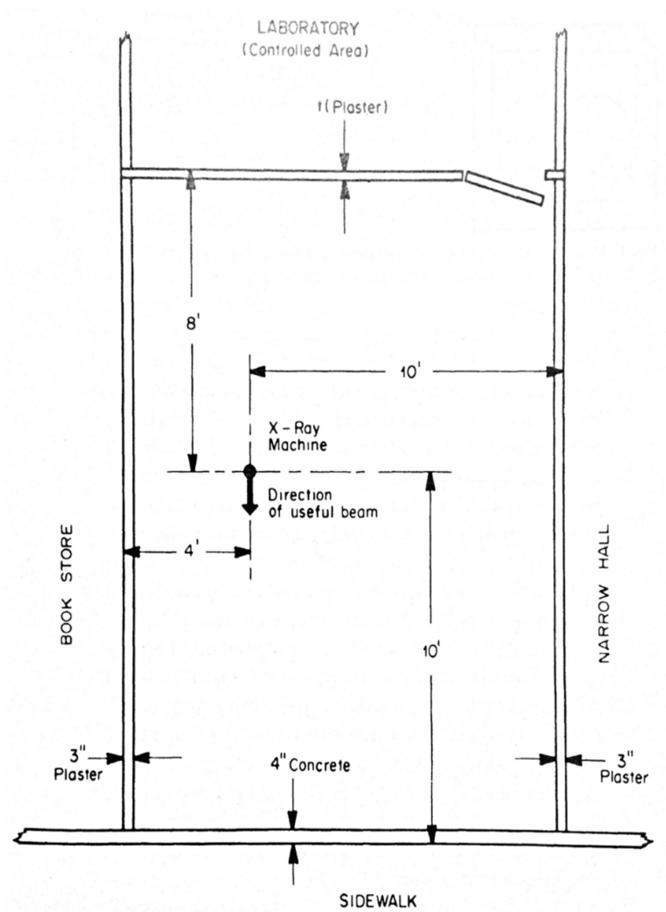
NPRE 441, Spring 2026 Quiz 6 (04-27-2026)

Your Name:

Q1. Design of the primary X-ray shielding (40 points)
For the X-ray facility shown in the figure below, please calculate the thickness of concrete shielding needed for the primary protective barrier.



Note:

1. The diagnostic X-ray source is operated at 200 kVp with a maximum current of 60 mA. It is used for 12 mins/day and 4 days/week.
2. The horizontal beam is *always* pointed toward the sidewalk that has an occupancy factor of 0.75.
3. The maximum exposure rate allowed for the sidewalk is 0.01 R/week.
4. The thickness of shielding needed can be estimated using the following figure.

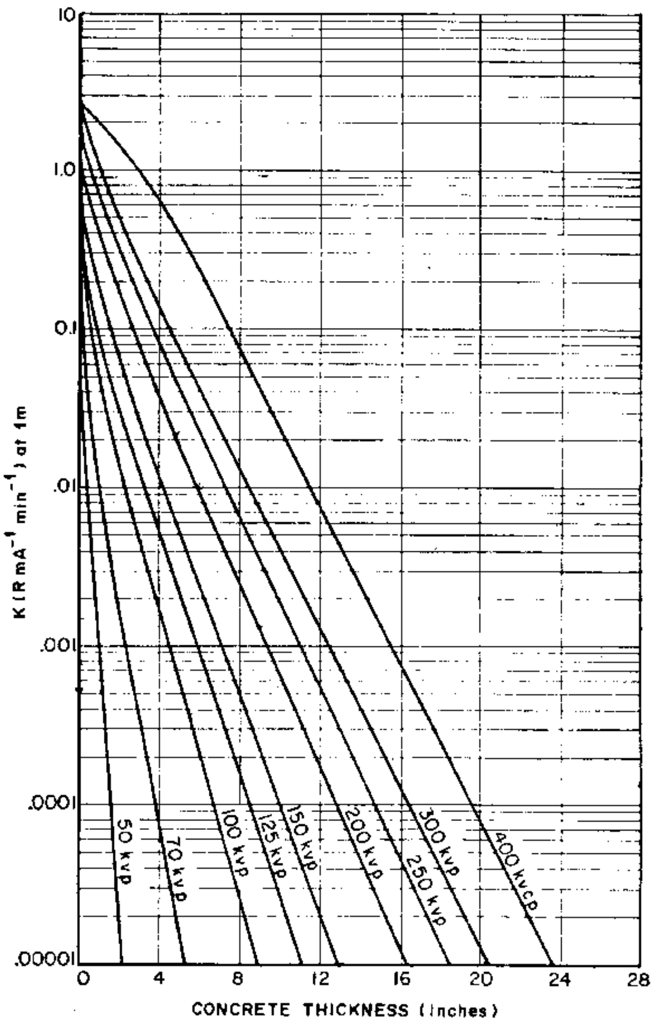


FIGURE 15.6. Attenuation in concrete of X rays produced with (peak) potential differences from 50 kVp to 400 kVp. (*National Bureau of Standards Handbook 76*, 1961, Washington, DC)

Solution:

For this question,

- The tube is operated at 200 kVp,
- The workload of the X-ray source is

$$W = 60 \text{ mA} \times 12 \text{ min/day} \times 4 \text{ days/week}$$
$$W = 2880 \text{ mA} \cdot \text{min/week}$$

- The occupancy factor is $T = 0.75$ for the sidewalk,
- The use factor is $U = 1$, since the horizontal beam is always pointed toward the sidewalk,
- The distance between the source and the area of interest is

$$d = 10 \text{ feet} = 3.05 \text{ m}$$

- The maximum allowed exposure rate is

$$P = 0.01 \text{ R/week}$$

For the primary barrier, the normalized shielded-source output factor can be evaluated with the following equation:

$$K = \frac{d^2 P}{WTU}$$

$$K = \frac{(3.05\ m)^2 (0.01 \frac{R}{week})}{(2880 \text{ mA} \cdot \text{min/week})(0.75)(1)}$$

$$K = 4.3 \times 10^{-5} \frac{R}{\text{mA} \cdot \text{min}} \text{ at 1 m}$$

Using the 200 kVp curve in Figure 15.6, the required total concrete thickness is approximately, is about **14 inches**. Since the figure shows 4 inches of existing concrete, the additional concrete thickness needed is approximately

$$14 - 4 = 10 \text{ inches}$$

Therefore, the required primary barrier thickness is about **14 inches of concrete**. Since 4 inches of concrete already exists, approximately **10 inches of additional concrete is needed**.

Q2: Neutron shielding design (60 points)

For a ^{210}Po-Be point source that emits 7.5×10^7 neutrons/second placed in a spherical water tank of 35 cm radius, please estimate the dose rate at the surface of the shielding.

Notes:

1. Assuming that the Po-Be neutron source emits fast neutrons of around 4 MeV, and each thermal neutron capture by hydrogen leads to the emission of a gamma-ray of 2.26 MeV energy.
2. For a point source of fast neutrons placed at the center of a spherical shielding of nT (cm) radius, the fast neutron flux emerging from the surface of the shielding is given by

$$\dot{\phi} = \frac{BS}{4\pi(nT)^2}\frac{1}{2^n} \quad (neutrons/cm^2/s)\,,$$

   where B=5 is the build-up factor, S is the source strength in neutrons per second, and T = 3.7 cm is the half-value layer (HVL) for fast neutrons in water.

3. For a point source of thermal neutrons of strength S neutrons/second at the center of a spherically-shaped non-multiplying medium of radius R, the flux of thermal neutrons escaping from the surface of the medium is

$$\dot{\phi} = \frac{S}{4\pi RD} e^{-R/L} \quad (neutrons/cm^2/sec),$$

   where L = 2.88 cm is the thermal diffusion length in water, and D = 0.16 cm is the diffusion coefficient.

4. For a spherical volume of radius R, filled with a uniform gamma-ray activity of C ($Bq \cdot cm^{-3}$), the dose rate induced by the gamma-ray activity at the surface of the spherical volume is

$$\dot{D} = \frac{1}{2} \cdot C \cdot \Gamma \cdot \frac{4\pi}{\mu} \cdot (1 - \mathrm{e}^{-\mu r}) \cdot \left(34 \frac{J/kg}{Coulomb/kg}\right) \cdot \left(\frac{\mu_{water}/\rho_{water}}{\mu_{air}/\rho_{air}}\right),$$

   where μ = 0.046 cm^{-1} is the linear attenuation coefficient of water for gamma rays of 2.26 MeV, and $\left(\frac{\mu_{water}/\rho_{water}}{\mu_{air}/\rho_{air}}\right)$ =1.1. Γ=$7.2 \times 10^{-5} \left(\frac{C}{kg} \cdot cm^2 \cdot MBq^{-1} \cdot h^{-1}\right)$ is the specific gamma-ray constant for 2.26 MeV gamma-rays.

5. You may estimate the dose rate induced by fast and thermal neutrons using the table below.

TABLE 9.5. Values of Neutron Fluence Rates Which, in a Period of 40 H, Result in a Maximum Dose Equivalent of 1 mSv

Neutron Energy, MeV	Neutron Fluence Rate, $cm^{-2}s^{-1}$	
	Adapted from NCRP Report No. 38 (NCRP. 1971)[a]	Adapted from Cross and Ing. 1985[a]
5×10^{-8}	270	280
10^{-7}	340	—
10^{-6}	280	280
10^{-5}	280	280
10^{-4}	290	290
10^{-3}	340	280
5×10^{-3}	—	310
10^{-2}	350	300
2×10^{-2}	—	250
5×10^{-2}	—	110
10^{-1}	58	40
3×10^{-1}	—	20
3.8×10^{-1}	—	16
4.4×10^{-1}	—	13
5×10^{-1}	14	16
6×10^{-1}	—	15
8×10^{-1}	—	14
9×10^{-1}	—	13
1.0	10	9.7
1.20	—	12
2.00	—	11
2.30	—	12
2.50	10	11
3.00	—	11
3.50	—	8.5
4.50	—	9.9
5.00	8.0	9.7
6.25	—	9.2
7.00	8.5	9.0
10.0	8.5	8.0
14.0	6.0	6.8
14.7	—	6.5
20	5.5	
40	5.0	
60	5.5	
100	7.0	
200	6.5	
300	5.5	
400	5.0	

[a]The fluence rates presented here have been obtained from the cited references by dividing the respective reference values for thermal neutrons by 2.5 and the respective values for all other energies by 2.0. These adjustments have been made to reflect recommendations of the NCRP (1987) to increase the effective quality factors for thermal neutrons and more energetic neutrons by 2.5 and 2.0. respectively.
SOURCE: From NCRP Report No. 112. By permission.

Solution:

Given

$$S = 7.5 \times 10^7 \text{ n/s}$$
$$R = 35 \text{ cm}$$
$$B = 5$$
$$T = 3.7 \text{ cm}$$

1. Fast neutron flux at the surface

$$
\begin{aligned}
n &= \frac{R}{T} = \frac{35}{3.7} = 9.46 \\
\Rightarrow \dot{\phi}_{fast} &= \frac{BS}{4\pi R^2}\left(\frac{1}{2}\right)^n \\
\Rightarrow \dot{\phi}_{fast} &= \frac{(5)(7.5 \times 10^7)}{4\pi(35)^2}\left(\frac{1}{2}\right)^{9.46} \\
\Rightarrow \dot{\phi}_{fast} &\approx 34.6 \text{ n/cm}^2\text{/s}
\end{aligned}
$$

From the table, for around 4 MeV neutrons, the fluence rate giving 1 mSv in 40 h is about:

$$9.2 \text{ n/cm}^2\text{/s}$$

So,

$$
\begin{aligned}
\dot{D}_{fast} &= \frac{34.6}{9.2} \times \frac{1 \text{ mSv}}{40 \text{ h}} \\
\Rightarrow \dot{D}_{fast} &= \frac{34.6}{9.2} \times 25\ \mu\text{Sv/h} \\
\Rightarrow &\boxed{\dot{D}_{fast} \approx 94\ \mu\text{Sv/h}}
\end{aligned}
$$

2. Thermal neutron flux at the surface

Using:

$$\dot{\phi}_{th} = \frac{S}{4\pi RD} e^{-R/L}$$

where:

$$
\begin{aligned}
D &= 0.16 \text{ cm}, L = 2.88 \text{ cm} \\
\dot{\phi}_{th} &= \frac{7.5 \times 10^7}{4\pi(35)(0.16)} e^{-35/2.88} \\
\dot{\phi}_{th} &\approx 5.62 \text{ n/cm}^2\text{/s}
\end{aligned}
$$

For thermal neutrons, the table gives about:

$$280 \text{ n/cm}^2\text{/s}$$

for 1 mSv in 40 h.

$$
\begin{aligned}
\dot{D}_{th} &= \frac{5.62}{280} \times 25 \\
&\boxed{\dot{D}_{th} \approx 0.50\ \mu\text{Sv/h}}
\end{aligned}
$$

So the thermal neutron dose is very small compared with the fast neutron dose.

3. Capture gamma dose from hydrogen

Fast neutrons that do not escape are assumed to be captured by hydrogen, producing 2.26 MeV gamma rays.

Escaping fast-neutron rate:

$$S_{escape} = \dot{\phi}_{fast}(4\pi R^2)$$
$$S_{escape} = 34.6(4\pi)(35)^2$$
$$S_{escape} \approx 5.33 \times 10^5 \text{ n/s}$$

So captured neutron rate:

$$S_{cap} = 7.5 \times 10^7 - 5.33 \times 10^5$$
$$S_{cap} \approx 7.45 \times 10^7 \text{ s}^{-1}$$

Volume of sphere:

$$V = \frac{4}{3}\pi R^3$$
$$V = \frac{4}{3}\pi(35)^3 \approx 1.80 \times 10^5 \text{ cm}^3$$

Uniform gamma activity:

$$C = \frac{S_{cap}}{V}$$
$$\Rightarrow C = \frac{7.45 \times 10^7}{1.80 \times 10^5}$$
$$\Rightarrow C \approx 415 \text{ Bq/cm}^3$$

Using the gamma dose equation:

$$\dot{D}_\gamma = \frac{1}{2}C\Gamma\frac{4\pi}{\mu}(1 - e^{-\mu R})(34)(1.1)$$

with:

$$\Gamma = 7.2 \times 10^{-5}\frac{C}{kg \cdot cm^2 \cdot MBq^{-1} \cdot h^{-1}}$$
$$\mu = 0.046 \text{ cm}^{-1}$$

$$C = 415 \text{ Bq/cm}^3 = 415 \times 10^{-6} \text{ MBq/cm}^3$$

Then:

$$\boxed{\dot{D}_{\gamma} \approx 0.122 \text{ mGy/h}}$$

For gamma rays:

$$1 \text{ mGy} \approx 1 \text{ mSv}$$

So:

$$\boxed{\dot{D}_{\gamma} \approx 122\ \mu\text{Sv/h}}$$

So the final estimated dose rate at the surface of the shielding is:

$$\dot{D}_{total} = \dot{D}_{fast} + \dot{D}_{th} + \dot{D}_{\gamma}$$
$$\dot{D}_{total} = 94 + 0.5 + 122$$
$$\boxed{\dot{D}_{total} \approx 217\ \mu\text{Sv/h}}$$
$$\boxed{\dot{D}_{total} \approx 0.22 \text{ mSv/h}}$$