Q1 (30 points)

(1) Please explain what the **Bragg-Gray principle** is? Please write down the equation and explain what each individual term in the equation stands for.

(2) Please explain what the **KERMA dose** is? What is the difference between the KERMA dose and the standard absorbed dose (energy deposition per unit mass)?

(3) What is the **absorption fraction** under the MIRD methodology?

Solution:

Part (1)

Considering a gas cavity is surrounded by a wall medium, where

- the dimension of the gas volume is small compared to the range of the secondary charged particles,
- the wall thickness > maximum range of secondary charged particles,
- the wall thickness is not great enough to significantly attenuate the incident radiation, and
- the wall and gas materials have similar atomic compositions.

In this case, we consider a form of electronic equilibrium is established between the wall and gas volume.

Then the energy absorbed per unit mass of the wall, $\frac{dE_m}{dM_m}$, is related to the energy absorbed per unit mass of gas, $\frac{dE_g}{dM_g}$, by

$$\frac{dE_m}{dM_m} = \frac{S_m}{S_g} \times \frac{dE_g}{dM_g}$$

where S_m is the main mass stopping power of the wall medium, and S_g is the mass stopping power of the gas to the secondary electrons.

Part (2)

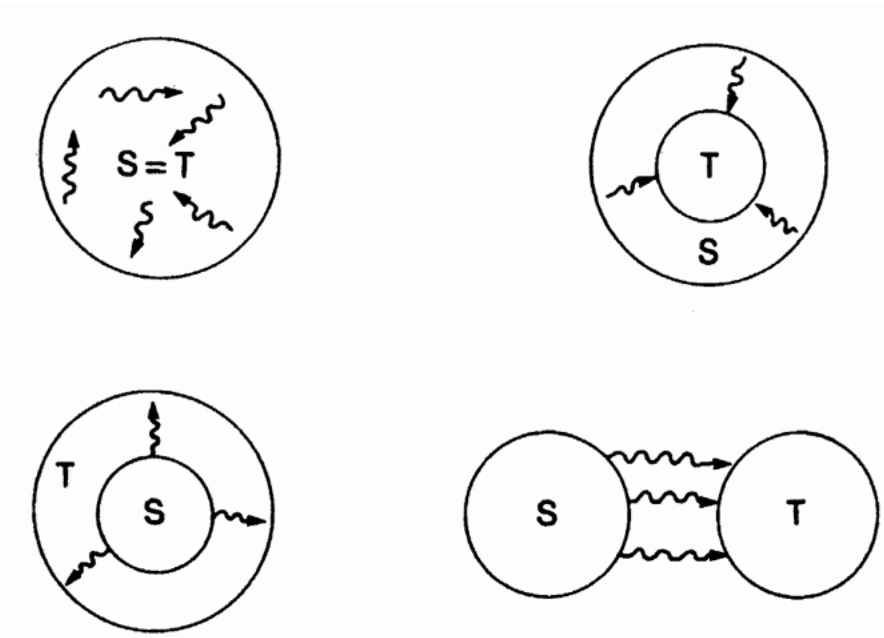
KERMA stands for "Kinetic Energy Released per Unit Mass". When a volume is irradiated by a given form of radiation, the KERMA dose accounts for the initial kinetic energy per unit mass carried by the "primary" ionizing particles (including the photoelectrons, positron-electron pairs, recoil electrons, and the scattered nuclei in case of fast neutrons) produced by the interaction of incident radiation per unit mass of the interacting medium.

By comparison, the regular radiation dose is calculated by the total energy absorbed in a unit mass.

Part (3)

To account for the partial absorption of gamma-ray energy in organs and tissues, the Medical Internal Radiation Dose (MIRD) Committee of the Society of Nuclear Medicine (SNM) has developed a formal system for calculating the dose to a target organ or tissue from a source organ containing a uniformly distributed radioisotope.

The absorption fraction: the fraction of the energy radiated by the source organ and absorbed by the target organ.



The absorbed fractions are calculated by the application of Monte Carlo methods.

$$\text{Absorbed Fraction} = \varphi = \frac{\text{energy absorbed by target}}{\text{energy emitted by source}}$$

Q2 (30 points)

What is the absorbed dose rate of a 70-kg person from a whole-body exposure to a mean thermal neutron flux of 1000 neutrons per cm^2 per second?

Note:

The radiation dose received from thermal neutrons mostly comes from thermal neutron capture by nitrogen ($^{1}_{0}n + {}^{14}_{7}N \rightarrow {}^{14}_{6}C + {}^{1}_{1}p$) and hydrogen ($^{1}_{0}n + {}^{1}_{1}H \rightarrow {}^{2}_{1}H + {}^{0}_{0}\gamma$) in tissue.

For the calculation of the dose rate due to thermal neutron capture by nitrogen, you can consider that

a. the number of nitrogen atoms per kg of tissue is 1.49×10^{24},
b. the absorption cross section for nitrogen is 1.75×10^{-25} cm^2, and
c. the energy released by the thermal neutron capture reaction is 0.63 MeV.

For the calculation of the dose by thermal neutron capture by hydrogen, consider that

a. the energy release through the thermal neutron capture by hydrogen is ~2.2 MeV,
b. the number of hydrogen atoms per kg of tissue is 5.98×10^{25},
c. the absorption cross section for hydrogen is 0.33×10^{-25} cm^2, and
d. the absorption fraction for 2.2 MeV gamma rays uniformly generated throughout the human body and then partially absorbed in the same volume is 0.278.

Solution:

The dose rate due to the (n, p) reaction is calculated from the following equation,

$$\dot{D}_{np} = \frac{\phi N_N \sigma_N Q \times 1.6 \times 10^{-13} \text{ J/MeV}}{1 \text{ J/kg} \cdot \text{Gy}}$$

where ϕ is the thermal neutron flux in neutrons/cm²·s,

N_N is the number of nitrogen atoms per kg of tissue $= 1.49 \times 10^{24}$ atoms/kg,

σ_N is the absorption cross section for nitrogen $= 1.75 \times 10^{-25}$ cm^2, and

Q is the energy released by the reaction $= 0.63$ MeV.

Therefore,

$$\dot{D}_n = 1 \times 10^3 \times 1.49 \times 10^{24} \times 1.75 \times 10^{-25} \times 0.63 \times 1.6 \times 10^{-13}$$
$$\Rightarrow \dot{D}_n = 2.628 \times 10^{-11} \text{ Gy/s}$$

For the dose rate due to the gamma rays from the (n, γ) reaction, the apparent activity of the body can be given by the following equation,

$$A = \phi N_H \sigma_H$$

where ϕ is the thermal neutron flux in neutrons/cm²·s,

N_H is the number of hydrogen atoms per kg of tissue $= 5.98 \times 10^{25}$ atoms/kg, and

σ_H is the absorption cross section for hydrogen $= 0.33 \times 10^{-25}$ cm^2.

Thus,

$$A = 10^3\ \text{cm}^{-2}\text{s}^{-1} \times 5.98 \times 10^{25}\ \text{atoms/kg} \times 0.33 \times 10^{-25}\ \text{cm}^2/\text{atom}$$
$$A = 1973.4\ \text{Bq/kg}$$

Considering that the apparent gamma activity is uniformly distributed throughout the whole body, the dose rate to the whole body is given by

$$\dot{D}_H = A \times E_\gamma \times \varphi$$
$$\Longrightarrow \dot{D}_H = 1973.4\ \text{Bq/kg} \times 2.2\ \text{MeV/decay} \times 1.6 \times 10^{-13}\ \text{J/MeV} \times 0.278$$
$$\Longrightarrow \dot{D}_H = 1.93 \times 10^{-10}\ \text{Gy/s}$$

Adding $\dot{D}_n$ and $\dot{D}_H$, the total dose rate is

$$\dot{D}_{total} = \dot{D}_n + \dot{D}_H$$
$$\dot{D}_{total} = 2.628 \times 10^{-11} + 1.93 \times 10^{-10}$$
$$\dot{D}_{total} = 2.19 \times 10^{-10}\ \text{Gy/s}$$

Q3 (40 points)

A child drinks 1 liter of milk per day containing I-131 at a mean concentration of 33.3 Bq per liter over a period of 30 days. Assuming the child has no additional intake of I-131, calculate the thyroid dose at the end of the 30-day ingestion period and the dose commitment.

Note:

a. The radioactive half-life of I-131 is 8.5 days, and the biological half-life is 138 days.
b. Assuming 50% of administered I-131 is deposited in the thyroid.
c. The mass of the thyroid is 20g.
d. The gamma-ray emissions from I-131 decay, their relative yield, and corresponding absorbed fraction (specific absorption) are given in the table below:

MeV	*f*	Spec. Abs	MeV/t
0.723	0.016	0.00166	1.92E-05
0.637	0.069	0.00166	7.3E-05
0.503	0.003	0.00166	2.5E-06
0.326	0.002	0.00155	1.01E-06
0.177	0.002	0.00155	5.49E-07
0.365	0.853	0.00155	0.000483
0.284	0.051	0.00155	2.25E-05
0.08	0.051	0.0429	0.000175
0.164	0.006	0.00155	1.53E-06
		Sum	0.000778

e. The beta emissions from I-131 decay and their relative yield are given in the table below:

Energy, MeV/t	Yield, *f*
0.0701	0.016
0.0955	0.069
0.1428	0.005
0.1917	0.904
0.2856	0.006

Solution:

Step 1: Derive the effective half-life of I-131

First calculate the effective half-life of the I-131 in the body:

$$T_R = 8.5 \text{ d}$$

$$T_B = 138 \text{ d}$$

$$T_E = \frac{T_R \times T_B}{T_R + T_B} = \frac{8.5 \text{ d} \times 138 \text{ d}}{8.5 \text{ d} + 138 \text{ d}} = 8.0d$$

which is the effective half-life of ^{131}I in body.

Converting to effective elimination constant:

$$\lambda = \frac{0.693}{T} = \frac{0.693}{8.0\ \text{d}} = 0.087d^{-1}$$

Note that we use λ to symbolize the effective decay constant in the following derivations.

Step 2: Derive the absorbed dose to the thyroid per I-131 decay

The average energy of each ^{131}I beta particle is found, and the yield from each decay is also tabulated from the table below:

Energy, MeV/t	Yield, f
0.0701	0.016
0.0955	0.069
0.1428	0.005
0.1917	0.904
0.2856	0.006

The mean beta energy per transformation is:

$$\bar{E}_e(\beta) = \sum \bar{E} \times f_{\beta i}$$

$$\Rightarrow \bar{E}_e(\beta) = (0.0701 \times 0.016) + (0.0955 \times 0.069) + (0.1428 \times 0.005) + (0.1917 \times 0.904) + (0.2856 \times 0.006)$$

$$\Rightarrow \bar{E}_e(\beta) = 0.184\ \text{MeV/t}$$

This represents the beta energy absorbed in the thyroid per I-131 decay.

For this case, the contribution from the γ is not significant and can be ignored, especially since the child's thyroid is small.

Step 3: Derive the I-131 activity in the thyroid as a function of t

The intake is $q = 33.3$ Bq/day, however, only 50% of the iodine is directly deposited in the thyroid.

$$K = 33.3 \frac{Bq}{d} \times \frac{1\ \text{deposited}}{2\ \text{administered}} = 16.65 \frac{Bq}{d}$$

Which is the rate of intake.

Some of the iodine is eliminated daily, so find the concentration in the thyroid at any time:

$$\frac{dq}{dt} = \text{deposition} - \text{disappearance}$$

$$\frac{dq}{dt} = K - \lambda q$$

Separating the variables, we have

$$\int_0^q \frac{dq}{K - \lambda q} = \int_0^t dt$$

After integration, and solving for q as a function of t,

$$q(t) = \frac{K}{\lambda}\left(1 - e^{-\lambda t}\right)$$

As $t \to \infty$, q approaches

$$q_\infty = \frac{K}{\lambda}$$

$$q_\infty = \frac{16.65 \text{ Bq/day}}{0.087 \text{ day}^{-1}} = 191.38 \text{ Bq}$$

**Noting that,

$$\int \frac{1}{ax+b}\, dx = \frac{1}{a}\ln\left(ax+b\right) + C$$

and considering the following conditions:

$$a = -\lambda_{eff}$$
$$b = K$$
$$q(t=0) = 0$$

If the uptake of I-131 continues, the **dose rate as a function of time** is

$$\dot{D}(t) = \frac{\bar{E}}{m}\cdot\frac{K}{\lambda}\left(1 - e^{-\lambda t}\right)$$

$$\Rightarrow \dot{D}(t) = \dot{D}_\infty\left(1 - e^{-\lambda t}\right)$$

If the uptake of I-131 continues, the **saturation dose rate** is given by

$$\dot{D}_\infty = \frac{\bar{E}}{m}\cdot\frac{K}{\lambda}$$

Step 4: Derive the accumulated dose received within the first 30 days.

If the uptake continues, the accumulated dose received by a given time $t$is

$$D = \int_0^t \dot{D}\,(t')\cdot dt'$$

where

$$\dot{D}(t) = \frac{\bar{E}}{m}\cdot\frac{K}{\lambda}\cdot\left(1 - e^{-\lambda t}\right)$$

and $\bar{E}$ is the mean absorbed energy in the organ per decay of I-131 in the thyroid.
So, the accumulated dose is given by

$$D = \int_0^t \dot{D}\,(t')\cdot dt'$$

$$\Rightarrow D = \int_0^t \frac{\bar{E}}{m} \cdot \frac{K}{\lambda} \cdot \left(1 - e^{-\lambda t'}\right) \cdot dt'$$

$$\Rightarrow D = \frac{\bar{E}}{m} \cdot \frac{K}{\lambda} \cdot \left[t + \frac{1}{\lambda}\left(e^{-\lambda t} - 1\right)\right]$$

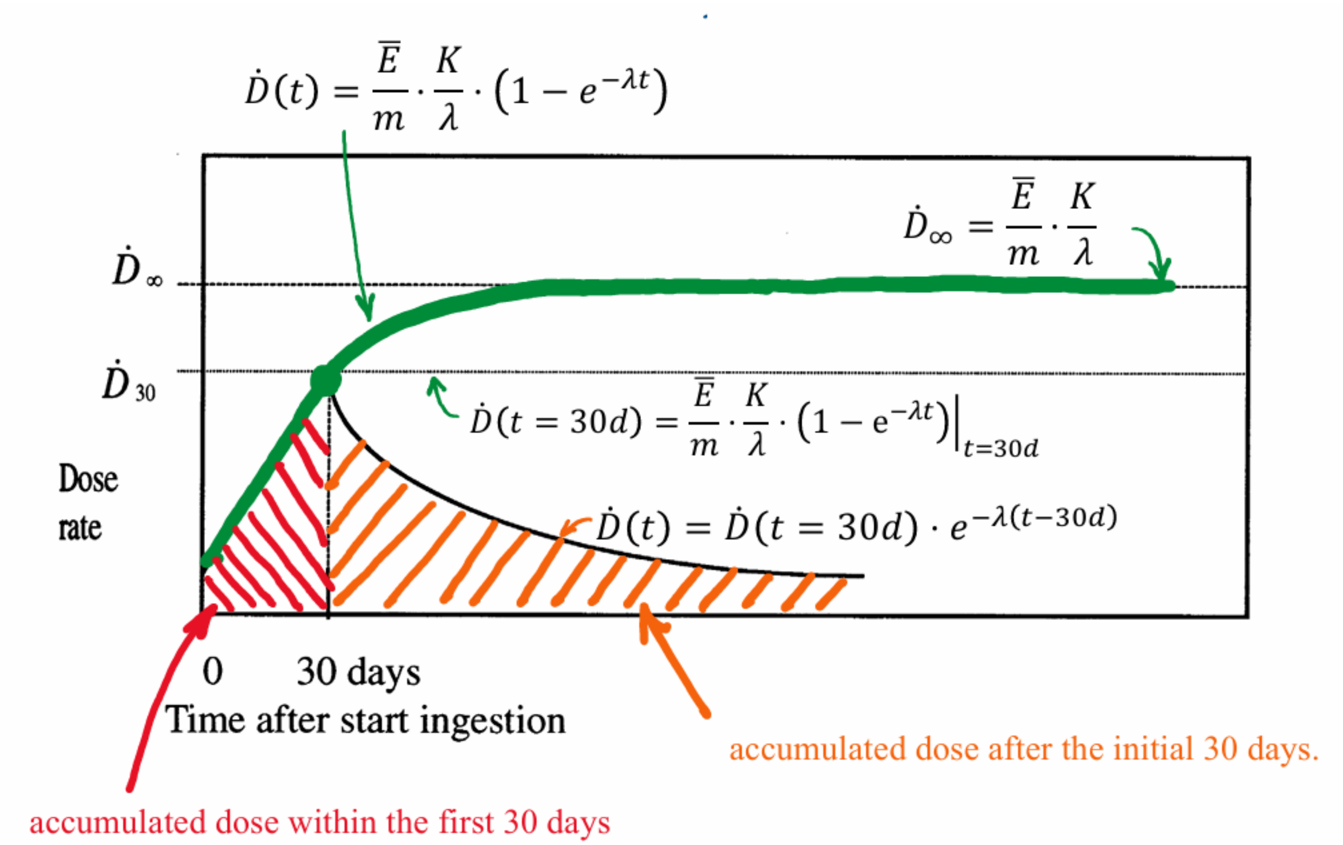
**For integration,

$$\int_0^t \left(1 - e^{-\lambda t}\right) dt$$

$$= \left[t - \left(-\frac{1}{\lambda}\right) e^{-\lambda t}\right]_0^t$$

$$= \left[t + \frac{1}{\lambda} e^{-\lambda t}\right] - \left[0 + \frac{1}{\lambda}\right]$$

$$= t + \frac{1}{\lambda}\left(e^{-\lambda t} - 1\right)$$



Using the following equation for accumulated dose till time t,

$$D = \dot{D}_\infty \left[t + \frac{1}{\lambda}\left(e^{-\lambda t} - 1\right)\right]$$

and

$$\dot{D}_\infty = \frac{\bar{E}}{m} \cdot \frac{K}{\lambda}$$

$$\Rightarrow \dot{D}_\infty = \frac{\bar{E}}{m} \cdot \frac{K}{\lambda} = \frac{0.184\ \text{MeV/t}}{20\ \text{g}} \cdot \frac{16.65\ \text{Bq/day}}{0.087 d^{-1}} = 0.0243\ \text{mGy/day}$$

and

the accumulated dose at 30 days is:

$$\dot{D}_\infty = 0.0243\ \text{mGy/day}$$
$$\lambda = 0.087 d^{-1}$$
$$t = 30\ \text{d}$$

Therefore,

$$D = 0.0243 \frac{mGy}{d} \times \left[30\ \text{d} + \frac{1}{0.087 d^{-1}} \times \left(e^{-0.0871\times(30)} - 1\right)\right] = \mathbf{0.47\ mGy}$$

$\mathbf{0.47}$ mGy is the accumulated dose at the end of the 30-day period.

Recalling that,

The total dose received during a time interval t after the deposition of the isotope is

$$D = \dot{D}_0 \int_0^t e^{-\lambda t}\, dt$$
$$\Rightarrow D = \frac{\dot{D}_0}{\lambda_E}\left(1 - e^{-\lambda t}\right)$$

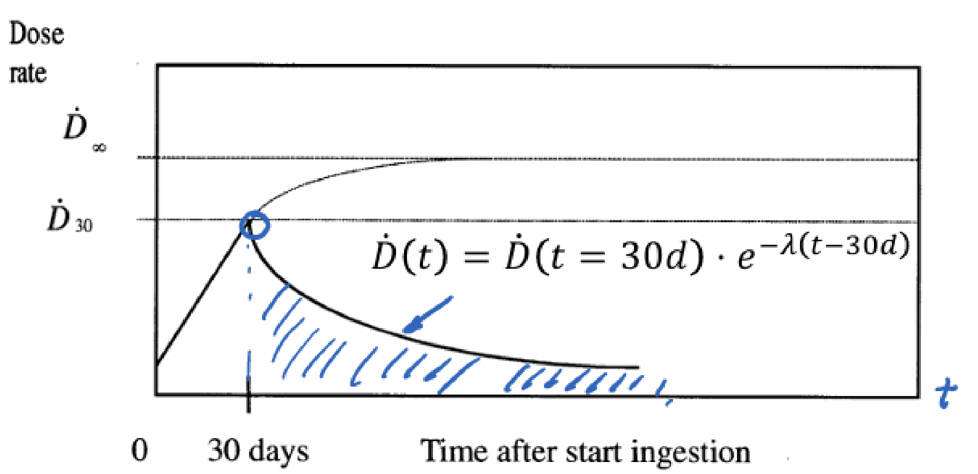
For an infinitely long time, that is, when the isotope is completely gone,

$$D = \frac{\dot{D}_0}{\lambda}$$

For practical purposes, an infinitely long time corresponding to about 6 half-lives. The total dose received from complete decay is called the dose commitment.

Step 5: Derive the total dose (dose commitment) received after $t = 30$ days.

The dose commitment is the sum of the dose accumulated during intake and then during elimination (washout).



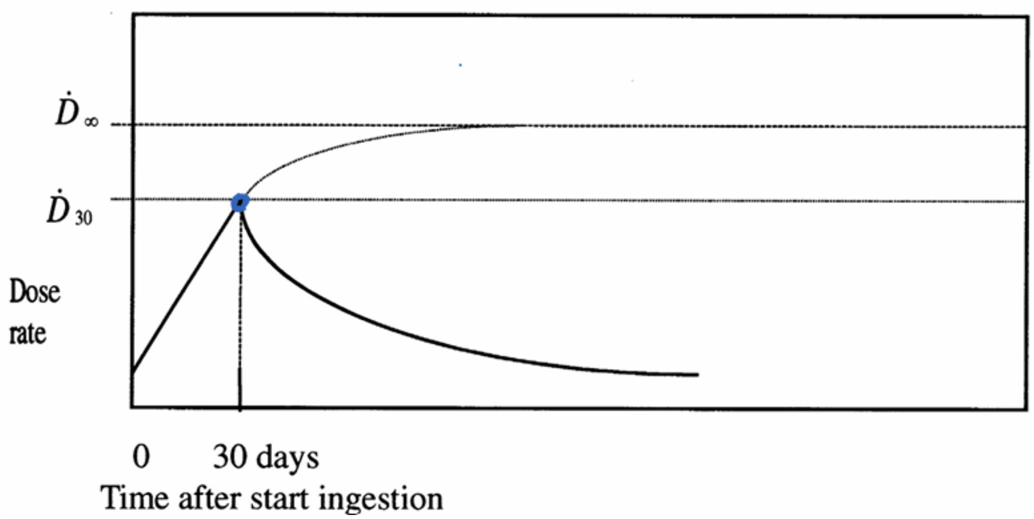
Find the dose rate at $t = 30$ days

$$\dot{D}_{30} = \dot{D}_{\infty}\left(1 - e^{-\lambda t}\right) = 0.0243\frac{mGy}{d} \times \left(1 - e^{-0.087 \times (30)}\right) = 0.0225\frac{mGy}{d}$$

So,

$$D = \frac{\dot{D}_{30}}{\lambda} = \frac{0.0225\frac{mGy}{d}}{0.087\frac{1}{d}} = \mathbf{0.259}$$ **mGy is the dose after ingestion stops.**

Step 6: Derive the total dose (dose commitment) from the initial intake of I-131



The total dose, from the time intake started to the end of the first 30 days, plus the dose after the intake stopped, will be:

0.47 mGy+0.259 mGy= 0.729 mGy total dose to the child's thyroid.