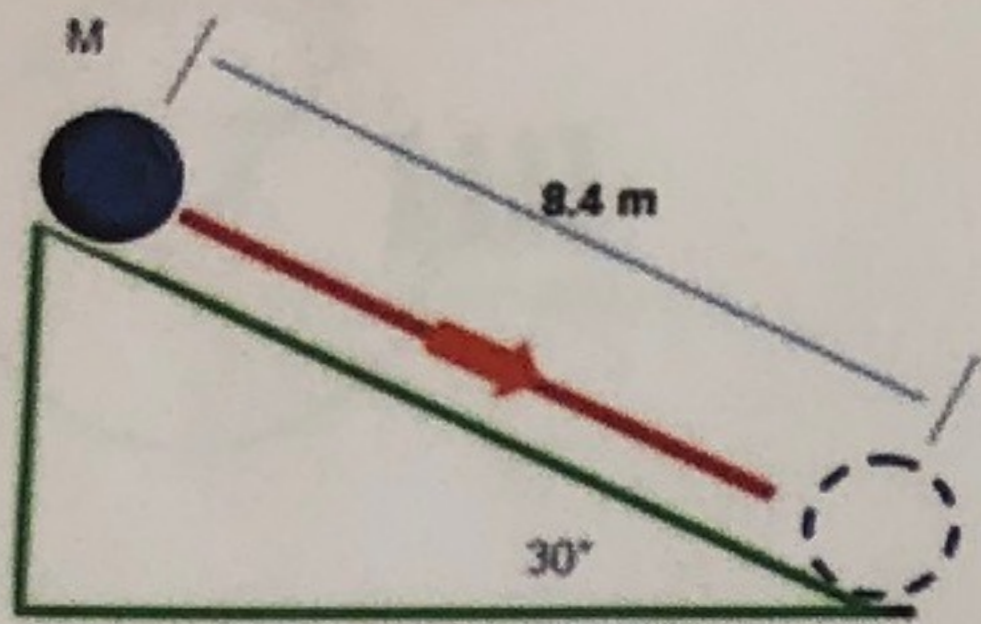
- a. $m_1 = m_2$
- b. $m_1 > m_2$
- c. $m_1 < m_2$

$$m_2 g r_2 = m_1 g r_1$$

$$\frac{m_2}{m_1} = \frac{r_1}{r_2} > 1 \Rightarrow m_1 < m_2$$

The next two questions pertain to the situation described below.

A hollow ball of mass $M = 0.5 \text{ kg}$ and radius 0.18 m rolls down without slipping an incline making a 30° angle with the horizontal. It rolls from rest, reaching the bottom 8.4 m along the incline. The ball is found to roll with linear acceleration equal to 2.94 m/s^2 . The moment of inertia of the ball is $I_{\text{ball}} = 0.011 \text{ kg m}^2$.



16) What is the angular acceleration of the ball?

- a. 0.953 rad/s^2
- b. 4.9 rad/s^2
- c. 16.3 rad/s^2
- d. 0.529 rad/s^2
- e. 5.06 rad/s^2

$$a = \alpha R \Rightarrow \alpha = \frac{a}{R} = \frac{2.94 \text{ m/s}^2}{0.18 \text{ m}} = 16.3 \text{ rad/s}^2$$

17) What is the linear speed of the ball just before it reaches the bottom of the incline?

- a. 7 m/s
- b. 38.9 m/s
- c. 16.5 m/s
- d. 47.8 m/s
- e. 4.97 m/s

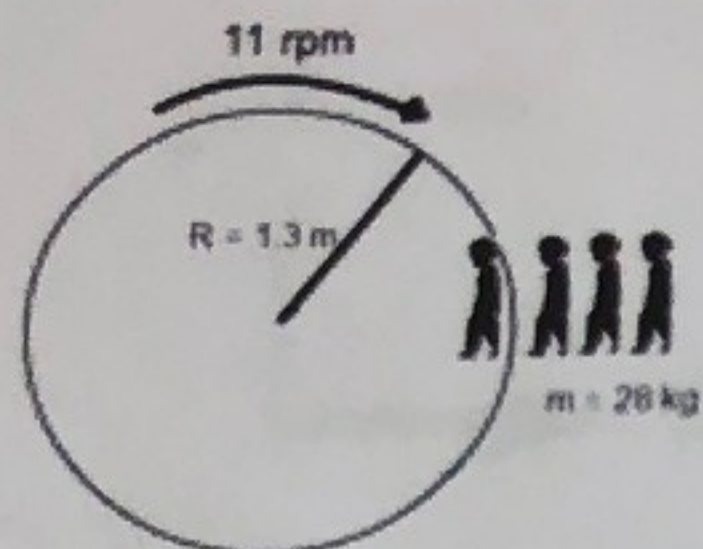
$$v^2 = v_0^2 + 2a\Delta x$$

$$v = 0 + 2 \times 2.94 \text{ m/s}^2 \times 8.4 \text{ m}$$

$$v = \sqrt{2 \times 2.94 \text{ m/s}^2 \times 8.4 \text{ m}} = 7.03 \text{ m/s}$$

The next three questions pertain to the situation described below.

A merry-go-round of radius $R = 1.3 \text{ m}$ has moment of inertia $I_m = 290 \text{ kg}\cdot\text{m}^2$ and is rotating freely at $\omega_0 = 11 \text{ rpm}$. A boy of mass $m = 28 \text{ kg}$ leaps onto the edge of the merry-go-round radially (heading directly towards the center). Assume the boy is a point object.



18) The angular momentum of the merry-go-round and the boy is conserved

- a. false
- b. true
- c. we cannot tell

19) What is the new angular speed of the merry-go-round ω with the boy at its edge? (Hint: calculate the new moment of inertia of the system including the boy)

- a. 11 rpm
- b. 10 rpm
- c. 0.99 rpm
- d. 4.73 rpm
- e. 9.46 rpm

$$I_1 \omega_1 = I_2 \omega_2$$

$$290 \text{ kg}\cdot\text{m}^2 \times 11 \text{ rpm} = (290 \text{ kg}\cdot\text{m}^2 + 28 \text{ kg} \times (1.3 \text{ m})^2) \omega_2$$

$$\omega_2 = \frac{290}{337.32} \times 11 \text{ rpm} = 9.46 \text{ rpm}$$

20) Now the boy starts to slowly walk radially and stops at the center of the merry-go-round. What is the new angular speed of the merry-go-round ω with the boy at its center?

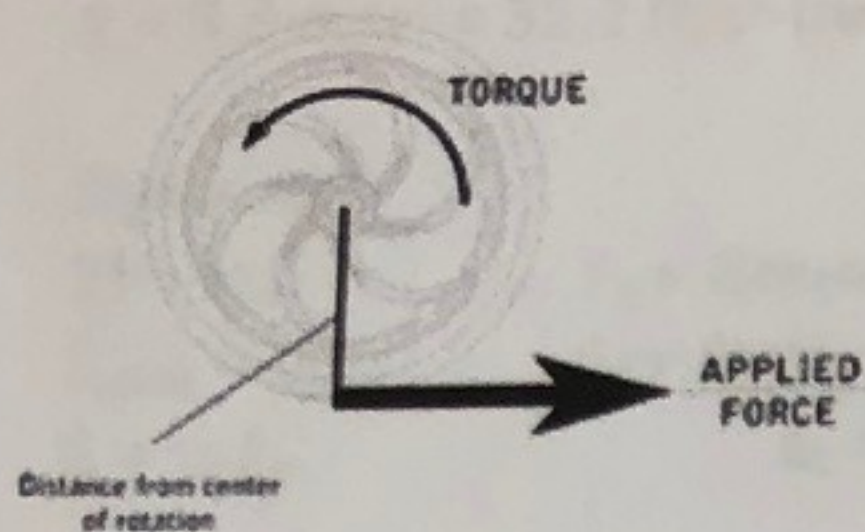


- a. 16.2 rpm
- b. 11 rpm
- c. 15.1 rpm
- d. 4.73 rpm
- e. 9.46 rpm

I returns to its original value

The next three questions pertain to the situation described below.

The radius of a wheel is 46 cm, and its rotational inertia I is $72 \text{ kg}\cdot\text{m}^2$. A tangential force F is applied to the outer rim at $t = 0$ such that it gives the wheel 0.639 rad/s^2 angular acceleration. The wheel starts from rest and rotates about its axis with no friction.



21) Determine magnitude of the applied force F .

- a. 200 N
- b. 100 N
- c. 50 N
- d. 150 N
- e. 25 N

$$\tau = RF = I\alpha \Rightarrow F = \frac{I\alpha}{R} = \frac{72 \text{ kg}\cdot\text{m}^2 \times 0.639 \text{ rad/s}^2}{0.46 \text{ m}} = 100 \text{ N}$$

22) Determine the kinetic energy K.E. of the wheel at $t = 10 \text{ s}$

- a. 1670 J
- b. 1070 J
- c. 1270 J
- d. 1470 J
- e. 870 J

$$KE = \frac{1}{2} I \omega^2 \quad \omega = \omega_0 + \alpha t = 0 + 0.639 \text{ rad/s}^2 \times 10 \text{ s} = 6.39 \text{ rad/s}$$

$$KE = \frac{1}{2} 72 \text{ kg}\cdot\text{m}^2 \times (6.39 \text{ rad/s})^2 = 1470 \text{ J}$$

23) Which of the following is true?

- a. The final rotational kinetic energy of the disk is equal to the work done by the force
- b. The final rotational kinetic energy of the disk is less than the work done by the force
- c. The final rotational kinetic energy of the disk is greater than the work done by the force

Work - Kinetic Energy theorem

Physics 101 Formulas

Kinematics

$$\begin{aligned} \mathbf{v}_{ave} &= \Delta \mathbf{x} / \Delta t & \mathbf{a}_{ave} &= \Delta \mathbf{v} / \Delta t \\ v &= v_0 + at & x &= x_0 + v_0 t + \frac{1}{2} at^2 & v^2 &= v_0^2 + 2a\Delta x \\ g &= 9.8 \text{ m/s}^2 = 32.2 \text{ ft/s}^2 & & & & \text{(near Earth's surface)} \end{aligned}$$

Dynamics

$$\begin{aligned} \Sigma \mathbf{F} &= m\mathbf{a} & F_g &= Gm_1m_2 / R^2 & F_g &= mg \text{ (near Earth's surface)} \\ f_{s,max} &= \mu_s F_N & \text{Gravitational constant, } G &= 6.7 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2 \\ f_k &= \mu_k F_N & a_c &= v^2 / R = \omega^2 R \end{aligned}$$

Work & Energy

$$\begin{aligned} W_F &= F D \cos(\theta) & K &= \frac{1}{2} m v^2 = p^2 / 2m & W_{NET} &= \Delta K = K_f - K_i & E &= K + U \\ W_{nc} &= \Delta E = E_f - E_i = (K_f + U_f) - (K_i + U_i) \\ U_{grav} &= mgy \end{aligned}$$

Impulse & Momentum

$$\begin{aligned} \text{Impulse } I &= \mathbf{F}_{ave} \Delta t = \Delta \mathbf{p} \\ \mathbf{F}_{ave} \Delta t &= \Delta \mathbf{p} = m\mathbf{v}_f - m\mathbf{v}_i \\ \mathbf{F}_{ave} &= \Delta \mathbf{p} / \Delta t \end{aligned}$$

$$\begin{aligned} \Sigma \mathbf{F}_{ext} \Delta t &= \Delta \mathbf{P}_{total} = \mathbf{P}_{total,final} - \mathbf{P}_{total,initial} & \text{(momentum conserved if } \Sigma \mathbf{F}_{ext} = 0) \\ \mathbf{x}_{cm} &= (m_1 \mathbf{x}_1 + m_2 \mathbf{x}_2) / (m_1 + m_2) \end{aligned}$$

Rotational Kinematics

$$\begin{aligned} \omega &= \omega_0 + \alpha t & \theta &= \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 & \omega^2 &= \omega_0^2 + 2\alpha \Delta \theta \\ \Delta x_T &= R \Delta \theta & v_T &= R\omega & a_T &= R\alpha \\ \text{(rolling without slipping: } \Delta x &= R \Delta \theta & v &= R\omega & a &= R\alpha) \\ 1 \text{ revolution} &= 2\pi \text{ radians} \end{aligned}$$

Rotational Statics & Dynamics

$$\begin{aligned} \tau &= Fr \sin \theta \\ \Sigma \tau &= 0 \text{ and } \Sigma \mathbf{F} = 0 \text{ (static equilibrium)} \\ \Sigma \tau &= I\alpha \\ W &= \tau \theta \text{ (work done by a torque)} \\ L &= I\omega & \Sigma \tau_{ext} \Delta t &= \Delta L \\ \text{(angular momentum conserved if } \Delta \tau_{ext} &= 0) \\ K_{rot} &= \frac{1}{2} I \omega^2 = L^2 / 2I & K_{total} &= K_{trans} + K_{rot} = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 \end{aligned}$$

Moments of Inertia (I)

$$\begin{aligned} I &= \Sigma m r^2 \text{ (for a collection of point particles)} \\ I &= \frac{1}{2} M R^2 \text{ (solid disk or cylinder)} \\ I &= \frac{2}{5} M R^2 \text{ (solid ball)} \\ I &= \frac{2}{3} M R^2 \text{ (hollow sphere)} \\ I &= M R^2 \text{ (hoop or hollow cylinder)} \\ I &= \frac{1}{12} M L^2 \text{ (uniform rod about center)} \end{aligned}$$

Fluids

$$\begin{aligned} P &= F/A, & P(d) &= P(0) + \rho g d \text{ change in pressure with depth } d \\ \rho &= M/V \text{ (density)} \\ \text{Buoyant force } F_B &= \rho g V_{dis} = \text{weight of displaced fluid} \\ \text{Flow rate } Q &= v_1 A_1 = v_2 A_2 \text{ continuity equation (area of circle } A = \pi r^2) \\ P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 &= P_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2 \text{ Bernoulli equation} \end{aligned}$$

$$\rho_{water} = 1000 \text{ kg/m}^3$$

$$1 \text{ m}^3 = 1000 \text{ liters}$$

$$1 \text{ atm} = 1.01 \times 10^5 \text{ Pa}$$

$$1 \text{ Pa} = 1 \text{ N/m}^2$$

Simple Harmonic Motion

Hooke's Law: $F_s = -kx$

$$U_{\text{spring}} = \frac{1}{2}kx^2$$

$$x(t) = A \cos(\omega t) \quad \text{or} \quad x(t) = A \sin(\omega t)$$

$$v(t) = -A\omega \sin(\omega t) \quad \text{or} \quad v(t) = A\omega \cos(\omega t)$$

$$a(t) = -A\omega^2 \cos(\omega t) \quad \text{or} \quad a(t) = -A\omega^2 \sin(\omega t)$$

$$\omega^2 = k/m \quad T = 2\pi/\omega = 2\pi\sqrt{m/k} \quad f = 1/T$$

$$x_{\text{max}} = A \quad v_{\text{max}} = \omega A \quad a_{\text{max}} = \omega^2 A \quad \omega = 2\pi f$$

$$\text{For a simple pendulum } \omega^2 = g/L, T = 2\pi\sqrt{L/g}$$

Harmonic Waves

$$v = \lambda / T = \lambda f$$

$$v^2 = F/(m/L) \text{ for wave on a string}$$

$$v = c = 3 \times 10^8 \text{ m/s for electromagnetic waves (light, microwaves, etc.)}$$

$$I = P/(4\pi r^2) \text{ (sound intensity)}$$

Sound Waves

Loudness: $\beta = 10 \log_{10}(I/I_0)$ (in dB), where $I_0 = 10^{-12} \text{ W/m}^2$

$$f_{\text{observer}} = f_{\text{source}} \frac{v_{\text{wave}} - v_{\text{observer}}}{v_{\text{wave}} - v_{\text{source}}} \text{ (Doppler effect)}$$

Temperature and Heat

Temperature: Celsius (T_C) to Fahrenheit (T_F) conversion: $T_C = (5/9)(T_F - 32)$

Celsius (T_C) to Kelvin (T_K) conversion: $T_K = T_C + 273$

$$\Delta L = \alpha L_0 \Delta T \quad \Delta V = \beta V_0 \Delta T \quad \text{thermal expansion}$$

$$Q = cM\Delta T \text{ specific heat capacity}$$

$$Q = L_f M \text{ latent heat of fusion (solid to liquid)}$$

$$Q = L_v M \text{ latent heat of vaporization}$$

$$Q = \kappa A \Delta T t / L \text{ conduction}$$

$$Q = e\sigma T^4 A t \text{ radiation } (\sigma = 5.67 \times 10^{-8} \text{ J/(s}\cdot\text{m}^2\cdot\text{K}^4))$$

$$P_{\text{net}} = e\sigma A(T^4 - T_0^4) \text{ (surface area of a sphere } A = 4\pi r^2)$$

Ideal Gas & Kinetic Theory

$$N_A = 6.022 \times 10^{23} \text{ molecules/mole} \quad \text{Mass of carbon-12} = 12.000 \text{ u}$$

$$PV = nRT = Nk_B T \quad R = 8.31 \text{ J/(mol}\cdot\text{K)} \quad k_B = R/N_A = 1.38 \times 10^{-23} \text{ J/K}$$

$$KE_{\text{ave}} = \frac{3}{2}k_B T = \frac{1}{2}m v_{\text{rms}}^2 \quad U = \frac{3}{2}Nk_B T \text{ (internal energy of a monatomic ideal gas)}$$

$$v_{\text{rms}}^2 = 3k_B T / m = 3RT / M \text{ (M = molar mass = kg/mole)}$$

Thermodynamics

$$\Delta U = Q + W \text{ (1st law)}$$

$$U = (\frac{3}{2})nRT \text{ (internal energy of a monatomic ideal gas for fixed n)}$$

$$C_v = (\frac{3}{2})R = 12.5 \text{ J/(mol}\cdot\text{K)} \text{ (specific heat at constant volume for a monatomic ideal gas)}$$

$$Q_H + Q_C + W = 0 \text{ (heat engine or refrigerator)}$$

$$e = -W/Q_H = 1 + Q_C/Q_H \quad e_{\text{max}} = 1 - T_C/T_H \text{ (Carnot engine)}$$

$$-Q_C/Q_H = T_C/T_H \text{ at maximum efficiency (2nd law)}$$

$$W = -P\Delta V \text{ (work done by expanding gas)}$$

$$\Delta S = Q/T \text{ (entropy)}$$